MEASURING EFFICIENCY & PRODUCTIVITY

COURSE: INNOVATION DEVELOPMENT IN COMMERCIAL BANKS

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OUTLINE FOR TODAY

- **INTRODUCTION**
- **CONCEPTS**
  - **PRODUCTION TECHNOLOGY**
  - **DISTANCE FUNCTIONS**
    - **OUTPUT AND INPUT ORIENTED DISTANCE FUNCTIONS**
- **TECHNIQUES FOR EFFICIENCY AND PRODUCTIVITY MEASUREMENT:**
  - **INDEX NUMBER METHODS**

Efficiency:

(i) How much more can we produce with a given level of inputs?

(ii) How much input reduction is possible to produce a given level of observed output?

(iii) How much more revenue can be generated with a given level of inputs? Similarly how much reduction in input costs be achieved?

Productivity:

- We wish to measure the level of output per unit of input and compare it with other firms
- Partial productivity measures – output per person employed; output per hour worked; output per hectare etc.
- Total factor productivity measures – Productivity measure which involves all the factors of production
    - More difficult to conceptualise and measure

SIMPLE PERFORMANCE MEASURES

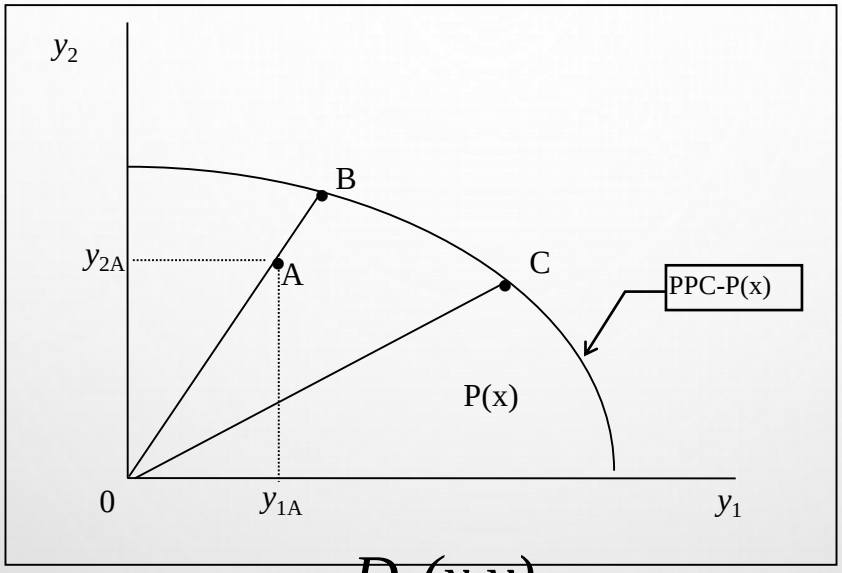
- **CAN BE MISLEADING**
- **CONSIDER TWO CLOTHING FACTORIES (*A* AND *B*)**
- ***LABOUR PRODUCTIVITY* COULD BE HIGHER IN FIRM *A* – BUT WHAT ABOUT USE OF CAPITAL AND ENERGY AND MATERIALS?**
- ***UNIT COSTS* COULD BE LOWER IN FIRM *B* – BUT WHAT IF THEY ARE LOCATED IN DIFFERENT REGIONS AND FACE DIFFERENT INPUT PRICES?**

TERMINOLOGY?

- **THE TERMS *PRODUCTIVITY* AND *EFFICIENCY* RELATE TO SIMILAR (BUT NOT IDENTICAL) THINGS**
- **PRODUCTIVITY = OUTPUT/INPUT**
- **EFFICIENCY GENERALLY RELATES TO SOME FORM BENCHMARK OR TARGET**
- **A SIMPLE EXAMPLE – WHERE FOR FIRM *B* PRODUCTIVITY RISES BUT EFFICIENCY FALLS:**

firm	year	input	output	productivity	efficiency
A	1	2	6	3	0.75
B	1	4	16	4	1.00
A	2	2	8	4	0.67
B	2	5	30	6	1.00

Output Distance Function



$D_o(x,y)$

The value of the distance function is equal to the ratio δ=0A/0B.

