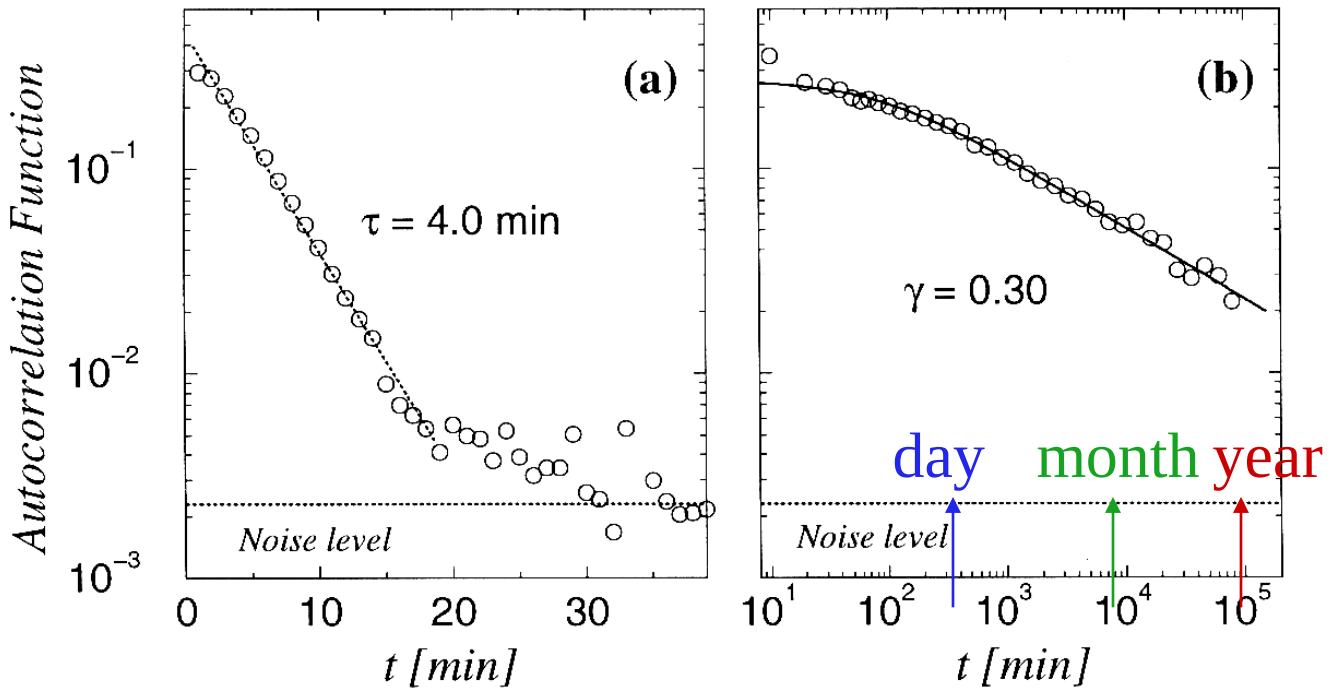
APPLIED FINANCE

CORRELATIONS IN S&P500

< δp(0) δp(t) > < |δp(0)| |δp(t)| >



from Y. Liu et al, 1999

THE LIST OF EMPIRICAL FACTS ABOUT PRICE FLUCTUATIONS

- Probability distribution $P(\delta_{\Delta t}\, p)$ is strongly non-gaussian especially when measured for $\Delta t <$ several days
- Crossover from Pareto-Levy distribution with $1+\mu \sim 2.4$ to power law tail with $1+\mu \sim 4$.
- Correlations in signs of price changes $< \delta p(0)\, \delta p(t) >$ quickly decay to zero
- Correlations in magnitudes of price changes persist from minutes to years (!) as manifested by by higher order correlators $< |\delta p(0)|\, |\delta p(t)| >$ or $< \delta p(0)^2\, \delta p(t)^2 >$

