

# Quantitative finance

---

## Lecture 10

# Measuring the Outcomes of Uncertainty and Risk

- Risk is a consequence of uncertainty. Although they are connected, they are distinct concepts.
- Emotions and feelings impact risk-related decisions. However, the focus of this chapter is financial metrics.
- A **metric** is a system of related measures that help quantify characteristics or qualities.

# Measuring the Outcomes of Uncertainty and Risk (continued)

- An individual or enterprise must be able to quantify a given risk to determine whether it is critical enough to warrant management through assets.
- Once resources are committed to manage a risk, one must be able to measure the progress (or lack thereof) towards obtaining the risk management objectives.

# Measuring the Outcomes of Uncertainty and Risk (continued)

- **Risk management techniques** provide tools for assessing profit opportunities and threats of loss.
- Without risk management techniques, one cannot tell which of several alternatives will optimize the risk-reward tradeoff.
- The **risk-reward tradeoff** is a cost-benefit analysis that takes uncertainty into account, helping us exploit profitable opportunities and avoid bankruptcy.

# Successful Applicability of Risk Metrics

- **Risk metrics** are critical to effective risk management programs. However, risk metrics are not effective in isolation.
- Risk metrics must vary depending on the situation, taking into account the risk appetite of the risk-bearer
- Optimal gain possibilities must be weighed against worst-case or ruin scenarios
- “One size ***does not*** fit all” in risk management

# Successful Application of Risk Metrics (continued)

- An individual or enterprise can become **insolvent** by underestimating:
  - the number (frequency) of losses in a given time frame, and/or
  - the magnitude (severity) of losses in that time frame.
- The **subprime mortgage crisis** involved the failure to properly estimate the frequency and severity of the losses actually experienced in the real world.

# Probability Theory

- Quantifying uncertainty is accomplished by using **probability theory**.
- Outcomes in the world are fairly regular, and remain stable over time.
- Each exposure unit has unique characteristics with respect to possible losses. One must consider these characteristics when estimating losses and planning to manage a given risk.

# Probability Theory (continued)

- Probability measures the relative frequency of particular outcomes over the course of repeated trials.
- Some outcomes have an equal chance of occurring, such as a coin landing on heads vs. tails.
- Most outcomes in the world have an unequal chance of occurring, such a hurricane vs. a moderate rain storm hitting a given city. The range of outcomes does, however, conform to certain patterns.

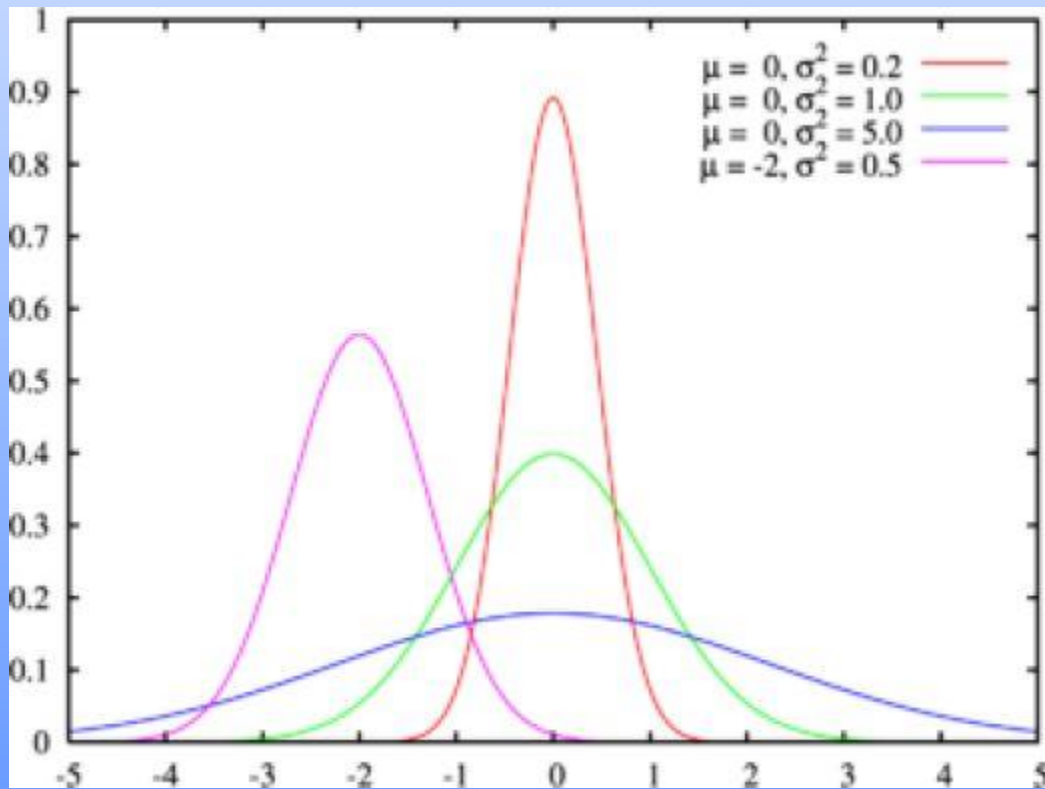
# Statistical Techniques Used in Risk Management

