

GRAPH COLORING

Interference Reduction in Wireless Networks Using Graph Coloring Methods

The problem of reducing interference in wireless networks is becoming increasingly important with the continuous deployment of larger and more sophisticated wireless networks. In this report we analyze the interference problem from a graph theoretical viewpoint. The interference reduction problem is modelled as a graph coloring problem. However, additional constraints to graph coloring scenarios that account for various networking conditions result in additional complexity to standard graph coloring. This lecture discusses a variety of algorithmic solutions for specific network topologies and makes recommendations for implementation.

1. Introduction

Signal interference is a major drawback of wireless networks. The aim of reducing interference is to prevent adjacent or connected nodes, which are linked by radio signals, from receiving and transmitting signals which conflict or blend together. Thus, interference occurs when conflicting transmissions over one radio frequency are received by one or more nodes in a wireless network. This inhibits the ability of the receiver to decipher incoming signals. This concept is illustrated in Figure 1a, which shows a typical situation in which the broadcast areas of nodes A and C overlap in the vicinity (shaded in gray) of node B, causing B to receive a garbled signal composed of the signals from A and C. In such situations, it is difficult for B not only to decipher simultaneous signals, but also to reliably determine the source of the signal. Applying the graph coloring problem to reducing interference in wireless networks is practical compared to investing large amounts of money in radio transmitter technology or simply hard-wiring a temporary fix. The problem of reducing interference in arbitrary networks turns out to be very difficult, and for this reason, simpler network layouts have been investigated such as, multi-hop wireless mesh network layouts [1], triangular lattice topologies [9], unit disk graphs [3], hexagonal topologies [8], and other more general topologies [4]. Other key facets of the interference problem in wireless networks specify whether a proposed solution is contrived in a distributed or centralized setting, whether nodes in a given solution are self-aware of their location or whether this assumption is not necessary, and

whether or not minimum separation distance between nodes needs to be factored into algorithmic solutions.

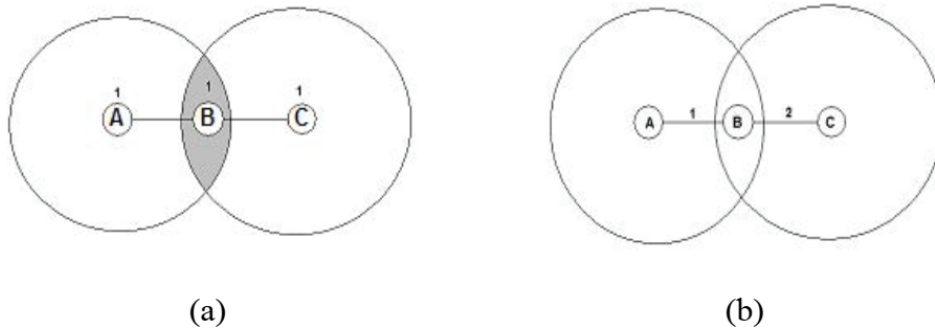


Figure 1. Interference resolved through channel assignment. (a) Interference occurs at B. (b) No interference.

Numerous methods for reducing interference exist, such as topology control [10], power control [7], and channel assignment [1, 2, 3, 5, 6, 8, 9]. This paper will focus exclusively on the latter method, which seeks to assign channels of different frequencies to interfering nodes or edges.

In Figure 1a, for instance, simultaneous transmissions of A and C result in interference at B. This problem is resolved in Figure 1b by having nodes A and C transmit over frequencies 1 and 2 respectively, equipping node B with two radios that can transmit and receive over frequencies 1 and 2. Signals from A and C can be demultiplexed, (that is, components of different frequencies can be extracted from one signal) at node B because of differing frequencies, and node B can clearly determine if node A transmits across frequency 1 or if node C transmits across frequency 2. Therefore the intersection node A and node C's broadcast areas no longer results in interference.

Through careful assignment of communication channels to nodes in a network, interference could be greatly reduced. It is important to note, however, that the number of radio frequencies is finite, and therefore, the problem of minimizing the number of channels allocated to a specific network is worthy of thorough investigation as well. In some instances, channel overlap is necessary if the number of assigned channels for a network is inadequate to connect all nodes [2].

