Quantitative Finance

Lecture 3

Time series models

A **time series model** specifies the joint distribution of the sequence $\{X_t\}$ of random variables.
For example:

$$P[X_1 \leq x_1, \ldots, X_t \leq x_t] \text{ for all } t \text{ and } x_1, \ldots, x_t.$$

Notation:
$X_1, X_2, \ldots$ is a stochastic process.
$x_1, x_2, \ldots$ is a single realization.

We'll mostly restrict our attention to **second-order properties** only:
$\mathrm{E}X_t, \mathrm{E}(X_{t_1}, X_{t_2})$.

Time series models

Example: White noise: $X_t \sim WN(0, \sigma^2)$.
i.e., $\{X_t\}$ uncorrelated, $\mathrm{E}X_t = 0$, $\mathrm{Var}X_t = \sigma^2$.

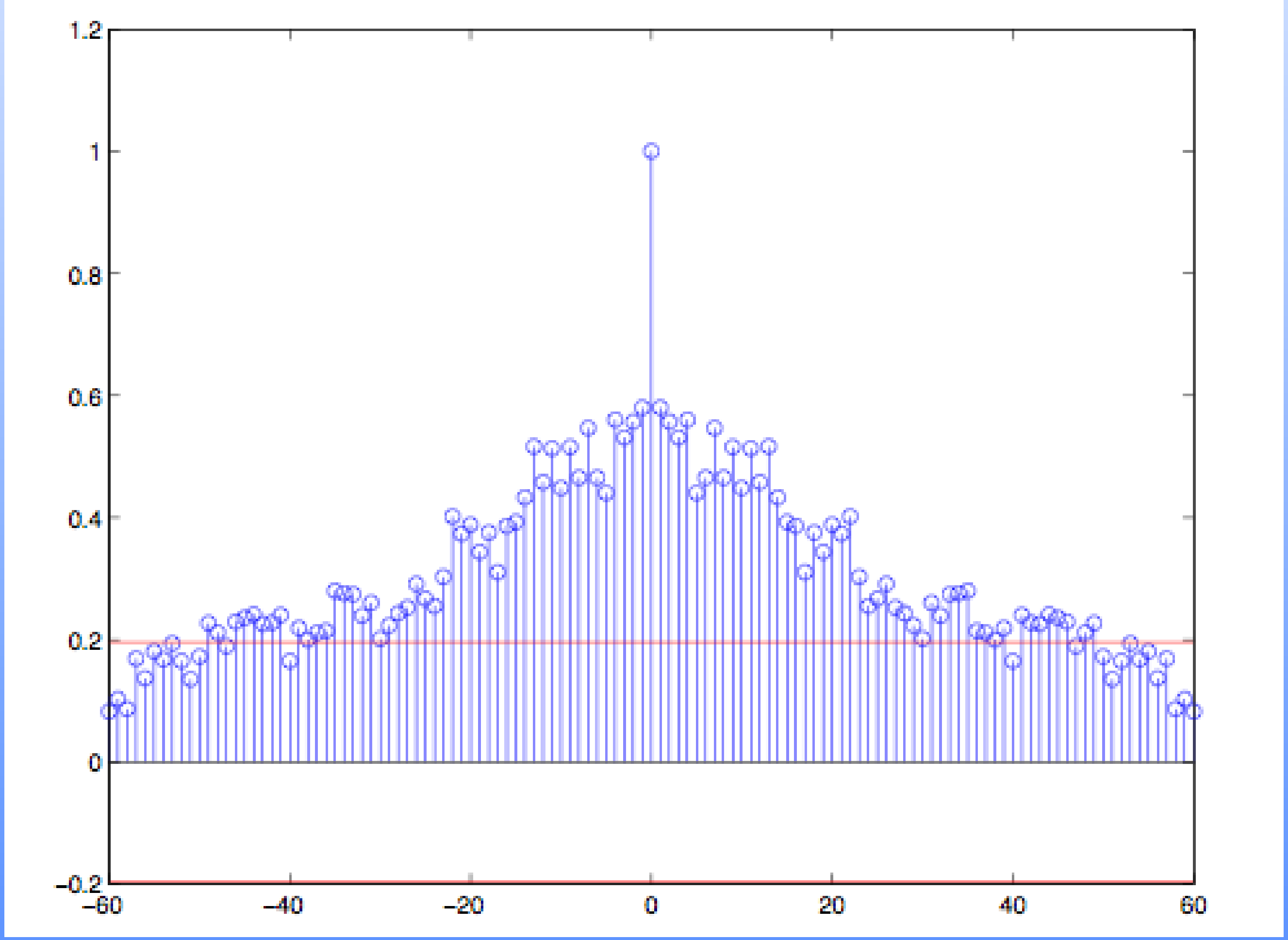
Example: i.i.d. noise: $\{X_t\}$ independent and identically distributed.

