Time series analysis

Lecture 3. Coefficient estimation in ARMA (p,q) processes. Box-Jenkins' approach

Dr. Khamidov Obidjon

Introduction

- Autoregressive Integrated Moving Average models (ARIMA models) were popularized by George Box and Gwilym Jenkins in the early 1970s.
- ARIMA models are a class of linear models that is capable of representing stationary as well as non-stationary time series.
- ARIMA models do not involve independent variables in their construction. They make use of the information in the series itself to generate forecasts.

Introduction

- ARIMA models rely heavily on autocorrelation patterns in the data.
- ARIMA methodology of forecasting is different from most methods because it does not assume any particular pattern in the historical data of the series to be forecast.
- It uses an interactive approach of identifying a possible model from a general class of models. The chosen model is then checked against the historical data to see if it accurately describe the series.

Introduction

- Recall that, a time series data is a sequence of numerical observations naturally ordered in time
  - Daily closing price of IBM stock
  - Weekly automobile production by the Pontiac division of general Motors.
  - Hourly temperatures at the entrance to Grand central Station.

Introduction

- Two question of paramount importance When a forecaster examines a time series data are:
  - Do the data exhibit a discernible pattern?
  - Can this be exploited to make meaningful forecasts?

Introduction

- The Box-Jenkins methodology refers to a set of procedures for identifying, fitting, and checking ARIMA models with time series data.Forecasts follow directly from the form of fitted model.
- The basis of BOX-Jenkins approach to modeling time series consists of three phases:
  - Identification
  - Estimation and testing
  - Application

Introduction

- Identification
  - Data preparation
    - Transform data to stabilize variance
    - Differencing data to obtain stationary series
  - Model selection
    - Examine data, ACF and PACF to identify potential models

Introduction

- Estimation and testing
  - Estimation
    - Estimate parameters in potential models
    - Select best model using suitable criterion
  - Diagnostics
    - Check ACF/PACF of residuals
    - Do portmanteau test of residuals
    - Are the residuals white noise?

Introduction

- Application
  - Forecasting: use model to forecast

Examining correlation in time series data

- The key statistic in time series analysis is the autocorrelation coefficient (the correlation of the time series with itself, lagged 1, 2, or more periods.)
- Recall the autocorrelation formula:

$$r_k = \frac{\sum_{t=k+1}^{n} (y_t - \bar{y})(y_{t-k} - \bar{y})}{\sum_{t=1}^{n} (y_t - \bar{y})^2}$$

Examining Correlation in Time Series Data

- Recall r_1 indicates how successive values of Y relate to each other, r_2 indicates how Y values two periods apart relate to each other, and so on.
- The auto correlations at lag 1, 2, …, make up the autocorrelation function or ACF.
- Autocorrelation function is a valuable tool for investigating properties of an empirical time series.

A white noise model

- A white noise model is a model where observations Y_t is made of two parts: a fixed value and an uncorrelated random error component.

