Financial market institutions

12. The Level and Structure of Interest Rates

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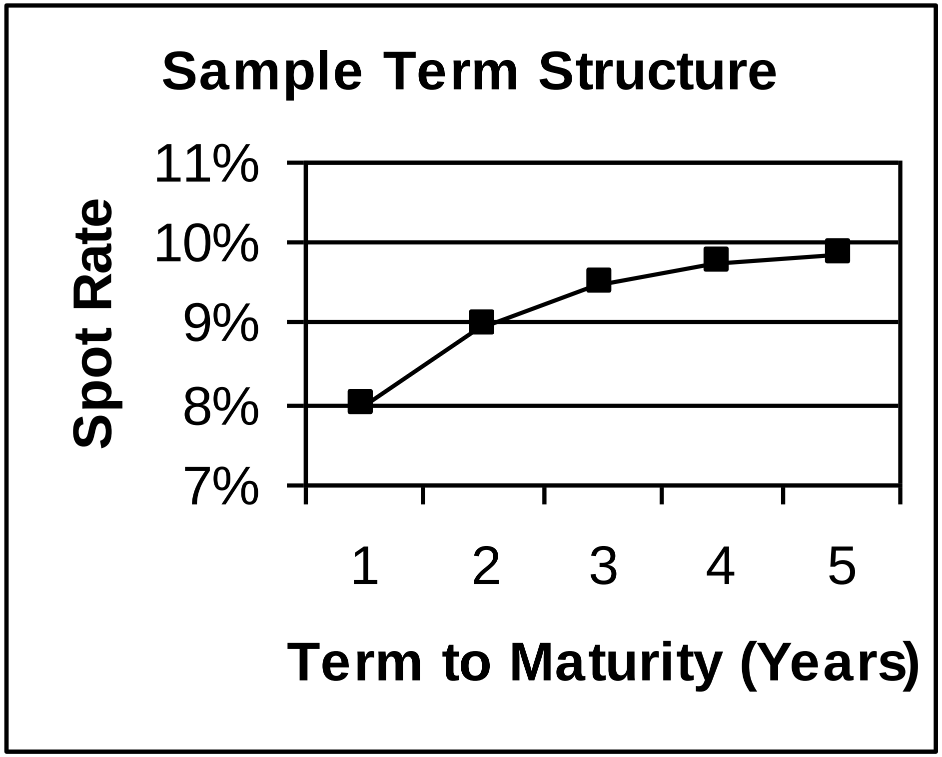
Introduction

- Recall that interest rates are the price of money (borrowing or lending) and, in equilibrium, interest rates equate the amount of borrowing to the amount of saving.
- The term structure of interest rates refers to different interest rates that exist over different term-to-maturity loans.
- In the most basic sense, theories to explain the term structure are still based on interest rates equating the supply and demand for loanable funds.
- Different rates may exist over different terms because of expectations of changing inflation and differing preferences regarding longer-term vs. shorter-term saving.
- Two main theories exist to enrich this explanation and help explain different rates over different maturity terms.

Introduction (continued)



The curve plotted through the above points is also called the “yield curve”

The term structure of interest rates is the relation between different interest rates for different term-to-maturity loans.

If we observe $r_1 = 8\%$, $r_2 = 9\%$, $r_3 = 9.5\%$, $r_4 = 9.75\%$ and $r_5 = 9.875\%$ then the current term structure of interest rates is represented by plotting these “spot rates” against their terms-to-maturity.

Definitions – Spot Rates

- The n-period current spot rate of interest denoted r_n is the current interest rate (fixed today) for a loan (where the cash is borrowed now) to be repaid in n periods. Note: all spot rates are expressed in the form of an effective interest rate per year. In the example above, r_1, r_2, r_3, r_4, and r_5, in the previous slide, are all current spot rates of interest.
- Spot rates are only determined from the prices of zero-coupon bonds and are thus applicable for discounting cash flows that occur in a single time period. This differs from the more broad concept of yield to maturity that is, in effect, an average rate used to discount all the cash flows of a level coupon bond.