$$P[X_1 \leq x_1, \ldots, X_t \leq x_t] = P[X_1 \leq x_1] \cdots P[X_t \leq x_t].$$

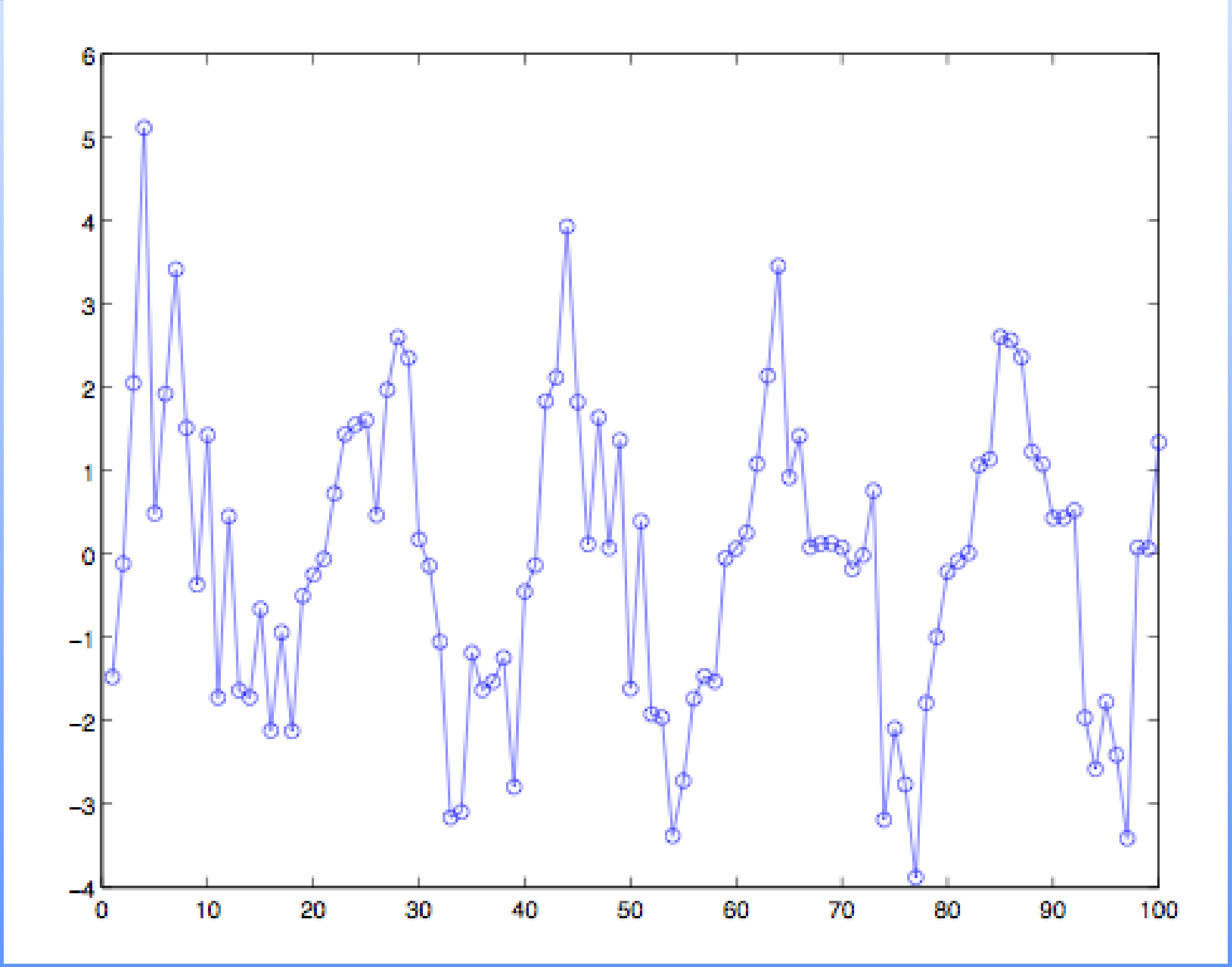
Not interesting for forecasting:

$$P[X_t \leq x_t | X_1, \ldots, X_{t-1}] = P[X_t \leq x_t].$$

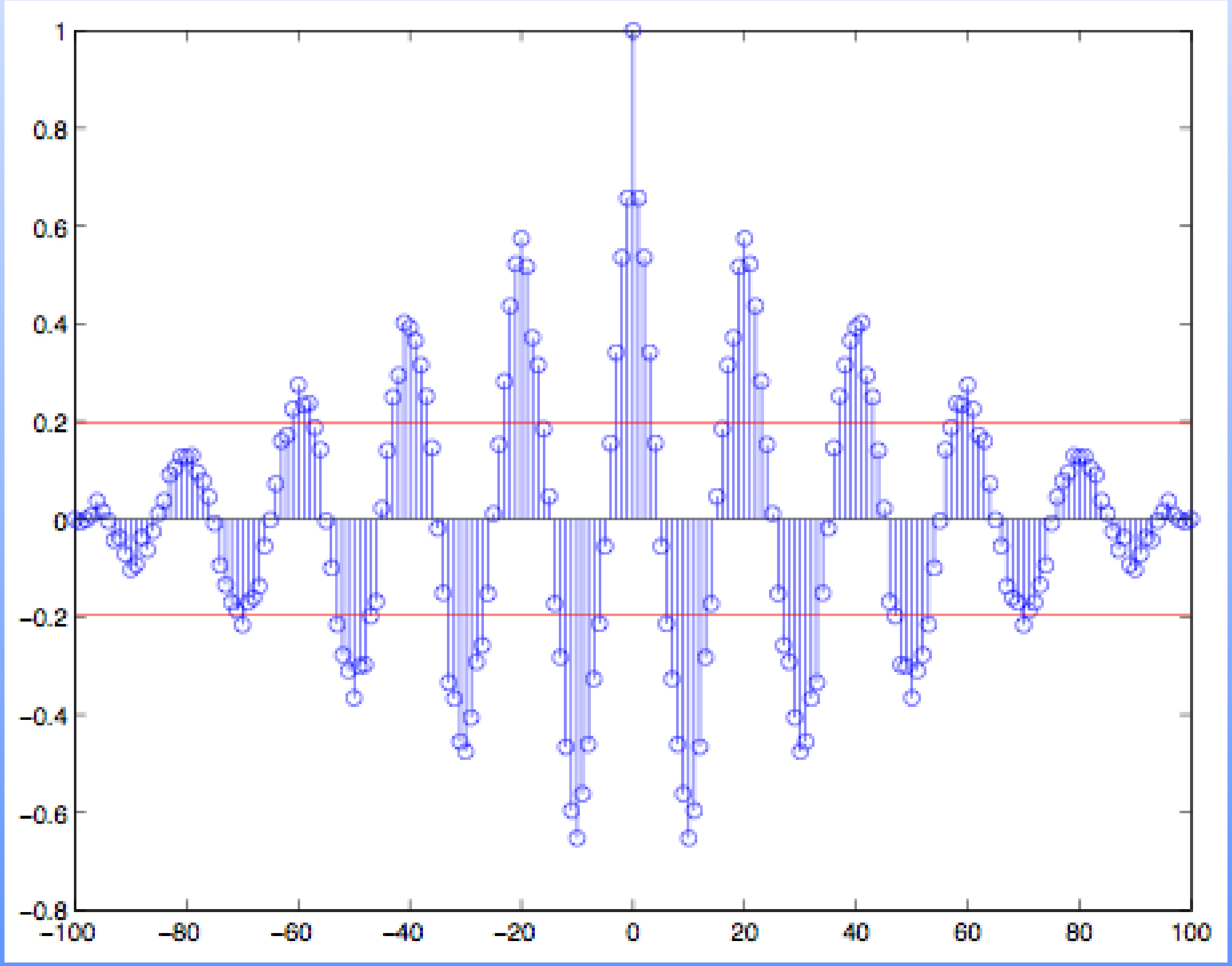
Sample ACF: Trend



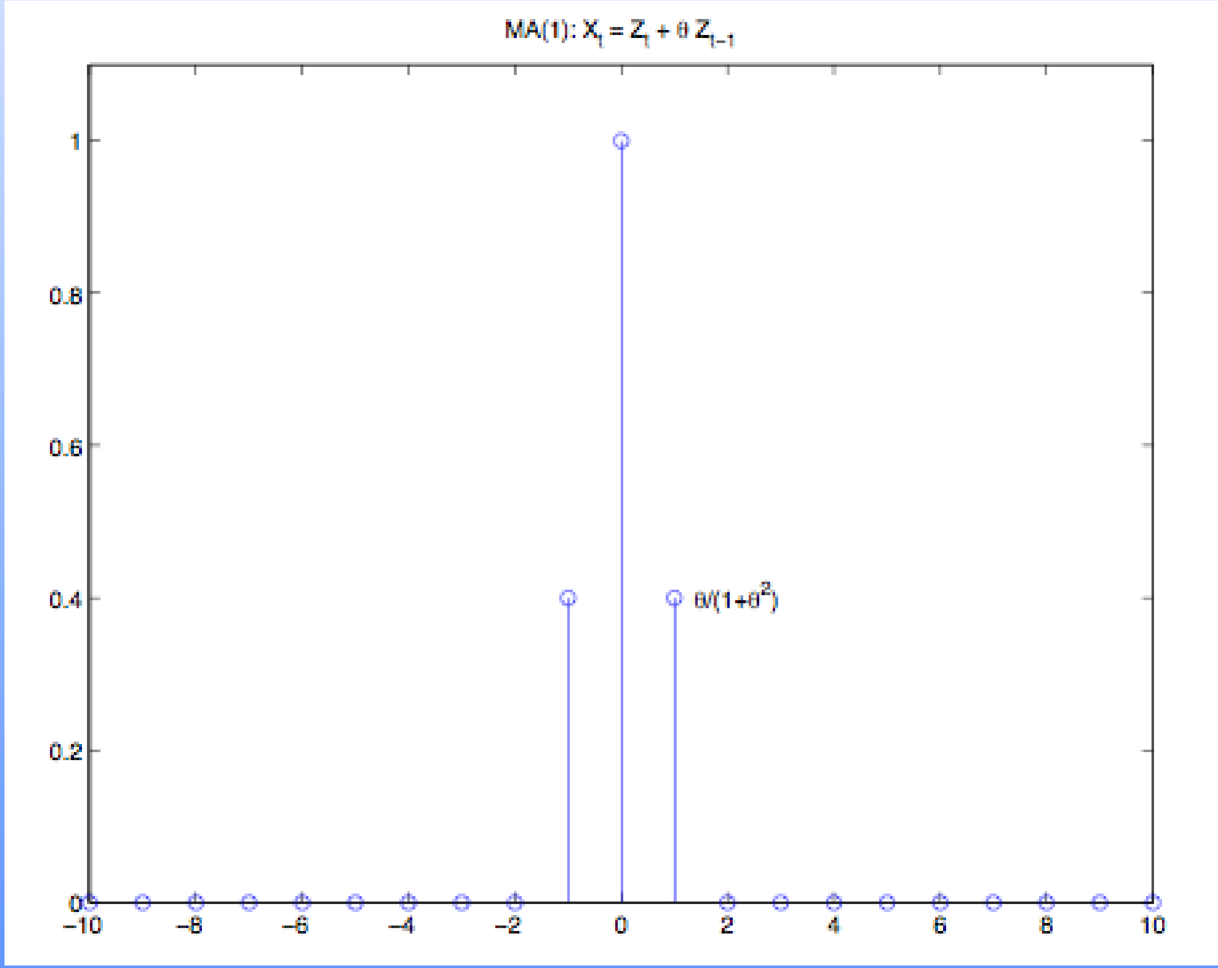
Periodic



Sample ACF: Periodic

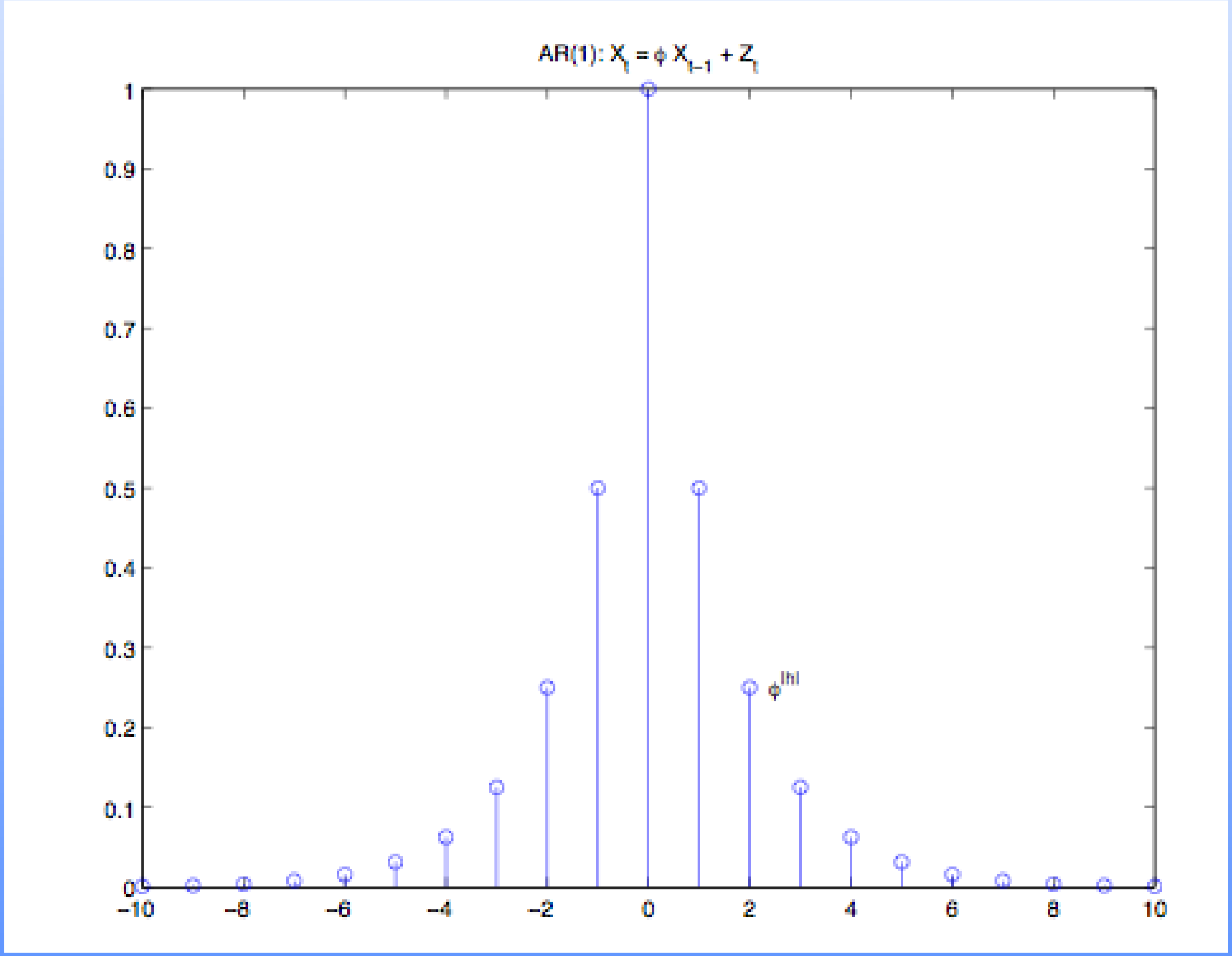


ACF: MA(1)



MA(1): $X_t = Z_t + \theta Z_{t-1}$
1
0.8
0.6
0.4
0.2
0
−10
−8
−6
−4
−2
0
2
4
6
8
10
$\theta/(1+\theta^2)$

ACF: AR


AR(1): $X_t = \phi X_{t-1} + Z_t$
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
$\phi^{|h|}$
−10
−8
−6
−4
−2
0
2
4
6
8
10

ARMA

An **ARMA(p,q) process** $\{X_t\}$ is a stationary process that satisfies

$$X_t - \phi_1 X_{t-1} - \cdots - \phi_p X_{t-p} = W_t + \theta_1 W_{t-1} + \cdots + \theta_q W_{t-q},$$

where $\{W_t\} \sim WN(0, \sigma^2)$.

Also, $\phi_p, \theta_q \neq 0$ and $\phi(z), \theta(z)$ have no common factors.