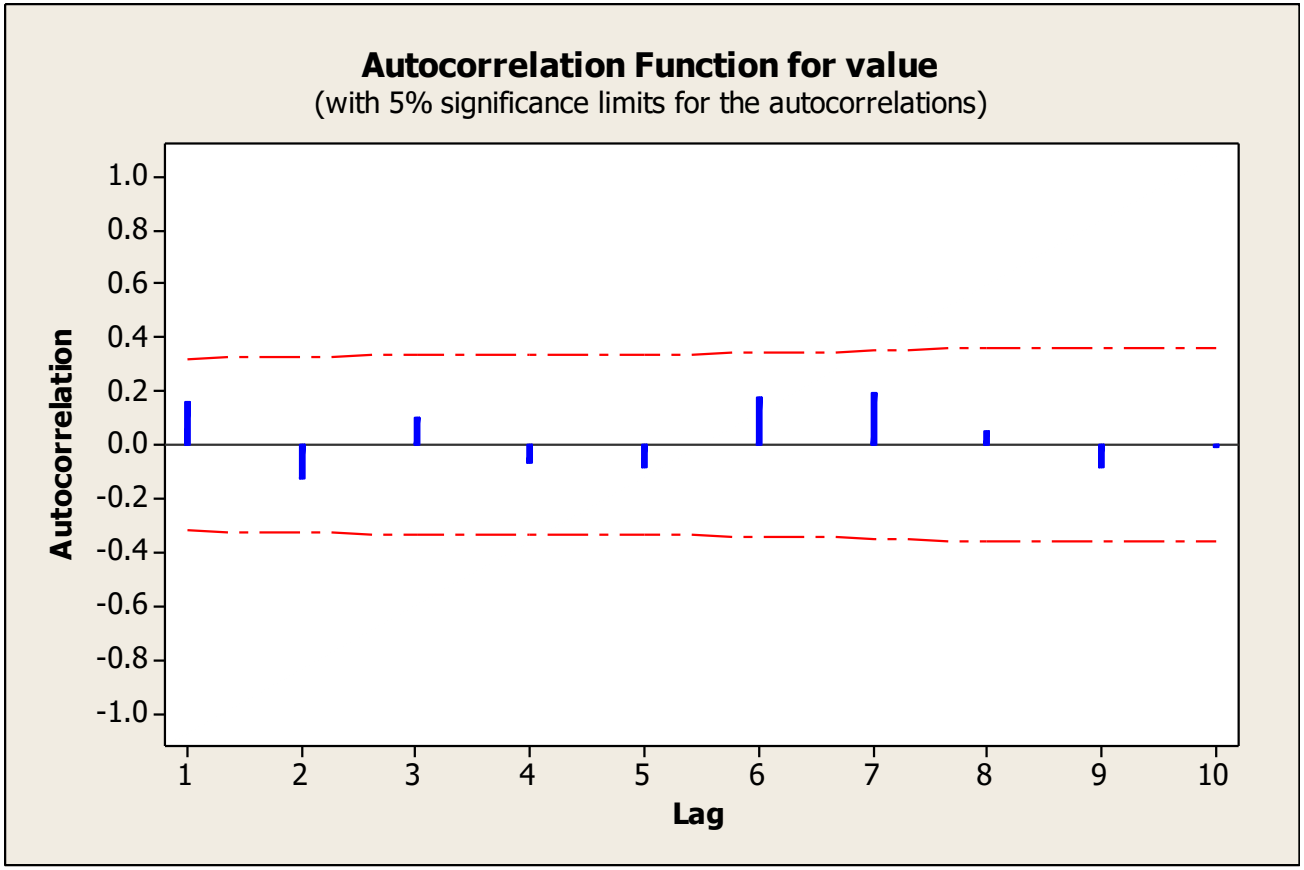
$$y_t = C + e_t$$

- For uncorrelated data (a time series which is white noise) we expect each autocorrelation to be close to zero.
- Consider the following white noise series.

White noise series

period	value	period	value
1	23	21	50
2	36	22	86
3	99	23	90
4	36	24	65
5	36	25	20
6	74	26	17
7	30	27	45
8	54	28	9
9	59	29	73
10	17	30	33
11	36	31	17
12	89	32	3
13	77	33	29
14	86	34	30
15	33	35	68
16	90	36	87
17	74	37	44
18	7	38	5
19	54	39	26
20	98	40	52

ACF for the white noise series



Sampling distribution of autocorrelation

- The autocorrelation coefficients of white noise data have a sampling distribution that can be approximated by a normal distribution with mean zero and standard error $1/\sqrt{n}$. where n is the number of observations in the series.
- This information can be used to develop tests of hypotheses and confidence intervals for ACF.

Sampling distribution of autocorrelation

- For example
  - For our white noise series example, we expect 95% of all sample ACF to be within

$$\pm 1.96 \frac{1}{\sqrt{n}} = \pm 1.96 \frac{1}{\sqrt{40}} = \pm .3099$$

  - If this is not the case then the series is not white noise.
  - The sampling distribution and standard error allow us to distinguish what is randomness or white noise from what is pattern.