Definitions – Forward Rates

- The one-period forward rate of interest denoted f_n is the interest rate (fixed today) for a one period loan to be repaid at some future time period, n.
- I.e., the money is borrowed in period n-1 and repaid in period n.
- E.g., given the data plotted previously, we find f_2 = 10.0092593% is the rate agreed upon today for a loan where the money is borrowed in one year and repaid the following year. Note: forward rates are also expressed as effective interest rates per year.
  - Investing $1,000 in the two year zero coupon bond @ r_2=9% gives $1,188.10 in 2 yrs. This is equivalent to investing in the one year bond at 8%, giving $1,080 after 1 year, and then investing in another 1 year bond @ X% for the second year to get $1,188.10.
  - Solve for X . . . the forward rate.

Definitions – Forward Rates (continued)

- To calculate a forward rate, the following equation is useful:

$$1 + f_n = (1+r_n)^n / (1+r_{n-1})^{n-1}$$

  - where f_n is the one period forward rate for a loan repaid in period n
    - (i.e., borrowed in period n-1 and repaid in period n)
  - Calculate f_2 given r_1=8% and r_2=9%
  - Calculate f_3 given r_3=9.5%

Forward Rates – Self Study

- The t-period forward rate for a loan repaid in period n is denoted $_{n-t}f_n$
  - E.g., $_2f_5$ is the 3-period forward rate for a loan repaid in period 5 (and borrowed in period 2)
- The following formula is useful for calculating t-period forward rates:

$$1+{}_{n-t}f_n = [(1+r_n)^n / (1+r_{n-t})^{n-t}]^{1/t}$$

- Given the data presented before, determine $_1f_3$ and $_2f_5$
- Results: $_1f_3$=10.2577945%; $_2f_5$=10.4622321%

Definitions – Future Spot Rates

- Current spot rates are observable today and can be contracted today.
- A future spot rate will be the rate for a loan obtained in the future and repaid in a later period. Unlike forward rates, future spot rates will not be fixed (or contracted) until the future time period when the loan begins (forward rates can be locked in today).
- Thus we do not currently know what will happen to future spot rates of interest. However, if we understand the theories of the term structure, we can make informed predictions or expectations about future spot rates.
- We denote our current expectation of the future spot rate as follows: $E[{}_{n-t}r_n]$ is the expected future spot rate of interest for a loan repaid in period n and borrowed in period n-t.

Future Spot Rates… Who Cares?

- You are considering locking in your mortgage rate for one year or for five years. You would use the longer term if you thought interest rates would be much higher in one year. I.e., if you expect future spot rates in one year to be much higher, you will choose the longer term mortgage right now … so, yes, it matters for your personal life.
- As a financial manager, you must decide whether your firm should borrow long term or short term. You would prefer to borrow short term as these rates are currently lower, however, you are concerned about what rates you will face when you refinance your loan. I.e., you are concerned about future spot rates at the time you refinance and your expectations will affect your current decision to finance long or short term… so, yes, it matters for the corporation.

Notation Review

r_n	spot rate for a loan negotiated today and repaid in period n
f_n	forward rate for a one-period loan repaid in period n
${}_{n-t}f_n$	forward rate for a t-period loan repaid in period n and borrowed in period n-t
$E[{}_{n-t}r_n]$	our current expectation for the future spot rate of interest for a t-period loan repaid in period n and borrowed (and contracted) in period n-t.

Term Structure Theories: Pure-Expectations Hypothesis

- The Pure-Expectations Hypothesis states that expected future spot rates of interest are equal to the forward rates that can be calculated today (from observed spot rates).
- In other words, the forward rates are unbiased predictors for making expectations of future spot rates.
- What do our previous forward rate calculations tell us if we believe in the Pure- Expectations Hypothesis?

