Course: Economics of Industry

Lecture 9: Duopoly

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Overview

- Duopoly
  - **Background**
  - Price competition
  - Quantity competition
  - Assessment

How the basic elements of the firm and of game theory are used

Basic ingredients

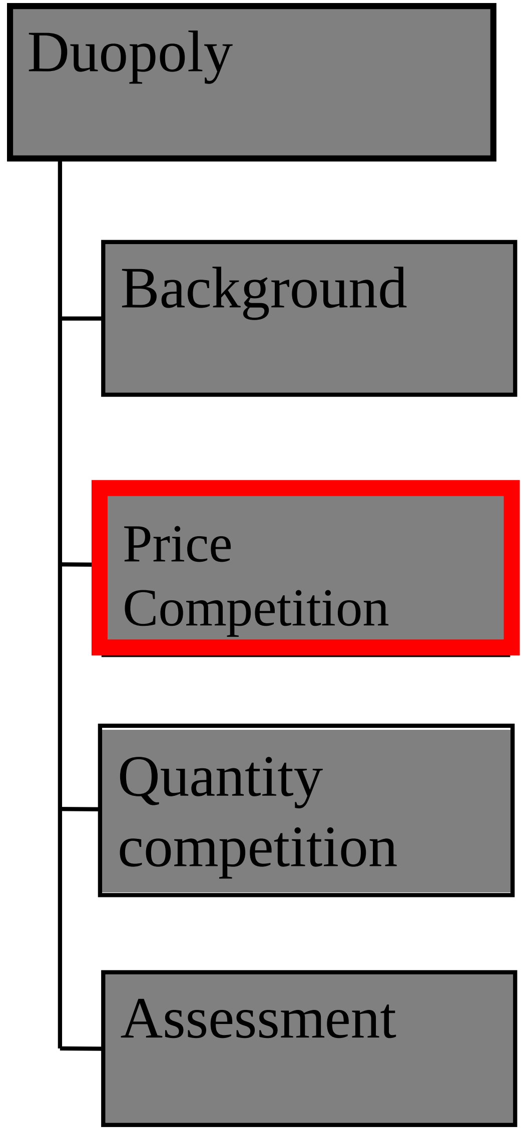
- Two firms:
  - issue of entry is not considered
  - but monopoly could be a special limiting case
- Profit maximisation
- Quantities or prices?
  - there's nothing within the model to determine which "weapon" is used
  - it's determined *a priori*
  - highlights artificiality of the approach
- Simple market situation:
  - there is a known demand curve

Reaction

- We deal with “competition amongst the few”
- Each actor has to take into account what others do
- A simple way to do this: *the reaction function*
- Based on the idea of “best response”
  - we can extend this idea
  - in the case where more than one possible reaction to a particular action
  - it is then known as a reaction *correspondence*
- We will see how this works:
  - where reaction is in terms of prices
  - where reaction is in terms of quantities

Overview

Introduction to a simple simultaneous move price-setting problem



Competing by price

- Simplest version of model:
  - there is a market for a single, homogeneous good
  - firms announce prices
  - each firm does not know the other's announcement when making its own
- Total output is determined by demand
  - determinate market demand curve
  - known to the firms
- Division of output amongst the firms determined by market "rules"
- Take a specific case with a clear-cut solution

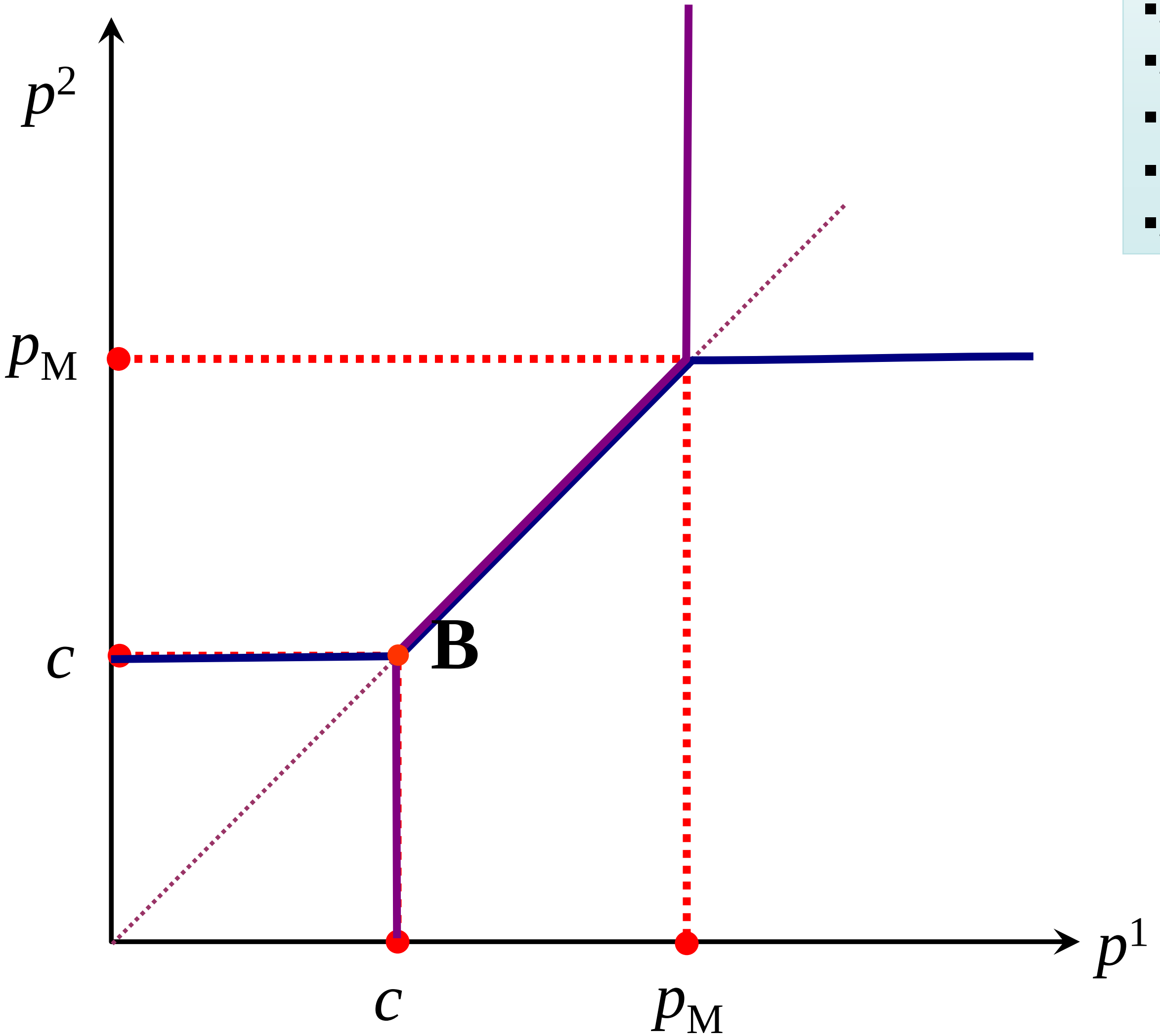
Bertrand – basic set-up

- Two firms can potentially supply the market
  - each firm: zero fixed cost, constant marginal cost c
  - if one firm alone supplies the market it charges monopoly price $p_M > c$
  - if both firms are present they announce prices
- The outcome of these announcements:
  - if $p^1 < p^2$ firm 1 captures the whole market
  - if $p^1 > p^2$ firm 2 captures the whole market
  - if $p^1 = p^2$ the firms supply equal amounts to the market
- What will be the equilibrium price?