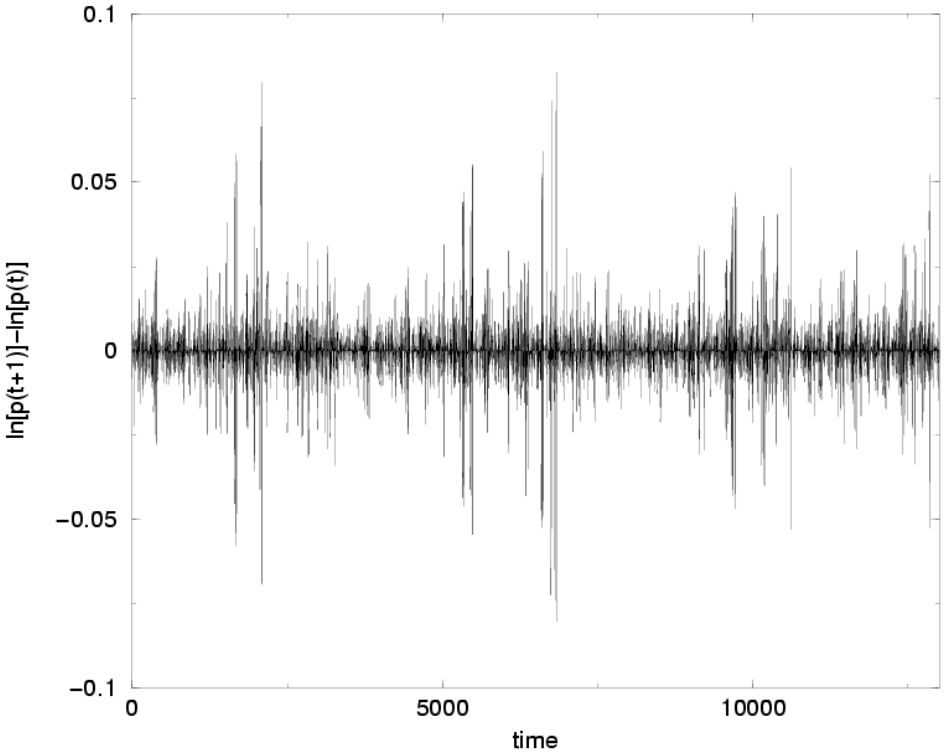
WHAT MODELS ARE USED TO EXPLAIN THESE EMPIRICAL LAWS?

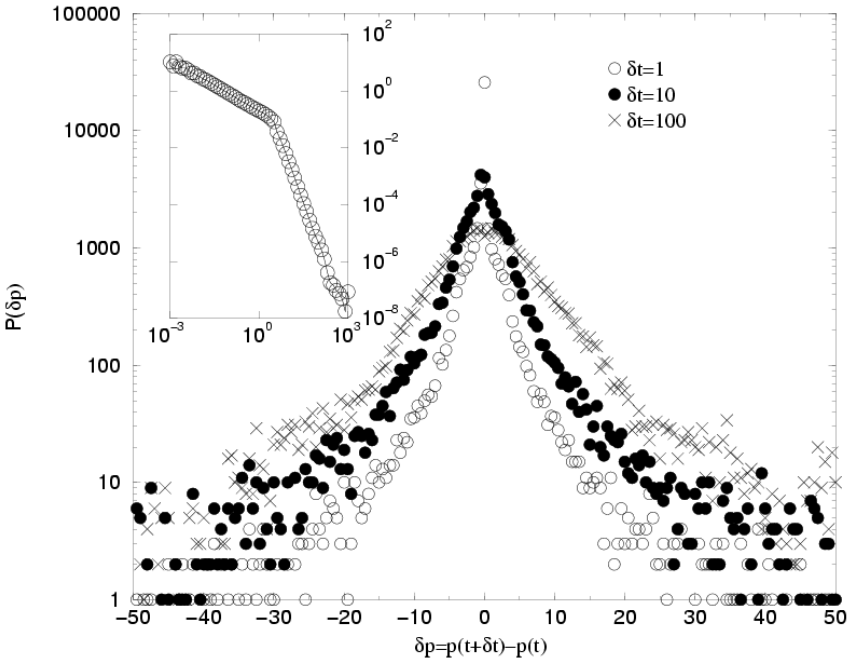
- Econometrists use models with ugly acronyms ARCH(n), GARCH(n,m) (AutoRegressive Conditional Heteroscedasticity): Variance of a random walk has some history dependence on its previous values
  - — Don't explain anything
  - — Not very successful in reproducing real data
- Physicists crave for simple models keeping only the essential ingredients:
  1. Models where players use and evolve strategies
  2. Models based just on the trading rules of the market (how supply/demand determines the price)