- Range = (the highest value) – (the lowest value) in a distribution of outcomes.
- In a business setting, the range of values includes both the possibility of gain and the possibility of loss.
- The **expected value** is the long-term average outcome, estimated using historical data. Expected value is also called the **average value** or the **mean value**.

# Statistical Techniques Used In Risk Management (continued)

- While the **expected value** is mathematically correct, typically no one outcome will be equal to the expected value.
- In a normal distribution, as shown in a bell-shaped curve, half of the outcomes will exceed the expected value and half will be lower than the expected value.
- Two or more data sets with a normal distribution can have identical expected values but differ in the range of outcomes.

# Bell Curves



# Variance and Standard Deviation

- **Variance**, or **dispersion**, describes the degree to which the distribution of results is spread out rather than concentrated around the expected value.
- The **standard deviation** is the square root of the variance, and is used to compare the degree of dispersion in two or more data sets.

# Steps in Calculating Standard Deviation

1. Calculate the distribution's (the data set's) expected value.
2. One by one, subtract the expected value from each outcome value in the data set, ending with a set of differences.
3. Square each of the differences calculated in Step 2.
4. Add together the squares calculated in Step 3.
5. Divide the sum in Step 4 by one less than the number of outcomes. This is the variance.
6. Take the square root of the variance from Step 5 to calculate one standard deviation.

# Key Statistical Risk Management Concepts

- The **expected value** of loss is equal to the average severity of loss occurrences that happen over time, as the consequence of uncertainty.
- Expected value is calculated by multiplying the probability of each loss by the size of each loss, then adding these calculations together to obtain a weighted average.
- **Frequency** is the expected number of loss occurrences.
- A **probability distribution** represents the relative frequency of losses.

# What if the averages of different distributions vary?

- In most cases, two or more distributions (or data sets) will not have an identical average, or mean. If the means are significantly different, one applies the **coefficient of variance** tool.
- To find the coefficient of variance, divide the standard deviation of each distribution by its unique mean. The higher the resulting value, the greater the degree of uncertainty and, accordingly, risk.

# Extreme Risk Concepts

- The **value at risk** concept (VaR) measures the worst-case dollar-value loss for an enterprise due to a specific set of risks, up to a specified probability level. Due to these probability limits, VaR may miss the most extreme levels of risk.
- The **maximum probable annual loss** (MPAL) works with a one-year probability loss distribution and chooses the selected lower percentile value as the MPAL.
- **Range** is another tool that works with extreme loss possibilities.

# Less Extreme Loss Expectations

- Variance and standard deviation are useful in dealing with less extreme loss expectations than are analyzed through VaR and MPAL.
- Scarce resources impact risk management programs. An enterprise cannot continually expend resources based on only the worst possible dollar-loss scenarios.

# Measuring Non-Diversifiable Portfolio Risk

- **Systemic risks, or non-diversifiable risks,** are those that affect all enterprises or portfolios to varying degrees.
- The **capital asset pricing model (CAPM)** assumes that investors expect to be compensated for the time-value of money and the systemic risk they bear.
- CAPM assumes that investors should not be compensated for idiosyncratic risk, since a well-balanced portfolio greatly reduces the impact of idiosyncratic risk.