Bertrand – best response?

- Consider firm 1's response to firm 2
- If firm 2 foolishly sets a price p^2 above p_M then it sells zero output
  - firm 1 can safely set monopoly price p_M
- If firm 2 sets p^2 above c but less than or equal to p_M then:
  - firm 1 can "undercut" and capture the market
  - firm 1 sets $p^1 = p^2 - \delta$, where $\delta > 0$
  - firm 1's profit always increases if δ is made smaller
  - but to capture the market the discount δ must be positive!
  - so strictly speaking there's no *best* response for firm 1
- If firm 2 sets price equal to c then firm 1 cannot undercut
  - firm 1 also sets price equal to c
- If firm 2 sets a price below c it would make a loss
  - firm 1 would be crazy to match this price
  - if firm 1 sets $p^1 = c$ at least it won't make a loss
- Let's look at the diagram

Bertrand model – equilibrium



- *Marginal cost for each firm*
- *Monopoly price level*
- *Firm 1's reaction function*
- *Firm 2's reaction function*
- *Bertrand equilibrium*

Bertrand – assessment

- Using “natural tools” – prices
- Yields a remarkable conclusion
  - – mimics the outcome of perfect competition
  - – price = MC
- But it is based on a special case
  - – neglects some important practical features
  - fixed costs
  - product diversity
  - capacity constraints
- Outcome of price-competition models usually sensitive to these

Overview

Duopoly

- Background
- Price competition
- Quantity competition
  - **Collusion**
  - The Cournot model
  - Leader-Follower
- Assessment

The link with monopoly and an introduction to two simple "competitive" paradigms

Quantity models

- Now take *output quantity* as the firms' choice variable
- Price is determined by the market once total quantity is known:
  - an auctioneer?
- Three important possibilities:

1. Collusion:
   - competition is an illusion
   - monopoly by another name
   - but a useful reference point for other cases
2. Simultaneous-move competing in quantities:
   - complementary approach to the Bertrand-price model
3. Leader-follower (sequential) competing in quantities

Collusion – basic set-up

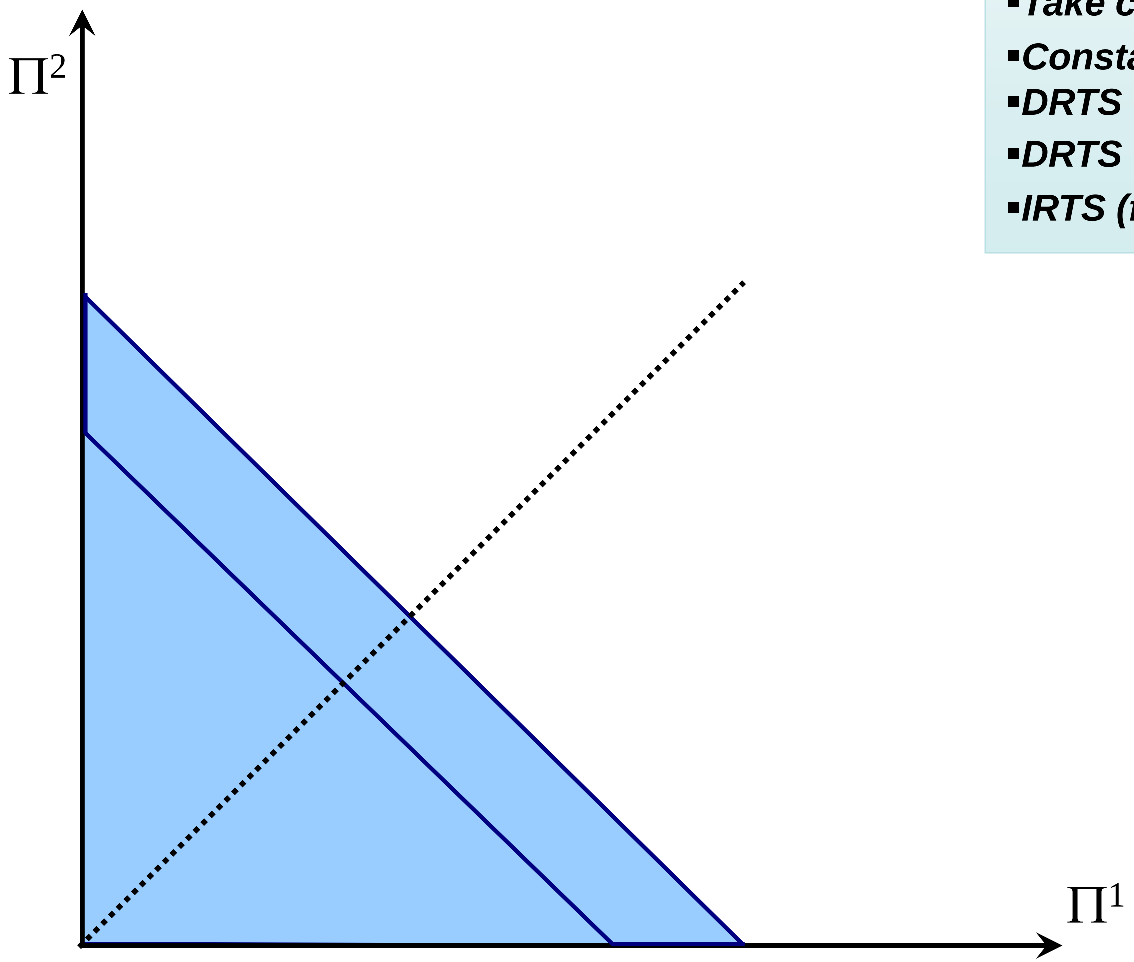
- Two firms agree to maximise joint profits
  - what they can make by acting as though they were a single firm
    - essentially a monopoly with two plants
- They also agree on a rule for dividing the profits
    - could be (but need not be) equal shares
- In principle these two issues are separate

The profit frontier

- To show what is possible for the firms
  - draw the profit frontier
- Show the possible combination of profits for the two firms
  - given demand conditions
  - given cost function
- Distinguish two cases