Portmanteau tests

- Instead of studying the ACF value one at a time, we can consider a set of them together, for example the first 10 of them (r_1 through r_{10}) all at one time.
- A common test is the Box-Pierce test which is based on the Box-Pierce Q statistics

$$Q = n\sum_{k=1}^{h} r_k^2$$

  - Usually h ≈ 20 is selected

Portmanteau tests

- This test was originally developed by Box and Pierce for testing the residuals from a forecast model.
- Any good forecast model should have forecast errors which follow a white noise model.
- If the series is white noise then, the Q statistic has a chi-square distribution with (h-m) degrees of freedom, where m is the number of parameters in the model which has been fitted to the data.
- The test can easily be applied to raw data, when no model has been fitted , by setting m = 0.

Example

- Here is the ACF values for the white noise example.

Lag	ACF
1	0.159128
2	-0.12606
3	0.102384
4	-0.06662
5	-0.08255
6	0.176468
7	0.191626
8	0.05393
9	-0.08712
10	-0.01212
11	-0.05472
12	-0.22745
13	0.089477
14	0.017425
15	-0.20049

Example

- The box-Pierce Q statistics for h = 10 is

$$Q = n\sum_{k=1}^{h} r_k^2 = 40[(.159)^2 + (-.126)^2 + \cdots + (-.0121)^2] = 5.66$$

- Since the data is not modeled m =0 therefore df = 10.
- From table C-4 with 10 df, the probability of obtaining a chi-square value as large or larger than 5.66 is greater than 0.1.
- The set of 10 r_k values are not significantly different from zero.

Portmanteau tests

- An alternative portmanteau test is the Ljung-Box test.

$$Q^* = n(n+2)\sum_{k=1}^{h}(n-k)^{-1}r_k^2$$

- Q^* has a Chi-square distribution with (h-m) degrees of freedom.
- In general, the data are not white noise if the values of Q or Q^* is greater than the the value given in a chi square table with α = 5%.

The Partial autocorrelation coefficient

- Partial autocorrelations measures the degree of association between y_t and y_{t-k}, when the effects of other time lags 1, 2, 3, …, k-1 are removed.
- The partial autocorrelation coefficient of order k is evaluated by regressing y_t against $y_{t-1},\ldots y_{t-k}$:

$$y_t = b_0 + b_1 y_{t-1} + b_2 y_{t-2} + \cdots + b_k y_{t-k}$$

  - α_k (partial autocorrelation coefficient of order k) is the estimated coefficient b_k.

The Partial autocorrelation coefficient

- The partial autocorrelation functions (PACF) should all be close to zero for a white noise series.
- If the time series is white noise, the estimated PACF are approximately independent and normally distributed with a standard error $1/\sqrt{n}$.
- Therefore the same critical values of