Output-oriented Technical Efficiency Measure:

$$TE = 0A/0B = d_o(x,q)$$

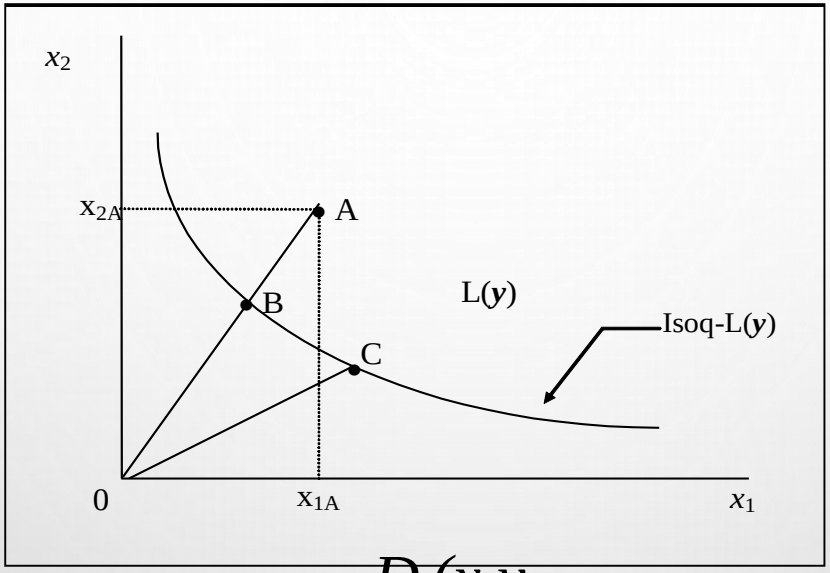
INPUT DISTANCE FUNCTION

- INPUT DISTANCE FUNCTION FOR TWO VECTORS **X** (INPUT)
- AND **Q** (OUTPUT) VECTORS IS DEFINED AS:

$$D_I(\mathbf{X},\mathbf{Q}) = MAX\{\rho: (\mathbf{X}/\rho) \in L(\mathbf{Q})\}$$

- PROPERTIES:
  - NON-NEGATIVE
  - NON-DECREASING IN **X**; NON-INCREASING IN **Q**
  - LINEARLY HOMOGENEOUS IN **X**
  - IF **X** BELONGS TO THE INPUT SET OF **Q** (I.E., $\mathbf{X} \in L(\mathbf{Q})$),
  - THEN $D_I(\mathbf{X},\mathbf{Q}) \geq 1$ AND THE DISTANCE IS EQUAL TO 1
  - ONLY IF **X** IS ON THE FRONTIER.

Input Distance Function



$D_i(x,y$

The value of the distance function is equal to the ratio ρ=0A/0B.

Technical Efficiency = TE = $1/d_i(x,q)$ = OB/OA

INPUT AND OUTPUT DISTANCE FUNCTIONS

- **WHAT IS THE RELATIONSHIP BETWEEN**
- **INPUT AND OUTPUT DISTANCE FUNCTIONS?**
- **IF BOTH INPUTS AND OUTPUTS ARE WEAKLY**
- **DISPOSABLE, WE CAN STATE THAT**

  $D_I(X,Q) \geq 1$ IF AND ONLY IF $D_O(X,Q) \leq 1$.
- **IF THE TECHNOLOGY EXHIBITS GLOBAL**
- **CONSTANT RETURNS TO SCALE THEN WE**
- **CAN STATE THAT:**

  $D_I(X,Q) = 1/D_O(X,Q)$, FOR ALL X AND Q

OBJECTIVES FOR THE FIRM

- **THE PRODUCTION TECHNOLOGY DEFINES**
- **THE TECHNOLOGICAL CONSTRAINT FACED BY THE FIRM**
- **THE OBJECTIVE OF THE FIRM COULD BE TO *MAXIMISE PROFIT***
- **OR *MINIMISE COSTS* WHEN OUTPUTS ARE FIXED**
- **OR *MAXIMISE REVENUE* WHEN INPUTS ARE FIXED**
- **OR ….**

PROFIT MAXIMISATION

- **FIRMS PRODUCE A VECTOR OF *M* OUTPUTS (Q) USING A VECTOR OF *K* INPUTS (X)**
- **THE PRODUCTION TECHNOLOGY (SET) IS:**