LIMIT ORDER MODEL

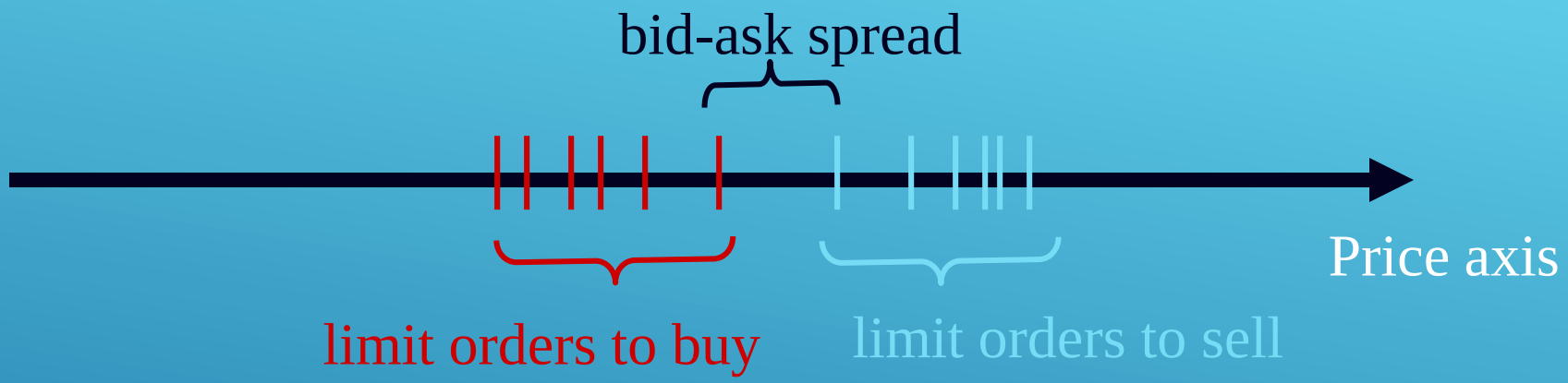
- No strategies (obvious oversimplification)
- No market makers (limit & market orders placed by buyers sellers themselves through a computerized trading system
- A buyer/seller has two options:
  - to place a limit order, i.e. the instruction to sell/buy a unit of stock if the price raises above/falls below a predetermined level
  - place a market order i.e. an instruction to buy/sell from the lowest bid/highest ask limit order
- Limit orders are placed with a random offset Δ from the latest transaction price