Pure-Expectations Hypothesis (continued)

- Consider that given expectations for inflation over the next year, investors require 4% for a one year loan.
- Suppose investors currently expect inflation for the next year (the second year) to be higher so that they expect to require 6% for a one year loan (starting one year from now).
- Then, the Pure-Expectations Hypothesis, is consistent with the current 2-year spot rate defined as follows:
  - $(1+r_2)^2=(1+r_1)(1+E[{}_1r_2]) = (1.04)x(1.06)$ so $r_2=4.995238\%$
  - Restated, if we observe $r_1=4\%$ and $r_2=4.995238\%$, then, under the Pure-Expectations Hypothesis, we would have $E[{}_1r_2]$ to be 6% (which is equal to f_2).

Liquidity-Preference Hypothesis

- Empirical evidence seems to suggest that investors have relatively short time horizons for bond investments. Thus, since they are risk averse, they will require a premium to invest in longer term bonds.
- The Liquidity-Preference Hypothesis states that longer term loans have a liquidity premium built into their interest rates and thus calculated forward rates will incorporate the liquidity premium and will overstate the expected future one-period spot rates.

Liquidity-Preference Hypothesis

- Reconsider investors' expectations for inflation and future spot rates. Suppose over the next year, investors require 4% for a one year loan and expect to require 6% for a one year loan (starting one year from now).
- Under the Liquidity-Preference Hypothesis, the current 2-year spot rate will be defined as follows:
  - $(1+r_2)^2=(1+r_1)(1+E[{}_1r_2]) + LP_2$ (LP_2 = liquidity premium: assumed to be 0.25% for a 2 year loan) $(1+r_2)^2 = (1.04)\text{x}(1.06) + 0.0025$ so $r_2=5.11422\%$

Liquidity-Preference Hypothesis

- Restated, if we don't know $E[{}_1r_2]$, but we can observe r_1=4% and r_2=5.11422%,
  - then, under the Liquidity-Preference Hypothesis, we would have $E[{}_1r_2] < f_2 = 6.24038\%$.
  - From this example, f_2 overstates $E[{}_1r_2]$ by 0.24038%
  - If we know LP_2 or the amount f_2 overstates $E[{}_1r_2]$, then we can better estimate $E[{}_1r_2]$.

Interpreting the Term Structure

- A flat term structure means constant forward rates equal to today's spot rates and thus …
  - Expectations for the same future spot rates as today if you believe in the Pure-Expectations Hypothesis
  - Expectations for declining future spot rates compared to today if you believe in the Liquidity-Preference Hypothesis
- A declining term structure means declining forward rates and thus …
  - Expectations for similarly declining future spot rates under the Pure-Expectations Hypothesis
  - Expectations for more sharply declining future spot rates under the Liquidity-Preference Hypothesis

Interpreting the Term Structure (continued)

- An increasing term structure means increasing forward rates and thus …
  - Expectations for similarly increasing future spot rates under the Pure-Expectations Hypothesis
  - Expectations for future spot rates that increase to a lesser degree or possibly remain flat or decrease (depending on the size of the Liquidity Premiums) under the Liquidity-Preference Hypothesis

Projecting Future Bond Prices

- Consider a three-year bond with annual coupons (paid annually) of \$100 and a face value of \$1,000 paid at maturity. Spot rates are observed as follows: r_1=9%, r_2=10%, r_3=11%
- What is the current price of the bond?
- What is its yield to maturity (as an effective annual rate)?
- What is the expected price of the bond in 2 years?
  - under the Pure-Expectations Hypothesis
  - under the Liquidity-Preference Hypothesis
    - assume f_3 overstates $E[{}_2r_3]$ by 0.5%

Term Structure of Interest Rates

- Bonds with identical risk, liquidity, and tax characteristics may have different interest rates because the time remaining to maturity is different