The rest of the paper is organized as follows. In Section 2, we define the interference problem as a graph coloring problem and discuss two coloring approaches, vertex and edge coloring. Open problems and future work are discussed in Section 3. Finally a summary of existing results and conclusions are presented in Section 4.

2. Vertex vs. Edge Coloring

Interference in wireless networks is commonly represented as an interference graph in which edges represent potential interference between the endpoint nodes. The problem

of reducing collisions and signal interference is modeled as a coloring problem on the interference graph. Nodes of different colors in the graph will be assigned separate channel of radio frequency. Efficient coloring algorithms will lead to an effective channel selection method that lowers the wireless interference.

The graph coloring problem's application is relevant because preventing vertices from connecting via radio frequency with other (conflicting) vertices is the quintessential task in reducing interference. Graph coloring algorithms are also reliable in that they are mathematically provable. Channel assignment, however, adds algorithmic complexity to standalone graph coloring problems, according to Khanna and Kumaran [8]. Channel assignment is indeed what ties the graph coloring problem together with the problem of reducing interference in wireless networks.

General graph coloring problems exist in two distinct varieties, edge and vertex coloring. The main condition for edge coloring problems is that no vertex in a graph can have more than one outgoing edge of a particular color. Edge coloring problems are separated into two classes, one corresponding to bipartite graphs and one corresponding to all other graph scenarios. Vertex graph coloring problems are not separated into classes, but merely require one condition that no vertex in a graph can be colored the same color as an adjacent vertex. Figure 2 below shows an example of vertex coloring using the minimum number of colors possible. Typically, all algorithms implementing the graph coloring problem strive to use the fewest colors possible.

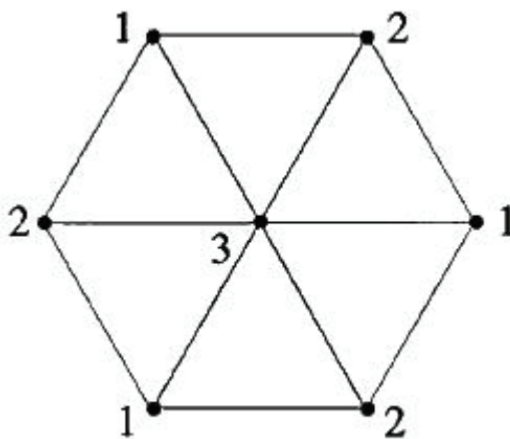


Figure 2 [Extracted from 9]. A sample 3-coloring.

There is ongoing research explicitly investigating vertex-coloring and edge-coloring graph methods that address the channel assignment problem for wireless networks [1, 2, 3, 5, 6, 8, 9]. The order in which colors or channels are assigned and to what nodes differs greatly from one solution to another. The creation of such an order is also quite varied among current proposed solutions. Weighted coloring is one method that deals

exclusively with assigning channels based on need to alleviate interference within the network.

2.1 Weighted Coloring

One variation of the graph coloring problem involves assigning weights to the interference graph. A weighted coloring implementation addresses interference chiefly in areas of greatest need. Once these problem areas are discovered, channel re-assignment can alleviate signal collision. McDiarmid and Reed [9] assign weights based on bandwidth demands, whereas Arbaugh et al. [2] assign weights that indicate the degree of channel interference between two nodes. McDiarmid and Reed [9] reduce interference by assigning to each node a number of colors equal to its weighted bandwidth demand value. Arbaugh et al. [2] devise algorithms, which they call Hminmax and Hsum, which greedily choose the frequency at each node which will locally at that node result in the greatest reduction in interference. McDiarmid and Reed [9] state that there is future work in determining an improved ratio for large demands. Arbaugh et al. [2] state, however, that for their algorithms Hminmax and Hsum, wireless-b and wireless-g environments were not studied in a mixed setting, and this remains an open issue.