Simulation examples

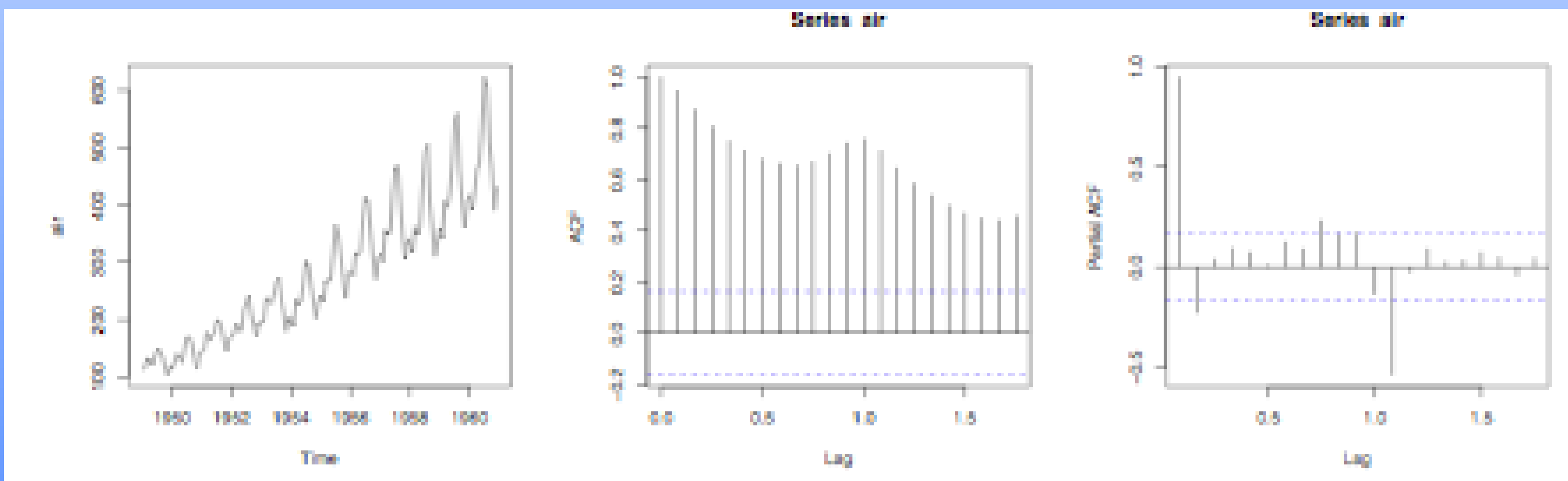
```
# some AR(1)
x1 = arima.sim(list(order=c(1,0,0), ar=.9), n=100)
x2 = arima.sim(list(order=c(1,0,0), ar=-.9), n=100)
par(mfrow=c(2,1))
plot(x1, main=(expression(AR(1)~~~phi==+.9)))     # ~ is a space and == is equal
plot(x2, main=(expression(AR(1)~~~phi==-.9)))
par(mfcol=c(2,2))
acf(x1, 20)
acf(x2, 20)
pacf(x1, 20)
pacf(x2, 20)
# an MA1
x = arima.sim(list(order=c(0,0,1), ma=.8), n=100)
par(mfcol=c(3,1))
plot(x, main=(expression(MA(1)~~~theta==.8)))
acf(x,20)
pacf(x,20)
# an AR2
x = arima.sim(list(order=c(2,0,0), ar=c(1,-.9)), n=100)
par(mfcol=c(3,1))
plot(x, main=(expression(AR(2)~~~phi[1]==1~~~phi[2]==-.9)))
acf(x, 20)
pacf(x, 20)
```

Simulation example

```
x = arima.sim(list(order=c(1,0,1), ar=.9, ma=-.5), n=100) # simulate some data
(x.fit = arima(x, order = c(1, 0, 1)))   # fit the model and print the results
tsdiag(x.fit, gof.lag=20)# diagnostics
x.fore = predict(x.fit, n.ahead=10)
# plot the forecasts
U = x.fore$pred + 2*x.fore$se
L = x.fore$pred - 2*x.fore$se
minx=min(x,L)
maxx=max(x,U)
ts.plot(x,x.fore$pred,col=1:2, ylim=c(minx,maxx))
lines(U, col="blue", lty="dashed")
lines(L, col="blue", lty="dashed")
```

Examples

```
library(tseries)
air <- AirPassengers
ts.plot(air)
acf(air)
pacf(air)
```



Examples

- For classical decomposition:

```
plot(decompose(air))
```

Fitting an ARIMA model:

```
air.fit<-
arima(air,order=c(0,1,1),seasonal=list(order=c(0,1,1),period=12))
tsdiag(air.fit)
```

Forecasting:

```
library(forecast) air.forecast <- forecast(air.fit)
plot.forecast(air.forecast)
```