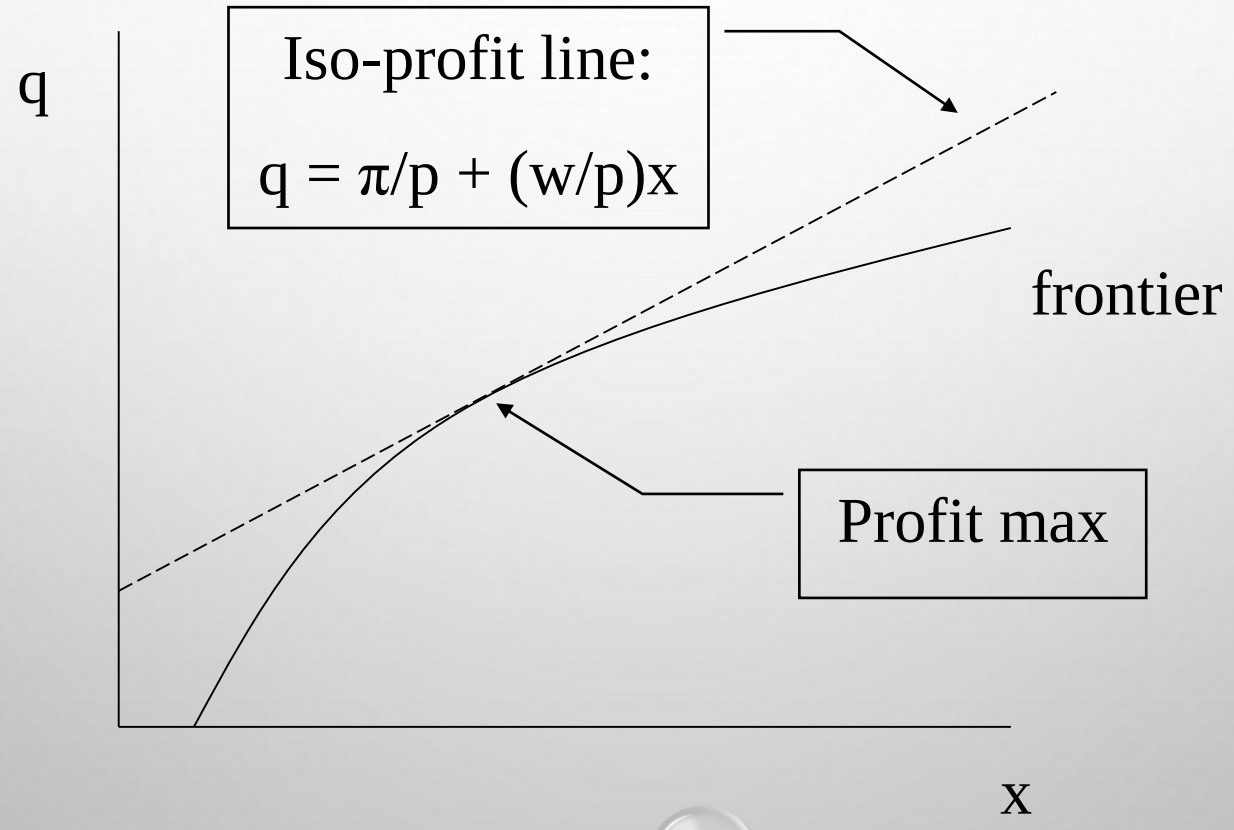
$$S = \{(\mathbf{x},\mathbf{q}) : \mathbf{x} \text{ can produce } \mathbf{q}\}$$

- **MAXIMUM PROFIT IS DEFINED AS:**

$$\pi(\mathbf{p},\mathbf{w}) = \max_{\mathbf{q},\mathbf{x}} \{(\mathbf{p}'\mathbf{q} - \mathbf{w}'\mathbf{x}) : (\mathbf{x},\mathbf{q}) \in S\}$$