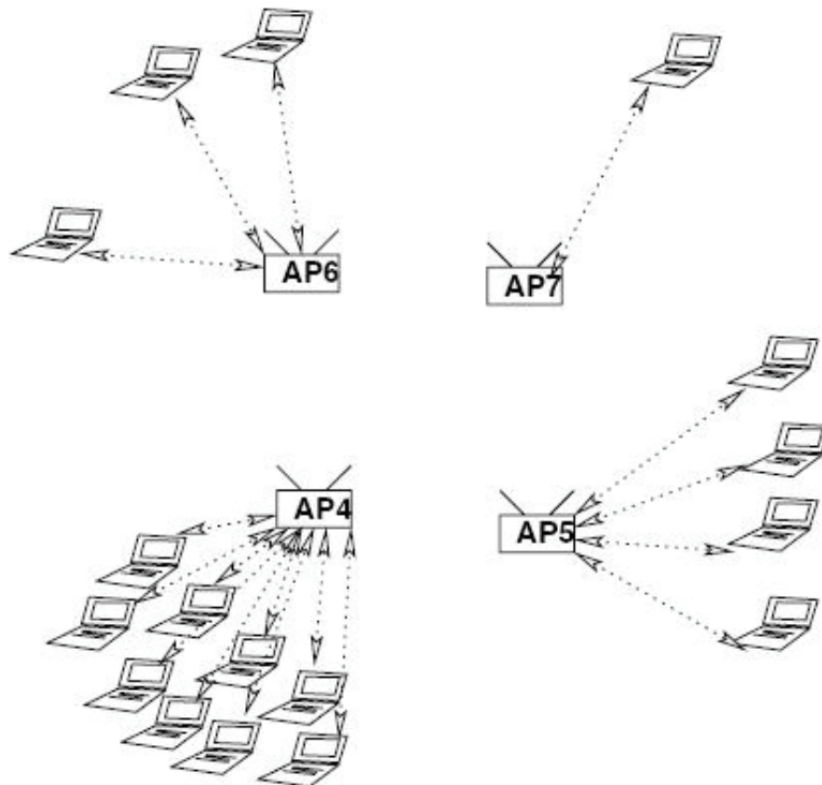


Figure 3. All Access Points (AP) are in interference range of each other [2].

Figure 3 depicts a network where there are only three non-overlapping channels for four wireless access points labeled AP4, AP5, AP6, and AP7. Each access point can

broadcast far enough to reach each wireless device attached to it. In this instance, two access points will need to share a communication channel [2].

A greedy weighted algorithm would assign AP4 and AP5 their own channel to broadcast over, because they have more devices connected than AP6 and AP7 do. AP6 and AP7 therefore will have to share one channel and endure the possibility of interference to spare the majority from the threat of interference. Provided that AP6 and AP7 are sufficiently far from each other, sharing one communication channel may result in no interference. This idea of channel reuse has been explored by several researchers aiming to reduce not only interference, but the number of communication channels as well.

2.2 Channel Re-use

Channel re-use involves imposing a minimum distance between two nodes operating at the same radio frequency. Re-using channels not only curtails interference but also improves overall efficiency in terms of channel use. Bertossi et al. [5] define the channel assignment problem with separation (CAPS) in order to incorporate such a re-use distance in channel assignment, resulting in a more efficient use of the given set of radio frequencies. An algorithm is presented by which a node, knowing its relative position in the network, can compute its channel assignment, with a specified re-use distance, in constant time for all network graph types except for binary trees, which require logarithmic time [5]. Bertossi et al. [5] however, identify an open issue exists in determining an optimal solution for general graphs and arbitrary channel re-use distance. An alternative for computing minimum distance would be to determine the minimum and maximum number of channels to use in a network.

2.3 Radio Frequency (RF) Spectrum

Investigating strict upper and lower bounds for the number of channels used in a given wireless network is of importance to the interference reduction problem. Khanna and Kumaran [8] define what they call the *wireless spectrum estimation problem*, where a node is assigned the smallest number of frequencies over which to broadcast and receive, such that interfering nodes do not share the same frequency. Bertossi et al. [5] later explore channel re-use governed by a minimum re-use distance rather than static assignment as delineated by upper and lower bounds (see Section 3.2). Additionally, the ability to compute minimum distance based on relative position is not always a necessity.

2.4 Location-Oblivious

Location-oblivious networks do not rely on each node knowing its relative geometric position. According to Barbeau et al. [3], location-aware networks, on the other hand, might require a Global Positioning System (GPS) in order to calculate nodes' relative positions. Therefore, location-oblivious networks are preferable. Barbeau et al. [3] introduce a distributed, location-oblivious unit disk graph coloring algorithm unlike Bertossi et al. [5] whose algorithms require nodes to be self-aware of their location.