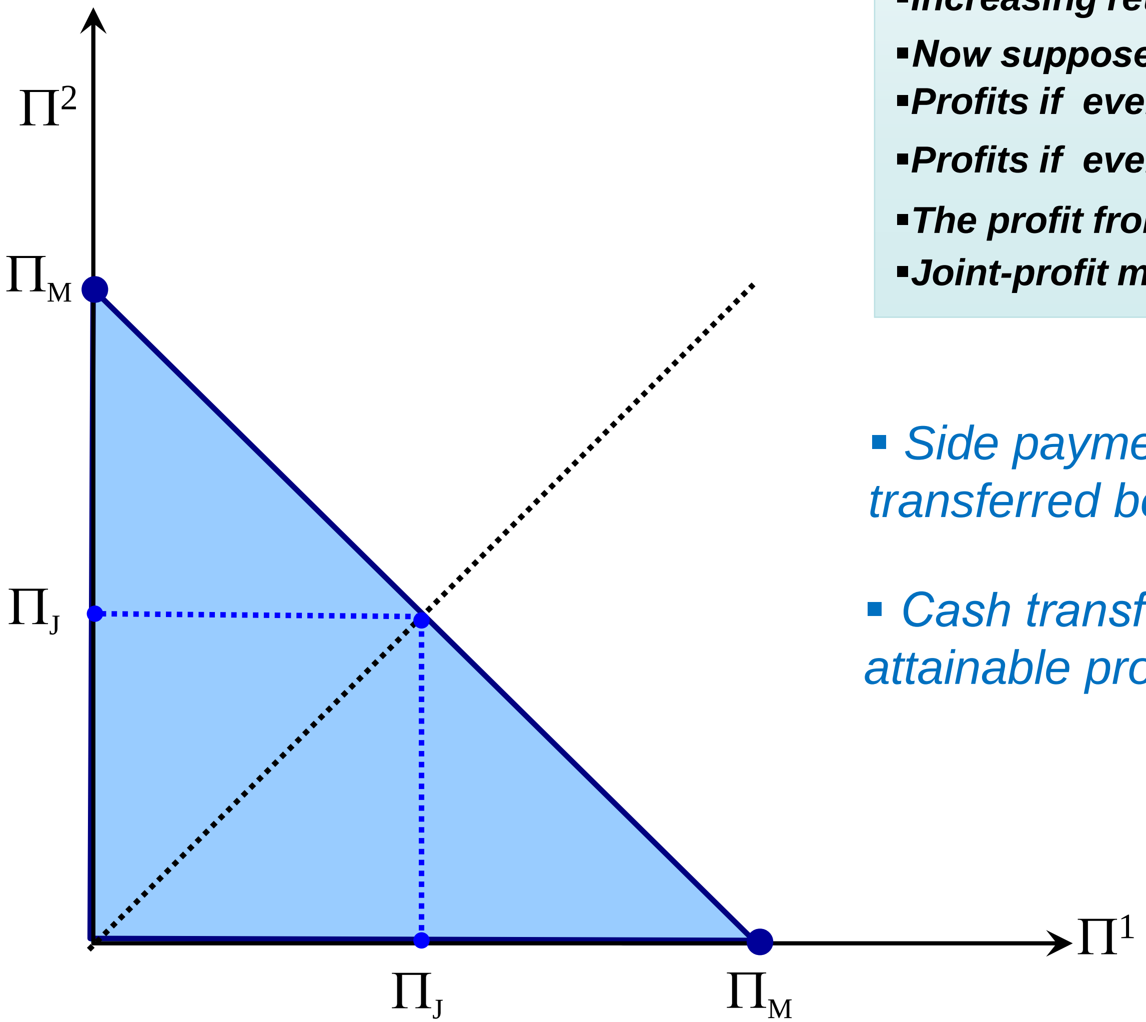
1. where cash transfers between the firms are not possible
2. where cash transfers are possible

Frontier – non-transferable profits



- ***Take case of identical firms***
- ***Constant returns to scale***
- ***DRTS (1): MC always rising***
- ***DRTS (2): capacity constraints***
- ***IRTS (fixed cost and constant MC)***

Frontier – transferable profits



- *Increasing returns to scale (without transfers)*
- *Now suppose firms can make "side-payments"*
- *Profits if everything were produced by firm 1*
- *Profits if everything were produced by firm 2*
- *The profit frontier if transfers are possible*
- *Joint-profit maximisation with equal shares*

- *Side payments mean profits can be transferred between firms*
- *Cash transfers "convexify" the set of attainable profits*

Collusion – simple model

- Take the special case of the “linear” model where marginal costs are identical:

$$c^1 = c^2 = c$$

- Will both firms produce a positive output?
  1. if unlimited output is possible then only one firm needs to incur the fixed cost
     - in other words a true monopoly
  2. but if there are capacity constraints then both firms may need to produce
     - both firms incur fixed costs
- We examine both cases – capacity constraints first

Collusion: capacity constraints

- If both firms are active total profit is

  $[a - bq]\, q \;-\; [C_0^1 + C_0^2 + cq]$

- Maximising this, we get the FOC:

  $a - 2bq - c = 0$

- Which gives equilibrium quantity and price:

  $$q = \frac{a - c}{2b}; \qquad p = \frac{a + c}{2}$$

- So maximised profits are:

  $$\Pi_M = \frac{[a - c]^2}{4b} \;-\; [C_0^1 + C_0^2]$$

- Now assume the firms are identical: $C_0^1 = C_0^2 = C_0$
- Given equal division of profits each firm's payoff is

  $$\Pi_J = \frac{[a - c]^2}{8b} \;-\; C_0$$

Collusion: no capacity constraints

- With no capacity limits and constant marginal costs
  - seems to be no reason for both firms to be active
- Only need to incur one lot of fixed costs C_0
  - C_0 is the smaller of the two firms' fixed costs
  - previous analysis only needs slight tweaking
  - modify formula for Π_J by replacing C_0 with $\frac{1}{2}C_0$
- But is the division of the profits still implementable?

Overview

Simultaneous move "competition" in quantities

- Duopoly
  - Background
  - Price competition
  - **Quantity competition**
    - Collusion
    - **The Cournot model**
    - Leader-Follower
  - Assessment

Cournot – basic set-up

- Two firms
  - assumed to be profit-maximisers
  - each is fully described by its cost function
- Price of output determined by demand
  - determinate market demand curve
  - known to both firms
- Each chooses the quantity of output
  - Single homogeneous output
  - neither firm *knows* the other’s decision when making its own
- Each firm makes an *assumption* about the other’s decision
  - firm 1 assumes firm 2’s output to be given number
  - likewise for firm 2

Cournot – model setup

- Two firms labelled $f = 1,2$
- Firm f produces output q^f
- So total output is:
  - $q = q^1 + q^2$
- Market price is given by:
  - $p = p\,(q)$
- Firm f has cost function $C^f(\cdot)$
- So profit for firm f is:
  - $p(q)\, q^f - C^f(q^f)$
- Each firm's profit depends on the other

Cournot – firm's maximisation

- Firm 1's problem is to choose q^1 so as to maximise

  $\Pi^1(q^1; q^2) := p\,(q^1 + q^2)\, q^1 - C^1(q^1)$

- Differentiate Π^1 to find FOC:

  $$\frac{\partial \Pi^1(q^1; q^2)}{\partial q^1} = p_q(q^1 + q^2)\, q^1 + p(q^1 + q^2) - C_q^1(q^1)$$