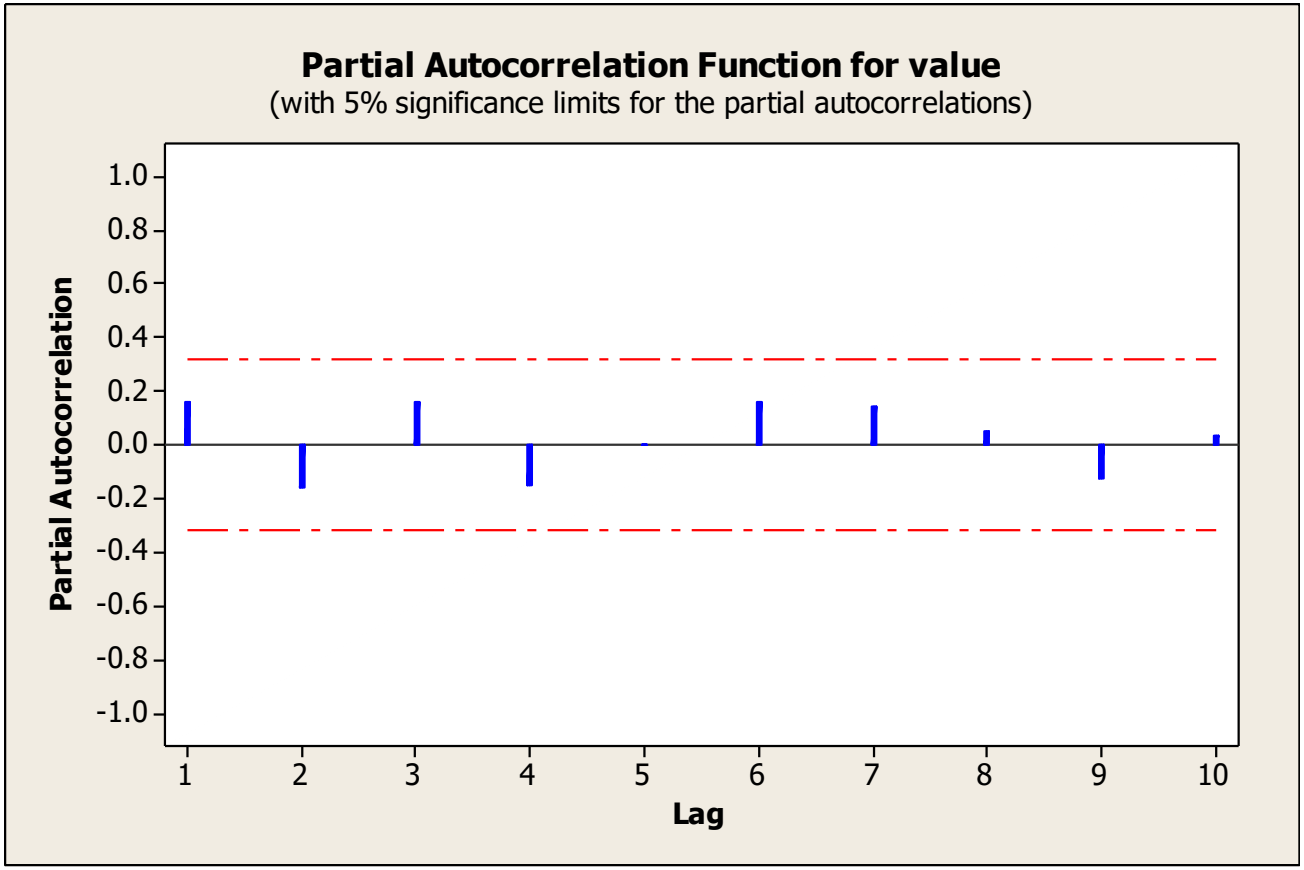
$$\pm 1.96 \frac{1}{\sqrt{n}}$$

Can be used with PACF to asses if the data are white noise.

The Partial autocorrelation coefficient

- It is usual to plot the partial autocorrelation function or PACF.
- The PACF plot of the white noise data is presented in the next slide.

PACF plot of the white noise series.