However, the algorithm presented by Barbeau et al. did not out-perform an algorithm that executes arbitrary coloring of a network graph [3]. Similarly to location-oblivious algorithms, dynamic channel assignment [2] does not require a node to know its relative position.

2.5 Dynamic Channel Assignment

Dynamic channel assignment can reduce interference in networks where the topology changes often and dramatically. Dynamic channel assignment involves continuously monitoring interference and re-assigning channels appropriately. Arbaugh et al. [2] employ a form of dynamic assignment in their multi-step, greedy approach algorithms Hminmax and Hsum that re-assign channels only to nodes experiencing the greatest interference (mentioned in Section 3.1). Almeroth et al. [1] introduce dynamic assignment in their algorithm, Breadth First Search - Channel Assignment (BFS-CA). BFS-CA utilizes breadth-first search to determine nodes of greatest connectivity which typically are subject to the greatest interference. Channels are re-assigned on-the-fly, and connectivity is maintained through a secure “default channel” [1]. Figure 4[1] below illustrates why a default channel is vital to the structure of the network.

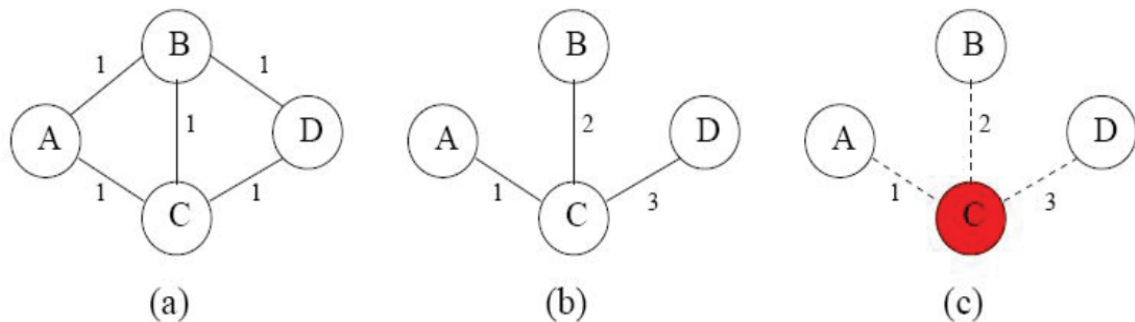


Figure 4. Network topology with varying channel assignments [1].

Figure 4(a) depicts a sample network topology where all nodes broadcast over one frequency. Suppose node C is selected for channel re-assignment via the addition of two new radios that broadcast and receive over channels 2 and 3. Figure 4(b) shows the result of this re-assignment. Through this process of eliminating the threat of interference, nodes A, B, and D lose direct connection to each other and instead must communicate using one-hop over node C. However, further complications arise with this topology change. Figure 4(c) shows the event of node C failing. The dotted lines represent broken connections. The consequence of this is that nodes A, B and D will not be able to communicate, even though they are within communication range of one another. For all changes in topology, node failure is a real threat in breaking communication. With a default channel in place, however, a severed connection due to node failure can be overcome with minimal downtime [1]. In addition to how BFS-CA [1] will perform in very dense topologies, addressing just how long the downtime due to node failure will

last is an open issue, and, in fact, graph coloring algorithms in general are works-in-progress because of their NP-hard and NP-complete algorithmic complexities.

3. Proposed Future Work

For future work, we propose to analyze the performance of the algorithms BFS-CA [1], Hminmax, and Hsum [2] on Unit Disk Graphs (UDG), both empirically (through implementation and experimentation) and analytically. UDGs are much simpler network models compared to typical mesh models, in the sense that only nodes that are no farther apart than the unit distance (after appropriate scaling of the maximum transmission range) can directly communicate to one another and therefore are connected by a link in the network graph.

We propose to take one step further and study the same algorithms on Quasi Unit Disk Graphs (qUDG), which are more realistic than UDGs, yet simpler than general networks. qUDGs take an input parameter α , with $0 < \alpha \leq 1$, and guarantee links between nodes that have a distance not greater than α ; nodes at distance greater than 1 are not connected by a link, and nodes at distance in the range $(\alpha, 1]$ may or may not be connected. This model attempts to account for errors in transmission, fading signal strength, and other imperfections.