  – for an interior solution this is zero

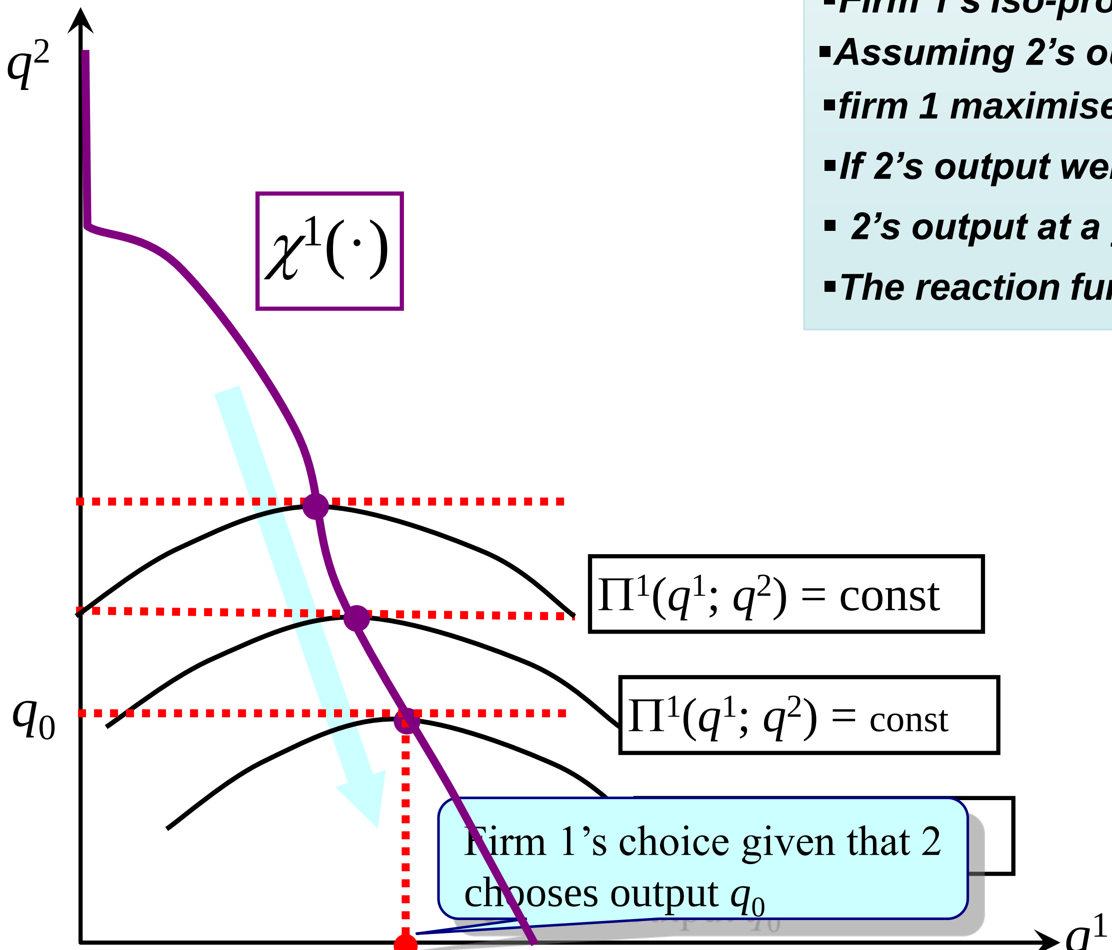
- Solving, we find q^1 as a function of q^2
- This gives us 1's *reaction function*, χ^1 :

  $q^1 = \chi^1(q^2)$

- Let's look at it graphically

Cournot – the reaction function

- ▪*Firm 1's Iso-profit curves*
- ▪*Assuming 2's output constant at* q_0
- ▪*firm 1 maximises profit*
- ▪*If 2's output were constant at a higher level*
- ▪ *2's output at a yet higher level*
- ▪*The reaction function*