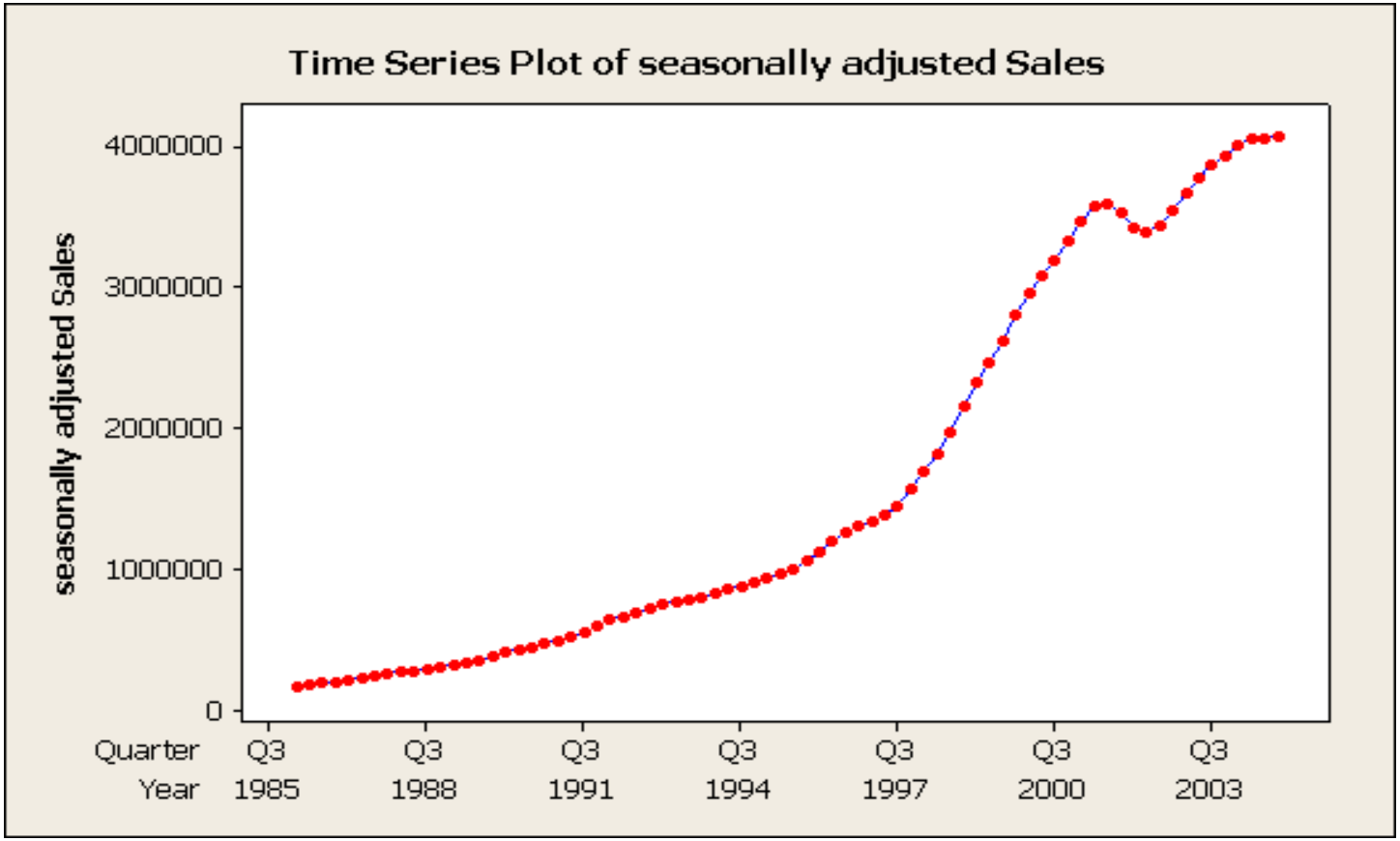
Examining stationarity of time series data

- Stationarity means no growth or decline.
- Data fluctuates around a constant mean independent of time and variance of the fluctuation remains constant over time.
- Stationarity can be assessed using a time series plot.
  - Plot shows no change in the mean over time
  - No obvious change in the variance over time.

Examining stationarity of time series data

- The autocorrelation plot can also show non-stationarity.
  - Significant autocorrelation for several time lags and slow decline in r_k indicate non-stationarity.
- The following graph shows the seasonally adjusted sales for Gap stores from 1985 to 2003.