There is much potential for greater interference reduction on UDGs and qUDGs than on general networks. In order to demonstrate such results, we plan to simulate BFS-CA, Hminmax, and Hsum in a wireless mesh network environment using Scalable Network Technologies' QualNet [11]. QualNet is a free network simulator that allows users to easily create, manipulate, and simulate any kind of network model through an intuitive graphical interface. Again, using QualNet, we will then implement BFS-CA, Hminmax, and Hsum and execute these algorithms on UDG graphs with varying number of nodes, say between 100 and 1000.

We will collect statistics such as maximum and average interference, retransmissions, channel reassignment, and propagation delays among others. We expect to see significant improvements in interference reduction by simulating BFS-CA, Hminmax, and Hsum in UDG and qUDG environments compared to simulation in various wireless mesh network environments. All results will be thoroughly documented and compiled onto a web page yet to be created.

4. Conclusion

The dynamic algorithm, BFS-CA [1], is one of the best algorithms for use in today's ever-changing wireless network topologies. It is the most implementation-ready compared to other graph coloring algorithms. BFS-CA was also shown to have a significant improvement over static assignment of channels [1]. The weighted Hminmax and Hsum [2] algorithms, despite resorting to greedy implementations, have achieved over a 40% average reduction in interference over one "state-of-the-art" method [2]. The

McDiarmid and Reed bandwidth-based weighted algorithms [9] bring together several novel ideas, however they seem difficult to eventually implement.

5. References

- [1] K.C. Almeroth, E. M. Belding, M. M. Buddhikot, K. N. Ramachandran, "Interference-Aware Channel Assignment in Multi-Radio Wireless Mesh Networks," Apr. 2006; <http://www.cs.ucsb.edu/~ebelding/txt/infocom06.pdf>.
- [2] W. Arbaugh, S. Banerjee, A. Mishra, "Weighted Coloring Based Channel Assignment for WLANs," *ACM SIGMOBILE Mobile Computing and Communications Review*, July 2005, vol. 9, no. 3, pp. 19-31.
- [3] M. Barbeau, P. Bose, P. Carmi, M. Couture, E. Kranakis, "Location Oblivious Distributed Unit Disk Graph Coloring" School of Computer Science Carleton University Jan 2006 pp. 1-20.
- [4] R. Battiti, A. A. Bertossi, M. A. Bonuccelli, "Assigning Codes in Wireless Networks: Bounds and Scaling Properties," *Wireless Networks*, May 1999, pp. 195-209.
- [5] A. A. Bertossi, C. M. Pinotti, R. B. Tan, "Efficient use of radio spectrum in wireless networks with channel separation between close stations," *Workshop on Discrete Algorithms and Methods for MOBILE Computing and Communications and Proceedings of the 4th International Workshop on Discrete Algorithms and Methods for Mobile Computing and Communications*, 2000, pp. 18-27.
- [6] T. Chiueh, K. Gopalan, A. Raniwala, "Centralized Channel Assignment and Routing Algorithms for Multi-Channel Wireless Mesh Networks," *ACM SIGMOBILE Mobile Computing and Communications Review*, Apr 2004, vol. 8 no. 2, pp. 50-65.
- [7] D. J. Goodman, N. B. Mandayam, C. U. Saraydar, "Efficient Power Control Via Pricing in Wireless Data Networks," *IEEE Transactions on Communications*, Feb 2002, vol. 50, issue 2, pp. 291-303.
- [8] S. Khanna, K. Kumaran, "On Wireless Spectrum Estimation and Generalized Graph Coloring," *INFOCOM '98. Seventeenth Annual Joint Conference of the IEEE Computer and Communications Societies. Proceedings. IEEE*, Apr 1998, vol. 3, pp. 1273-1283.
- [9] C. McDiarmid, B. Reed, "Channel Assignment and Weighted Coloring," *Networks*, Aug. 2000, vol. 36, no. 2, pp. 114-117.
- [10] R. Ramanathan, R. Rosales-Hain, "Topology Control of Multihop Wireless Networks Using Transmit Power Adjustment," *INFOCOM 2000. Nineteenth Annual Joint Conference of the IEEE Computer and Communications Societies. Proceedings. IEEE*, March 2000, vol. 2, pp. 404-413.
- [11] Scalable Network Technologies, Inc. QualNet Network Simulator. 2007. <http://www.scalable-networks.com/>