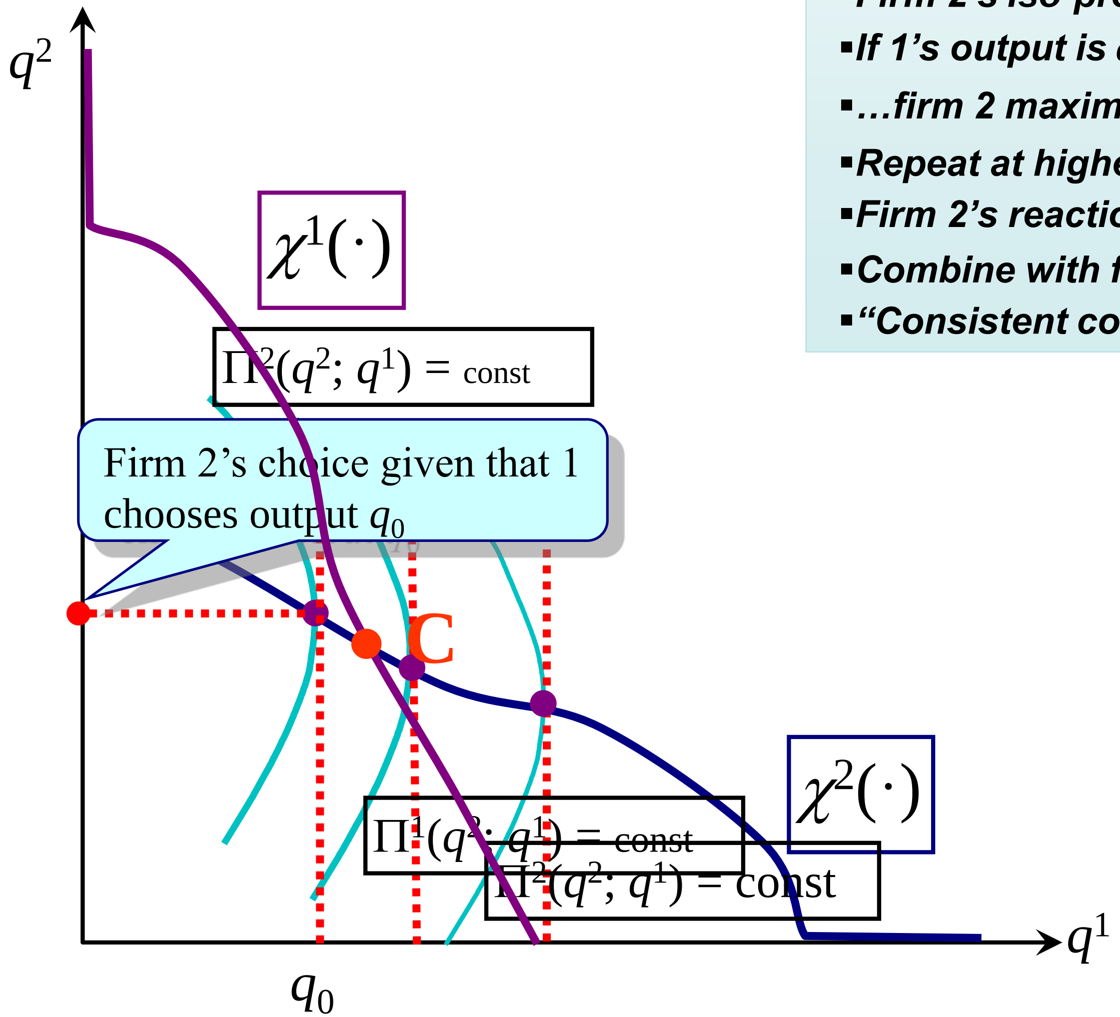


Cournot – solving the model

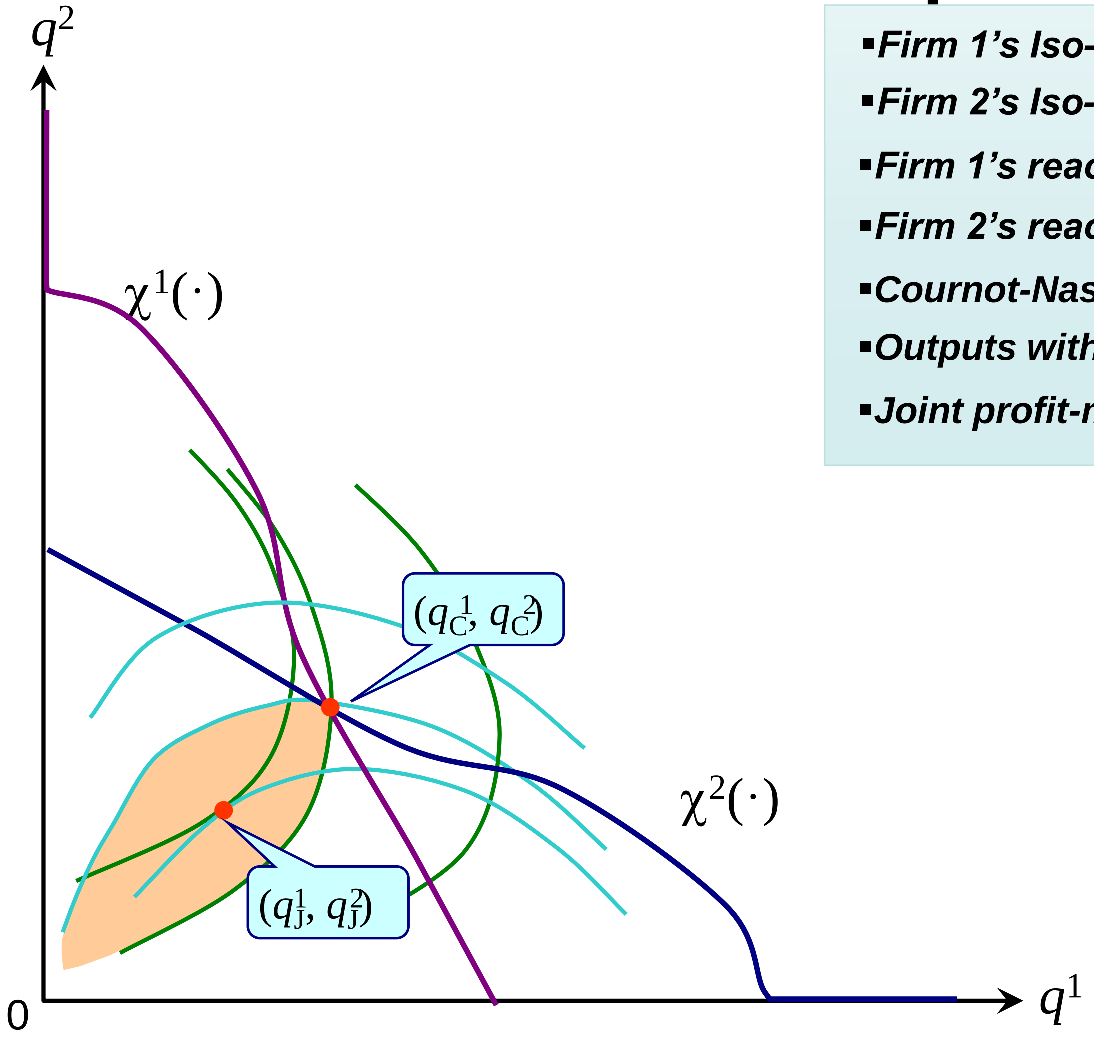
- $\chi^1(\cdot)$ encapsulates profit-maximisation by firm 1
- Gives firm's reaction 1 to fixed output level of competitor:
  - $q^1 = \chi^1 (q^2)$
- Of course firm 2's problem is solved in the same way
- We get q^2 as a function of q^1 :
  - $q^2 = \chi^2 (q^1)$
- Treat the above as a pair of simultaneous equations
- Solution is a pair of numbers $(q_C^{\ 1} , q_C^{\ 2})$
  - So we have $q_C^{\ 1} = \chi^1(\chi^2(q_C^{\ 1}))$ for firm 1
  - and $q_C^{\ 2} = \chi^2(\chi^1(q_C^{\ 2}))$ for firm 2
- This gives the *Cournot-Nash equilibrium* outputs

Cournot-Nash equilibrium (1)

- *Firm 2's Iso-profit curves*
- *If 1's output is q_0 …*
- *…firm 2 maximises profit*
- *Repeat at higher levels of 1's output*
- *Firm 2's reaction function*
- *Combine with firm 's reaction function*
- *"Consistent conjectures"*



Cournot-Nash equilibrium (2)



- *Firm 1's Iso-profit curves*
- *Firm 2's Iso-profit curves*
- *Firm 1's reaction function*
- *Firm 2's reaction function*
- *Cournot-Nash equilibrium*
- *Outputs with higher profits for both firms*
- *Joint profit-maximising solution*

The Cournot-Nash equilibrium

- Why "Cournot-Nash" ?
- It is the general form of Cournot's (1838) solution
- It also is the Nash equilibrium of a simple quantity game:
  - players are the two firms
  - moves are simultaneous
  - strategies are actions – the choice of output levels
  - functions give the best-response of each firm to the other's strategy (action)
- To see more, take a simplified example

Cournot – a “linear” example

- Take the case where the inverse demand function is:

$p = \beta_0 - \beta q$

- And the cost function for *f* is given by:

$C^f(q^f) = C_0^f + c^f q^f$

- So profits for firm *f* are:

$[\beta_0 - \beta q]\, q^f - [C_0^f + c^f q^f]$

- Suppose firm 1’s profits are Π
- Then, rearranging, the iso-profit curve for firm 1 is:

Cournot – solving the linear example

- Firm 1's profits are given by

$$\Pi^1(q^1; q^2) = [\beta_0 - \beta q]\, q^1 - [C_0^1 + c^1 q^1]$$

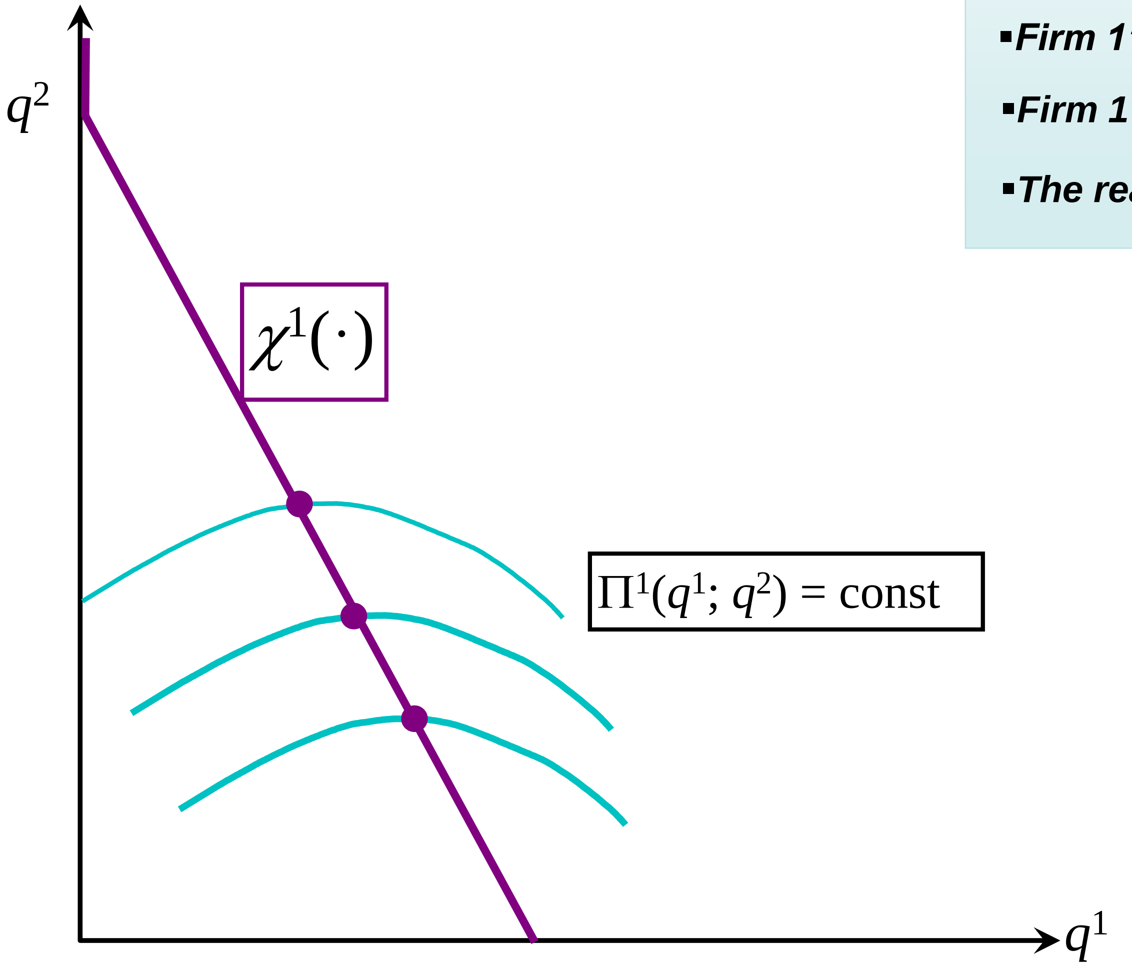
- So, choose q^1 so as to maximise this
- Differentiating we get:

$$\frac{\partial \Pi^1(q^1; q^2)}{\partial q^1} = -2\beta q^1 + \beta_0 - \beta q^2 - c^1$$