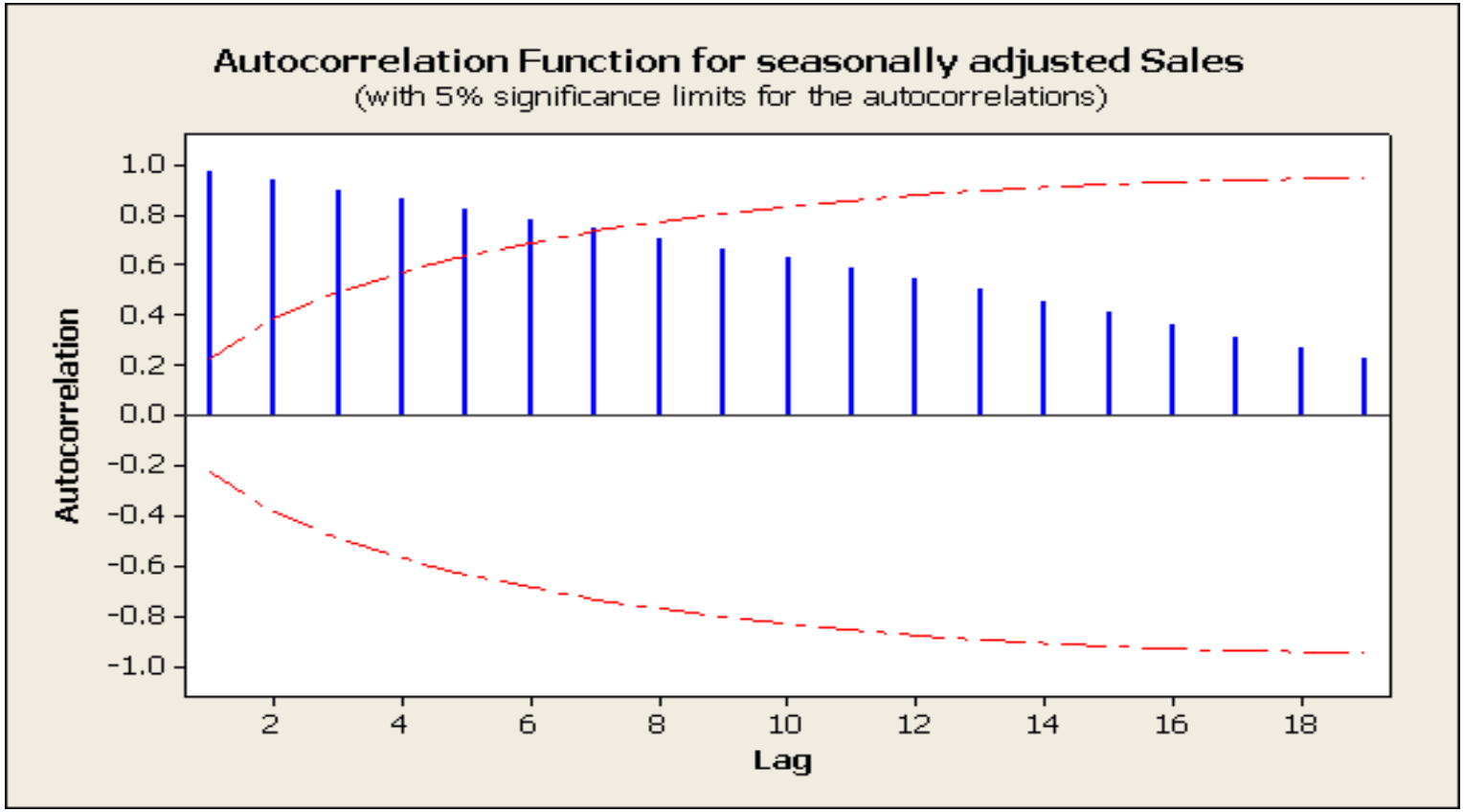
Examining stationarity of time series data



Examining stationarity of time series data

- The time series plot shows that it is non-stationary in the mean.
- The next slide shows the ACF plot for this data series.