Term Structure of Interest Rates

- Yield curve: a plot of the yield on bonds with differing terms to maturity but the same risk, liquidity and tax considerations
  - Upward-sloping: long-term rates are above short-term rates
  - Flat: short- and long-term rates are the same
  - Inverted: long-term rates are below short-term rates

Facts Theory of the Term Structure of Interest Rates Must Explain

1. Interest rates on bonds of different maturities move together over time
2. When short-term interest rates are low, yield curves are more likely to have an upward slope; when short-term rates are high, yield curves are more likely to slope downward and be inverted
3. Yield curves almost always slope upward

Three Theories to Explain the Three Facts

1. Expectations theory explains the first two facts but not the third
2. Segmented markets theory explains fact three but not the first two
3. Liquidity premium theory combines the two theories to explain all three facts

Expectations Theory

- The interest rate on a long-term bond will equal an average of the short-term interest rates that people expect to occur over the life of the long-term bond
- Buyers of bonds do not prefer bonds of one maturity over another; they will not hold any quantity of a bond if its expected return is less than that of another bond with a different maturity
- Bond holders consider bonds with different maturities to be perfect substitutes

Expectations Theory: Example

- Let the current rate on one-year bond be 6%.
- You expect the interest rate on a one-year bond to be 8% next year.
- Then the expected return for buying two one-year bonds averages (6% + 8%)/2 = 7%.
- The interest rate on a two-year bond must be 7% for you to be willing to purchase it.

Expectations Theory

For an investment of $1

i_t = today's interest rate on a one-period bond

i^e_{t+1} = interest rate on a one-period bond expected for next period

i_{2t} = today's interest rate on the two-period bond

Expectations Theory (cont'd)

Expected return over the two periods from investing \$1 in the two-period bond and holding it for the two periods

$$(1 + i_{2t})(1 + i_{2t}) - 1$$

$$= 1 + 2i_{2t} + (i_{2t})^2 - 1$$

$$= 2i_{2t} + (i_{2t})^2$$

Since $(i_{2t})^2$ is very small

the expected return for holding the two-period bond for two periods is

$$2i_{2t}$$

Expectations Theory (cont'd)

If two one-period bonds are bought with the $1 investment

$$(1+i_t)(1+i^e_{t+1})-1$$

$$1+i_t+i^e_{t+1}+i_t(i^e_{t+1})-1$$

$$i_t+i^e_{t+1}+i_t(i^e_{t+1})$$

$i_t(i^e_{t+1})$ is extremely small

Simplifying we get

$$i_t+i^e_{t+1}$$

Expectations Theory (cont'd)

Both bonds will be held only if the expected returns are equal

$$2i_{2t} = i_t + i^e_{t+1}$$

$$i_{2t} = \frac{i_t + i^e_{t+1}}{2}$$

The two-period rate must equal the average of the two one-period rates

For bonds with longer maturities

$$i_{nt} = \frac{i_t + i^e_{t+1} + i^e_{t+2} + \ldots + i^e_{t+(n-1)}}{n}$$

The *n*-period interest rate equals the average of the one-period interest rates expected to occur over the *n*-period life of the bond

Expectations Theory

- Explains why the term structure of interest rates changes at different times
- Explains why interest rates on bonds with different maturities move together over time (fact 1)
- Explains why yield curves tend to slope up when short-term rates are low and slope down when short-term rates are high (fact 2)
- Cannot explain why yield curves usually slope upward (fact 3)

Summary and Conclusions

- The Term Structure of Interest Rates shows the relation between interest rates for different term-to-maturity loans.
- Two theories to explain the Term Structure are the Pure-Expectations Hypothesis and the Liquidity-Preference Hypothesis.
- Empirical evidence is most consistent with the Liquidity-Preference Hypothesis.
- Knowledge of the Term Structure and the theories is useful for predicting future interest rates and future bond prices.
- This is useful for individuals and financial managers when deciding whether long- or short-term loans should be used for financing.

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