- FOC for an interior solution ($q^1 > 0$) sets this equal to zero
- Doing this and rearranging, we get the reaction function:

$$q^1 = \max \left\{ \frac{\beta_0 - c^1}{2\beta} - \tfrac{1}{2} q^2 \, , 0 \right\}$$

The reaction function again



- *Firm 1's Iso-profit curves*
- *Firm 1 maximises profit, given q^2*
- *The reaction function*

Finding Cournot-Nash equilibrium

- Assume output of both firm 1 and firm 2 is positive
- Reaction functions of the firms, $\chi^1(\cdot)$, $\chi^2(\cdot)$ are given by:

$$q^1 = \frac{a - c^1}{2b} - ½q^2 \;\; ; \qquad q^2 = \frac{a - c^2}{2b} - ½q^1$$

- Substitute from χ^2 into χ^1:

$$q^1_C = \frac{a - c^1}{2b} - ½ \left[\frac{a - c^2}{2b} - ½q^1_C \right]$$

- Solving this we get the Cournot-Nash output for firm 1:

$$q^1_C = \frac{a + c^2 - 2c^1}{3b}$$

- By symmetry get the Cournot-Nash output for firm 2:

$$q^2_C = \frac{a + c^1 - 2c^2}{3b}$$

Cournot – identical firms

- Take the case where the firms are *identical*
  - useful but very special
- Use the previous formula for the Cournot-Nash outputs

Reminder

$$q_C^1 = \frac{a + c^2 - 2c^1}{3b} \quad ; q_C^2 = \frac{a + c^1 - 2c^2}{3b}$$

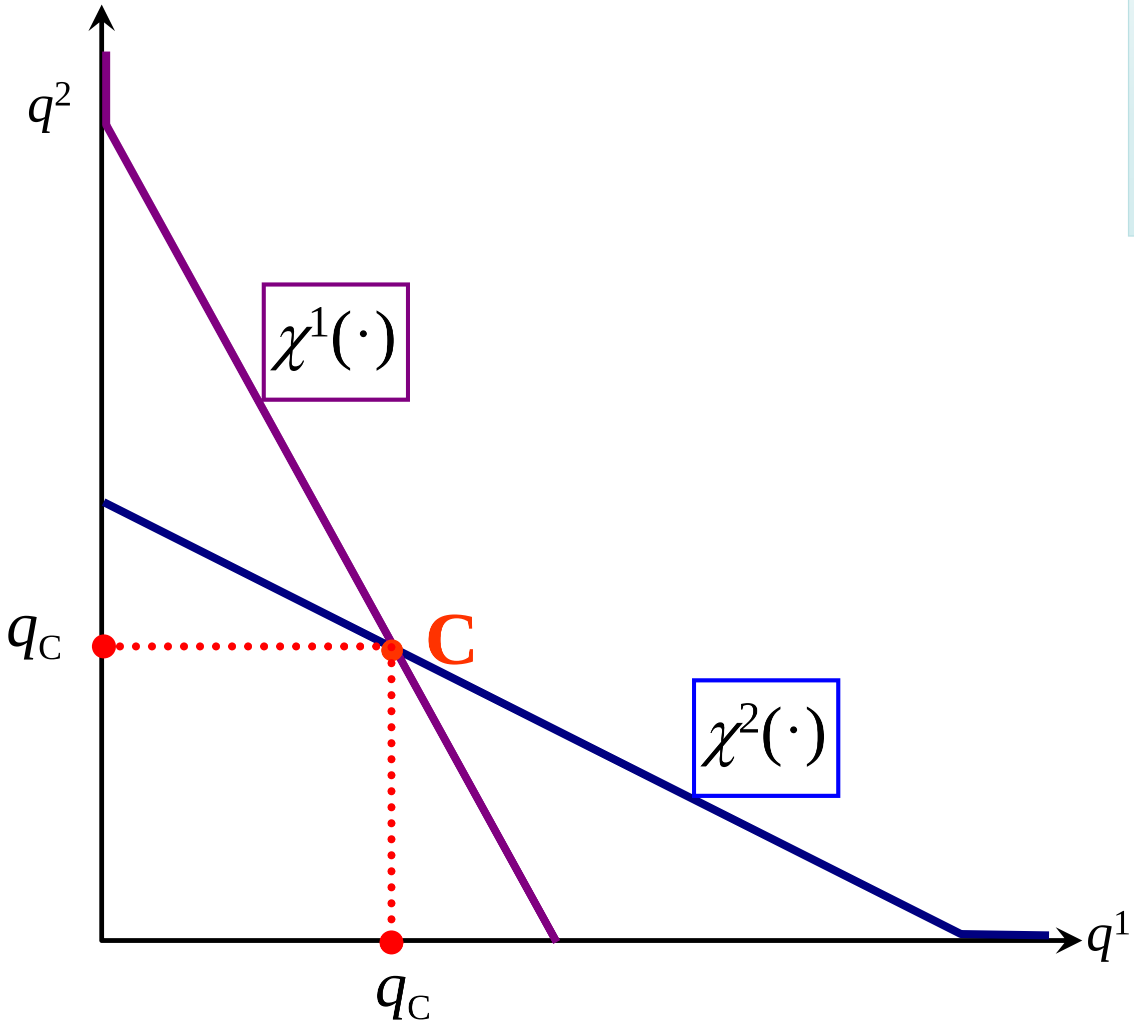
- Put $c^1 = c^2 = c$. Then we find $q_C^1 = q_C^2 = q_C$ where

$$q_C = \frac{a - c}{3b}$$

- From the demand curve the price in this case is ⅓[a+2c]
- Profits are

$$\Pi_C = \frac{[a - c]^2}{9b} - C_0$$

Symmetric Cournot



- *A case with identical firms*
- *Firm 1's reaction to firm 2*
- *Firm 2's reaction to firm 1*
- *The Cournot-Nash equilibrium*

Cournot – assessment

- Cournot-Nash outcome straightforward
  - usually have continuous reaction functions
- Apparently "suboptimal" from the selfish point of view of the firms
  - could get higher profits for all firms by collusion
- Unsatisfactory aspect is that price emerges as a "by-product"
  - contrast with Bertrand model
- Absence of time in the model may be unsatisfactory

Overview

Sequential "competition" in quantities

- Duopoly
  - Background
  - Price competition
  - Quantity competition
    - Collusion
    - The Cournot model
    - **Leader-Follower**
  - Assessment