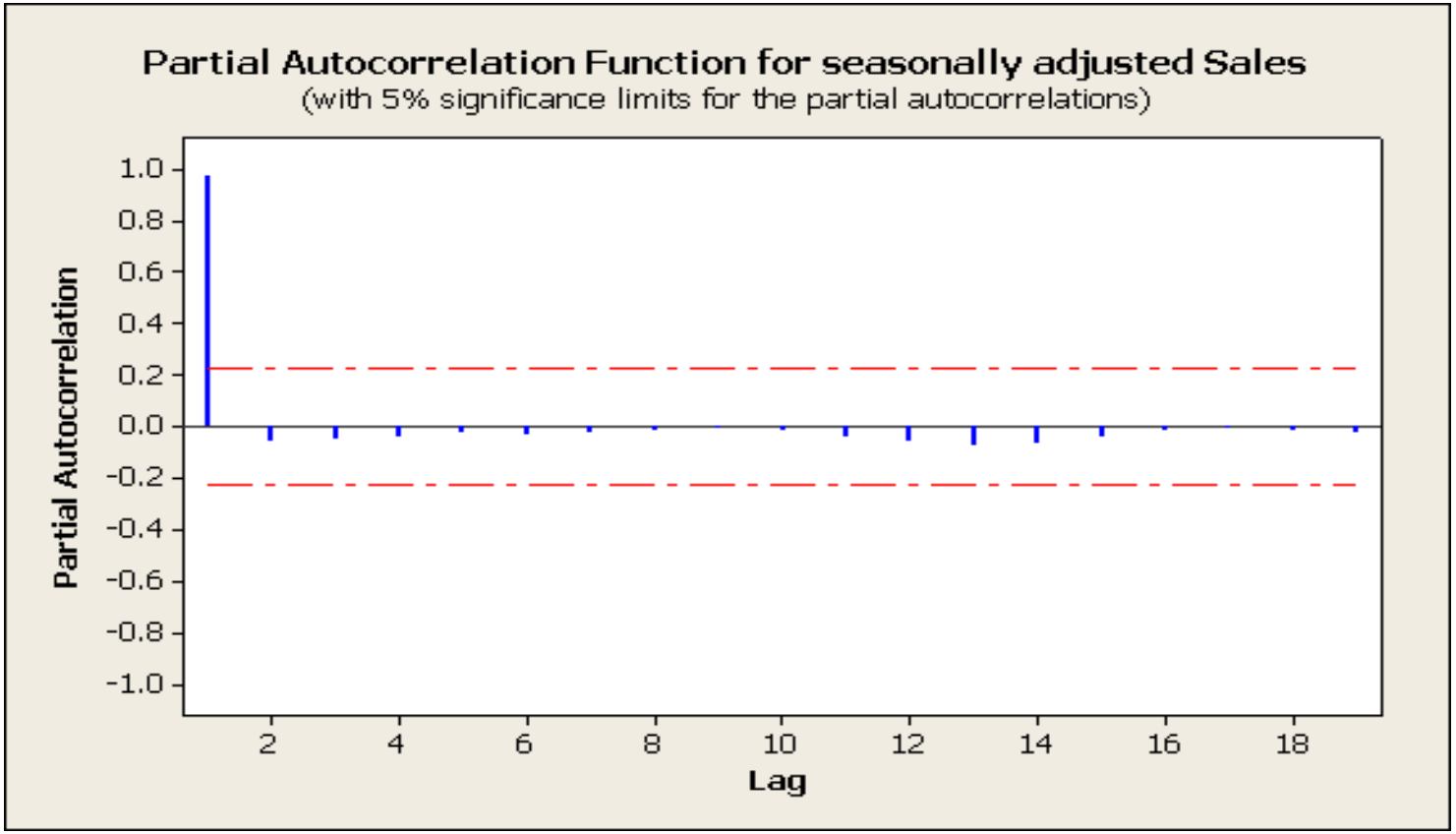
Examining stationarity of time series data



Examining stationarity of time series data

- The ACF also shows a pattern typical for a non-stationary series:
  - Large significant ACF for the first 7 time lag
  - Slow decrease in the size of the autocorrelations.
- The PACF is shown in the next slide.

Examining stationarity of time series data



Examining stationarity of time series data

- This is also typical of a non-stationary series.
  - Partial autocorrelation at time lag 1 is close to one and the partial autocorrelation for the time lag 2 through 18 are close to zero.

Reference and sourse:

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2. Time Series Analysis by James Douglas Hamilton
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