PRICE SIGNAL IN A LIMIT ORDER MODEL





LIMIT ORDER BOOK



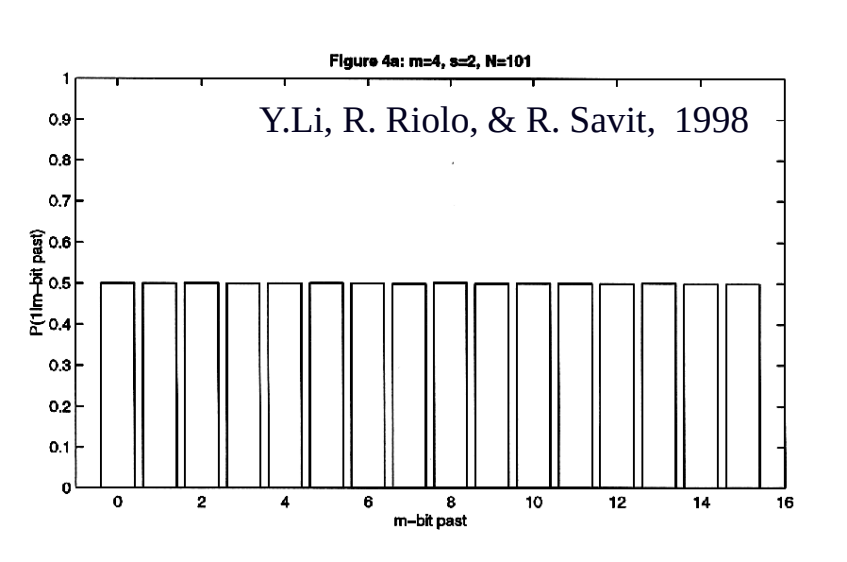
- Is there asymmetry between sell and buy sides of the book?
- How density of limit orders depends on the distance from the current price?
- How bid-ask spread scales with volume?

MINORITY GAME (MG)

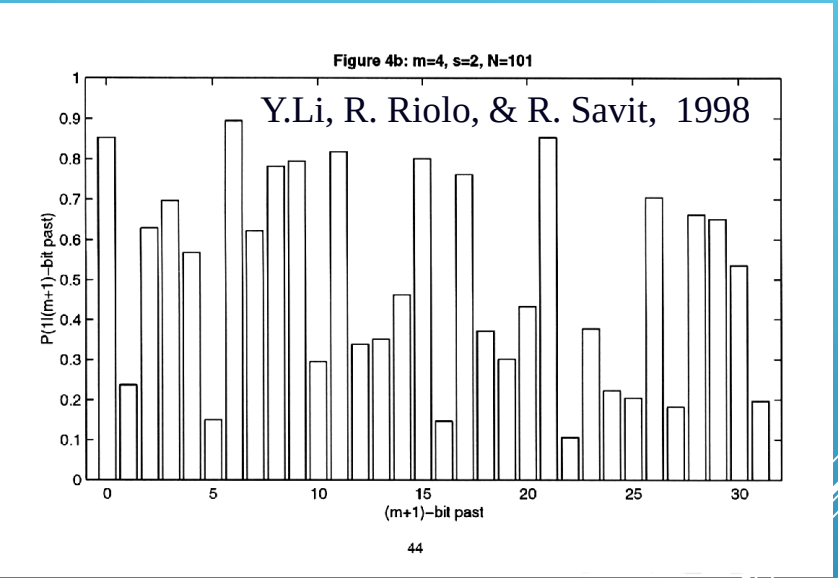
- Introduced by Brian Arthur as "El Farol" bar problem (1994)
- Reformulated for stock prices by D. Chalet and Y.-C. Zhang (1997)
  - at each time step N (odd) players (traders) choose one of two sides B (buy stock) or S (sell stock)
  - minority side wins and "stock price" reflects it as (-) if Buyers won or (+) if Sellers won
  - each player has a pool of S strategies based on outcomes (+ or -) of the last M steps of the game
  - a virtual score is kept for each strategy
  - each player always uses the best strategy from his pool

TESTING THE EFFICIENT MARKET HYPOTHESIS IN MG

All traders have 4 time-step memory:
i.e. all their strategies try to guess the next outcome based on 2^4=16 possible configurations of the last 4 outcomes



Prediction of the future based on 4 time-steps past



Prediction of the future based on 5 time-steps past

"I believe that microscopic market simulations have an important role to play in economics and finance. If it takes people from outside economics and finance - perhaps physicists -to demonstrate this role, it won't be for the first time that outsiders have made substantial contributions to these fields."

From the letter by Nobel Laureate in Economics Harry M. Markowitz to "econophysicist" Dietrich Stauffer