Leader-Follower – basic set-up

- Two firms choose the quantity of output
  - single homogeneous output
- Both firms know the market demand curve
- But firm 1 is able to choose first
  - it announces an output level
- Firm 2 then moves, knowing the announced output of firm 1
- Firm 1 knows the reaction function of firm 2

Leader-follower – model

- Firm 1 (the leader) knows firm 2's reaction
  - if firm 1 produces q^1 then firm 2 produces $c^2(q^1)$
- Firm 1 uses χ^2 as a feasibility constraint for its own action
- Building in this constraint, firm 1's profits are given by

$$p(q^1 + \chi^2(q^1))\, q^1 - C^1(q^1)$$

Reminder

- In the "linear" case firm 2's reaction function is

$$q^2 = \frac{a - c^2}{2b} - \tfrac{1}{2}q^1$$

- So firm 1's profits are

$$\left[a - b\left[q^1 + [a - c^2]/2b - \tfrac{1}{2}q^1\right]\right]q^1 - [C_0{}^1 + c^1 q^1]$$

Solving the leader-follower model

- Simplifying the expression for firm 1's profits we have:

  $\frac{1}{2}\,[a + c^2 - bq^1]\; q^1 \;-\; [C_0{}^1 + c^1 q^1]$

- The FOC for maximising this is:

$$\frac{1}{2}\,[a + c^2] \;-\; bq^1 \;-\; c^1 = 0$$

- Solving for q^1 we get:

$$q_S^1 = \frac{a + c^2 - 2c^1}{2b}$$

- Using 2's reaction function to find q^2 we get:

$$q_S^2 = \frac{a + 2c^1 - 3c^2}{4b}$$

Leader-follower – identical firms

Of course they still differ in terms of their strategic position – firm 1 moves first

- Again assume that the firms have the same cost function
- Take the previous expressions for the Leader-Follower outputs:

Reminder

$$q_S^1 = \frac{a + c^2 - 2c^1}{2b} \; ; \quad q_S^2 = \frac{a + 2c^1 - 3c^2}{4b}$$

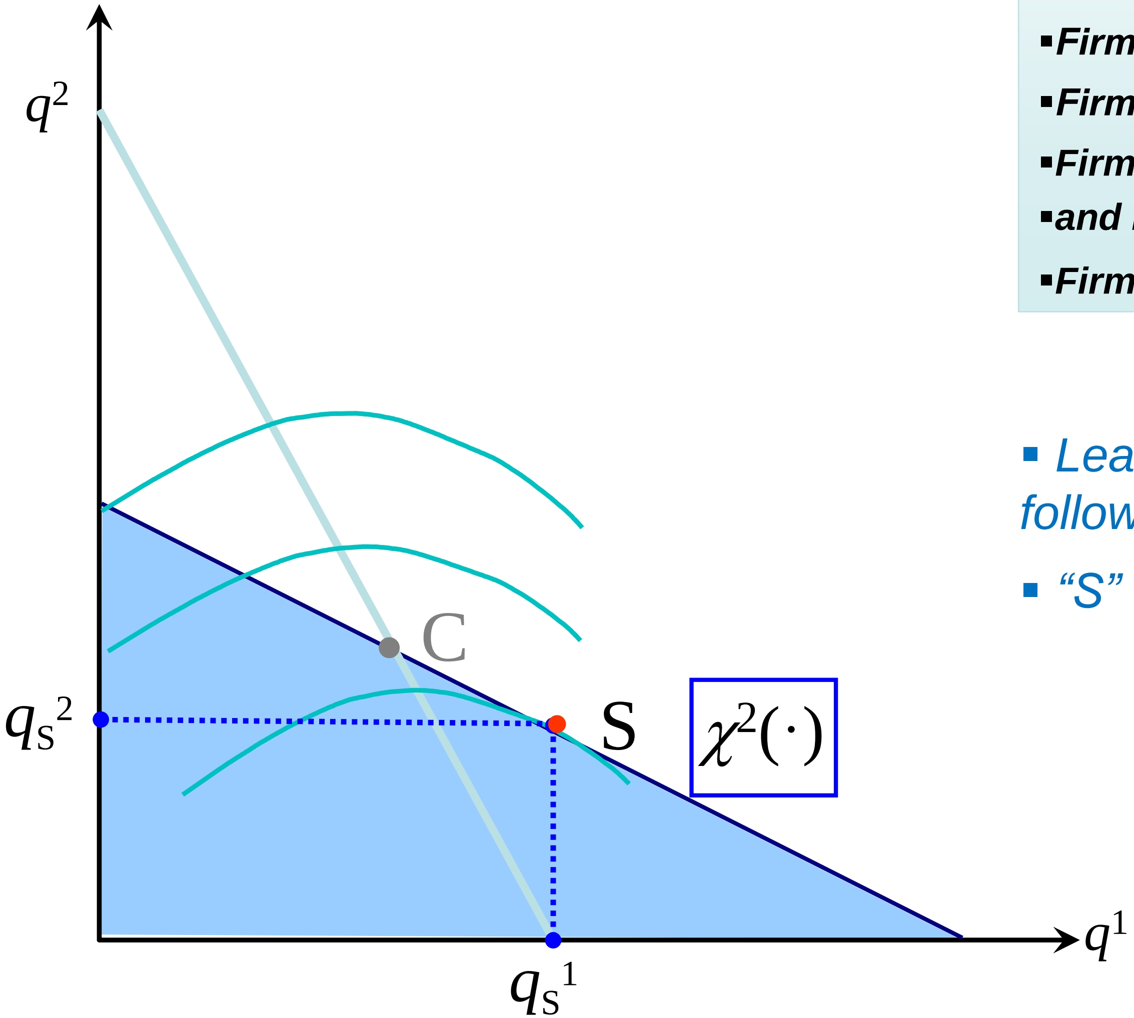
- Put $c^1 = c^2 = c$; then we get the following outputs:

$$q_S^1 = \frac{a - c}{2b} \; ; \quad q_S^2 = \frac{a - c}{4b}$$

- Using the demand curve, market price is ¼ $[a + 3c]$
- So profits are:

$$\Pi_S^1 = \frac{[a - c]^2}{8b} - C_0 \; ; \Pi_S = \frac{2[a - c]^2}{16b} - C_0$$

Leader-Follower



- ***Firm 1's Iso-profit curves***
- ***Firm 2's reaction to firm 1***
- ***Firm 1 takes this as an opportunity set***
- ***and maximises profit here***
- ***Firm 2 follows suit***

- *Leader has higher output (and follower less) than in Cournot-Nash*
- *"S" stands for <u>von Stackelberg</u>*

Overview

Duopoly

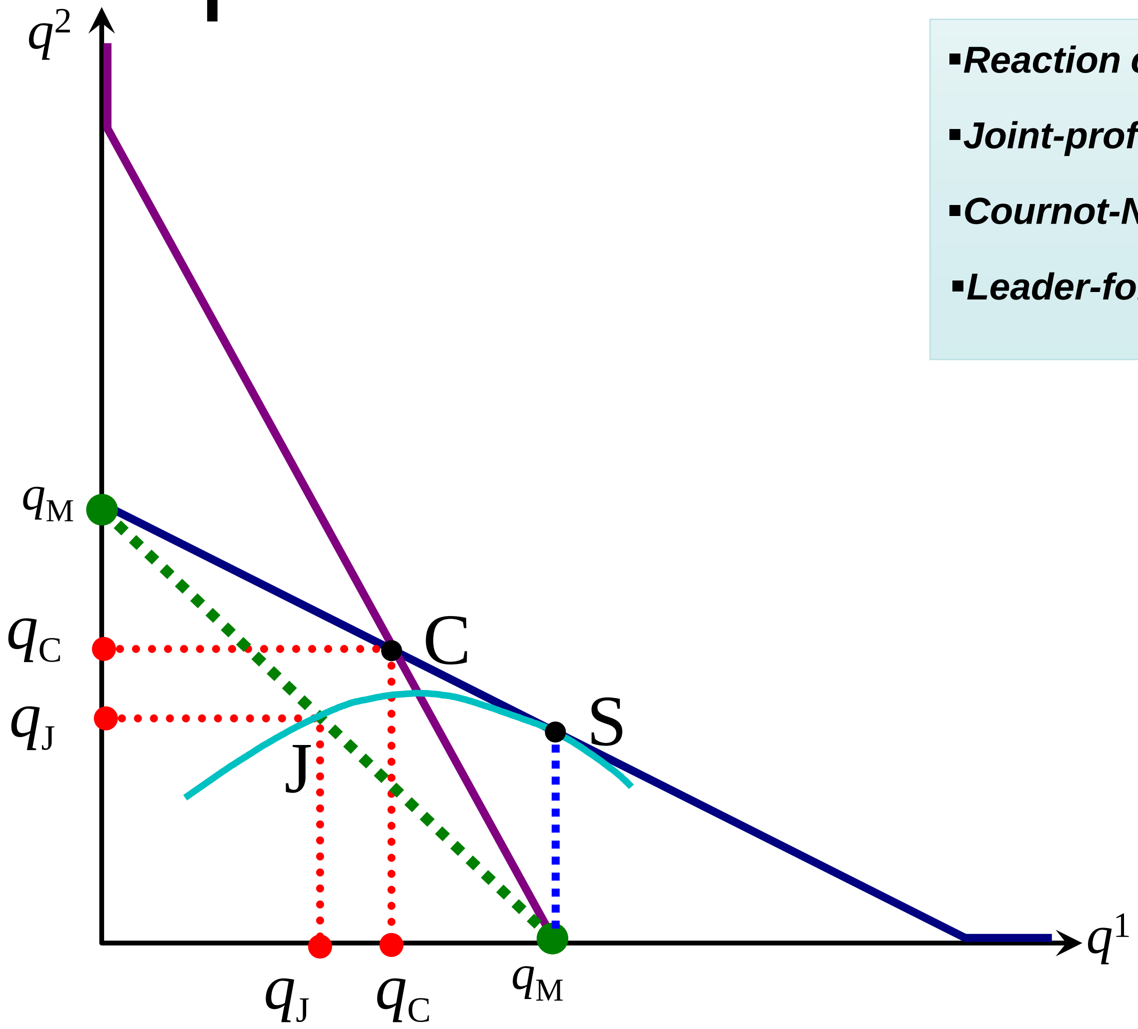
- Background
- Price competition
- Quantity competition
- Assessment

How the simple price- and quantity-models compare

Comparing the models

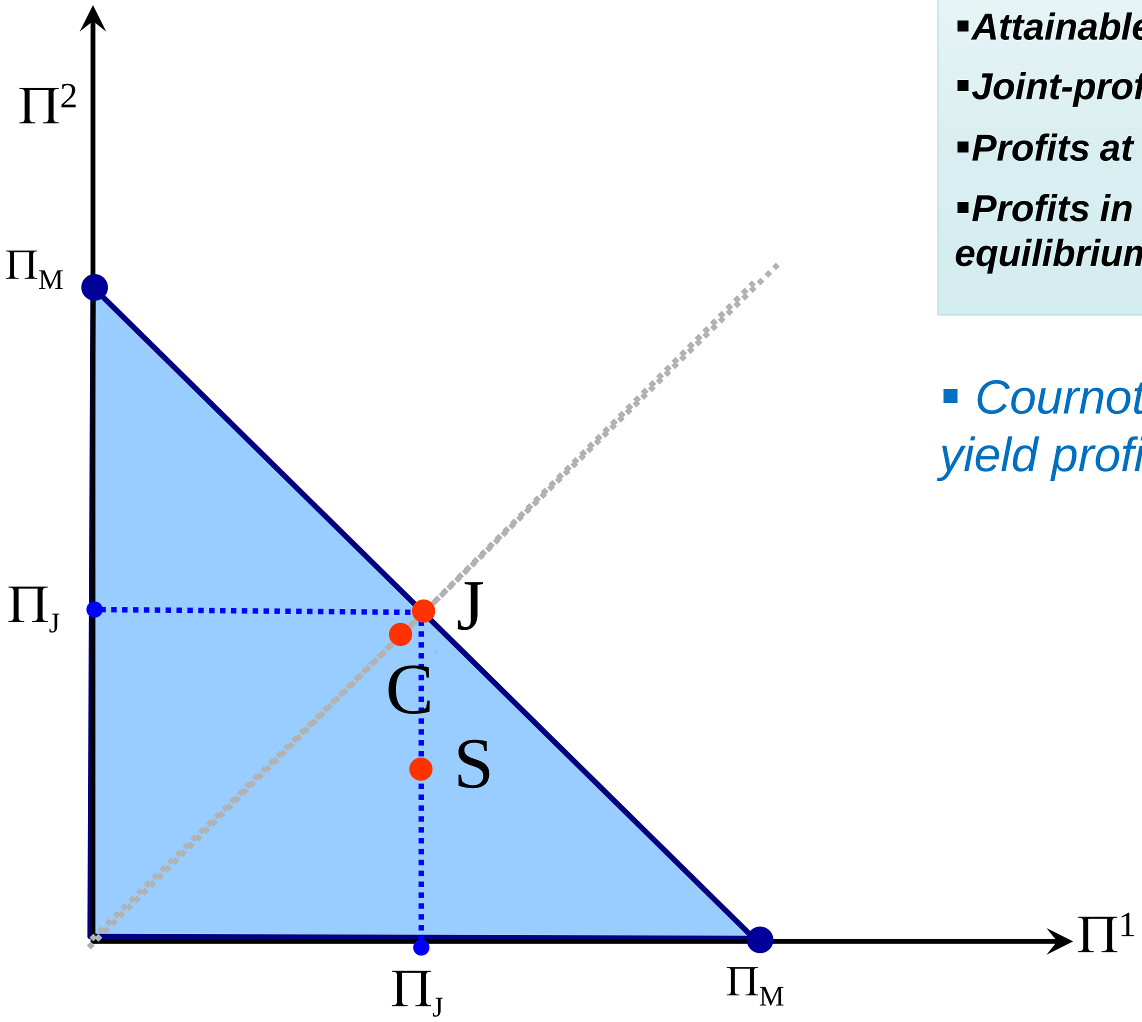
- The price-competition model may seem more “natural”
- But the outcome (p = MC) is surely at variance with everyday experience
- To evaluate the quantity-based models we need to:
  - compare the quantity outcomes of the three versions
  - compare the profits attained in each case

Output under different regimes



- *Reaction curves for the two firms*
- *Joint-profit maximisation with equal outputs*
- *Cournot-Nash equilibrium*
- *Leader-follower (Stackelberg) equilibrium*

Profits under different regimes



- ***Attainable set with transferable profits***
- ***Joint-profit maximisation with equal shares***
- ***Profits at Cournot-Nash equilibrium***
- ***Profits in leader-follower (Stackelberg) equilibrium***

- *Cournot and leader-follower models yield profit levels inside the frontier*

What next?

- Introduce the possibility of entry
- General models of oligopoly
- Dynamic versions of Cournot competition

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