WHERE P IS A VECTOR OF *M* OUTPUT PRICES AND W IS A VECTOR OF *K* INPUT PRICES

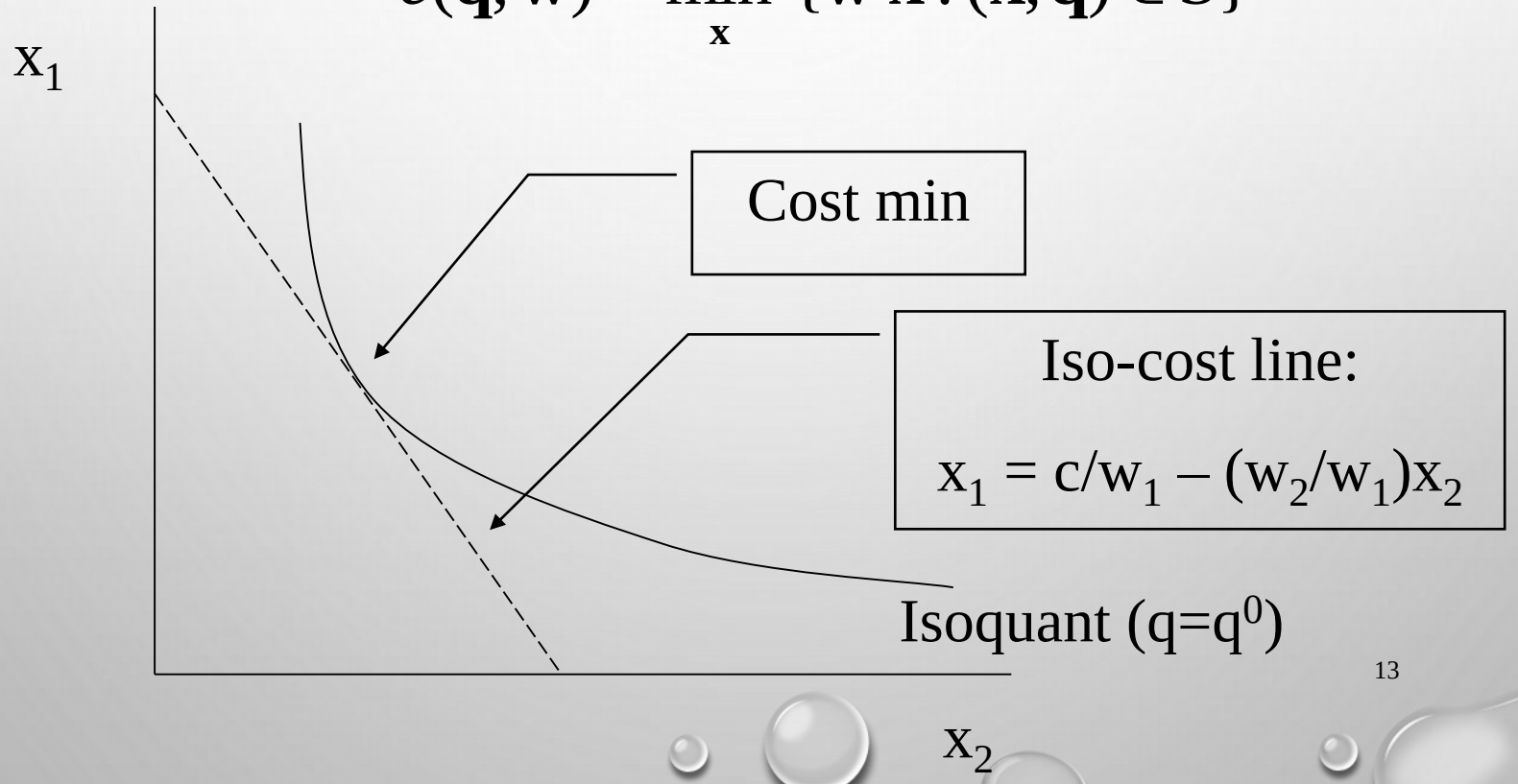
PROFIT MAXIMISATION



COST MINIMISATION

- **THE FIRM MUST PRODUCE OUTPUT, Q^0**
- **MINIMUM COST IS DEFINED AS:**

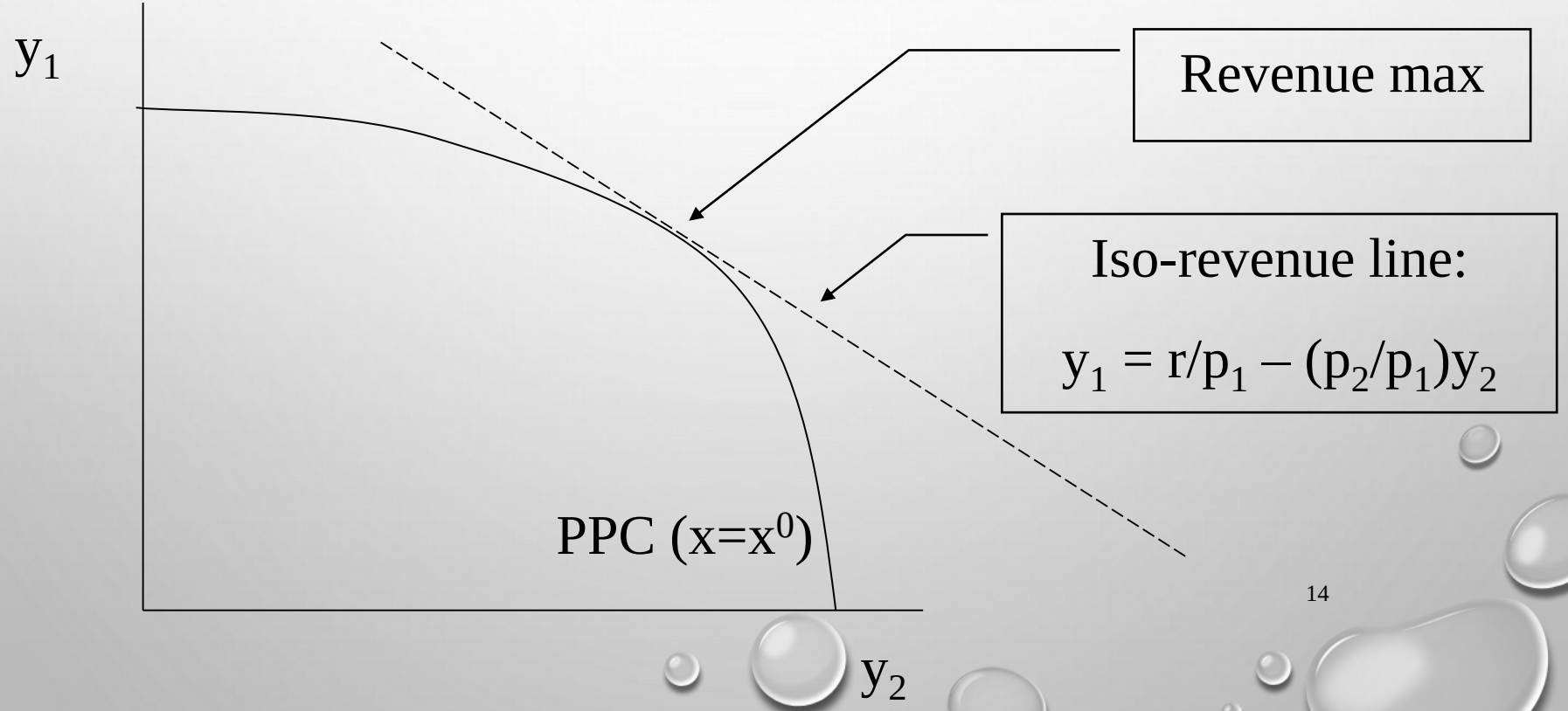
$$c(\mathbf{q}, \mathbf{w}) = \min_{\mathbf{x}} \{\mathbf{w}'\mathbf{x} : (\mathbf{x}, \mathbf{q}) \in S\}$$



REVENUE MAXIMISATION

- THE FIRM HAS INPUT ALLOCATION, X^0
- MAXIMUM REVENUE IS DEFINED AS:

$$r(\mathbf{p},\mathbf{x}) = \max_{\mathbf{q}} \{\mathbf{p}'\mathbf{q} : (\mathbf{x},\mathbf{q}) \in S\}$$



SHORT VERSUS LONG RUN

- **IN THE LONG RUN ALL THINGS CAN VARY**
- **IN THE SHORT RUN SOME THINGS ARE FIXED**
- **COST MIN CAN BE VIEWED AS PROFIT MAX IN THE SHORT RUN WHEN OUTPUTS ARE FIXED**
- **REVENUE MAX CAN BE VIEWED AS PROFIT MAX IN THE SHORT RUN WHEN INPUTS ARE FIXED**
- **ONE CAN ALSO FIX A SUBSET OF INPUTS (E.G., CAPITAL) AND LOOK AT SHORT RUN PROFIT MAX OR SHORT RUN COST MIN, ETC.**

Production function $q = f(\mathbf{x})$

Marginal product $MP_n = \frac{\partial f(\mathbf{x})}{\partial x_n}$

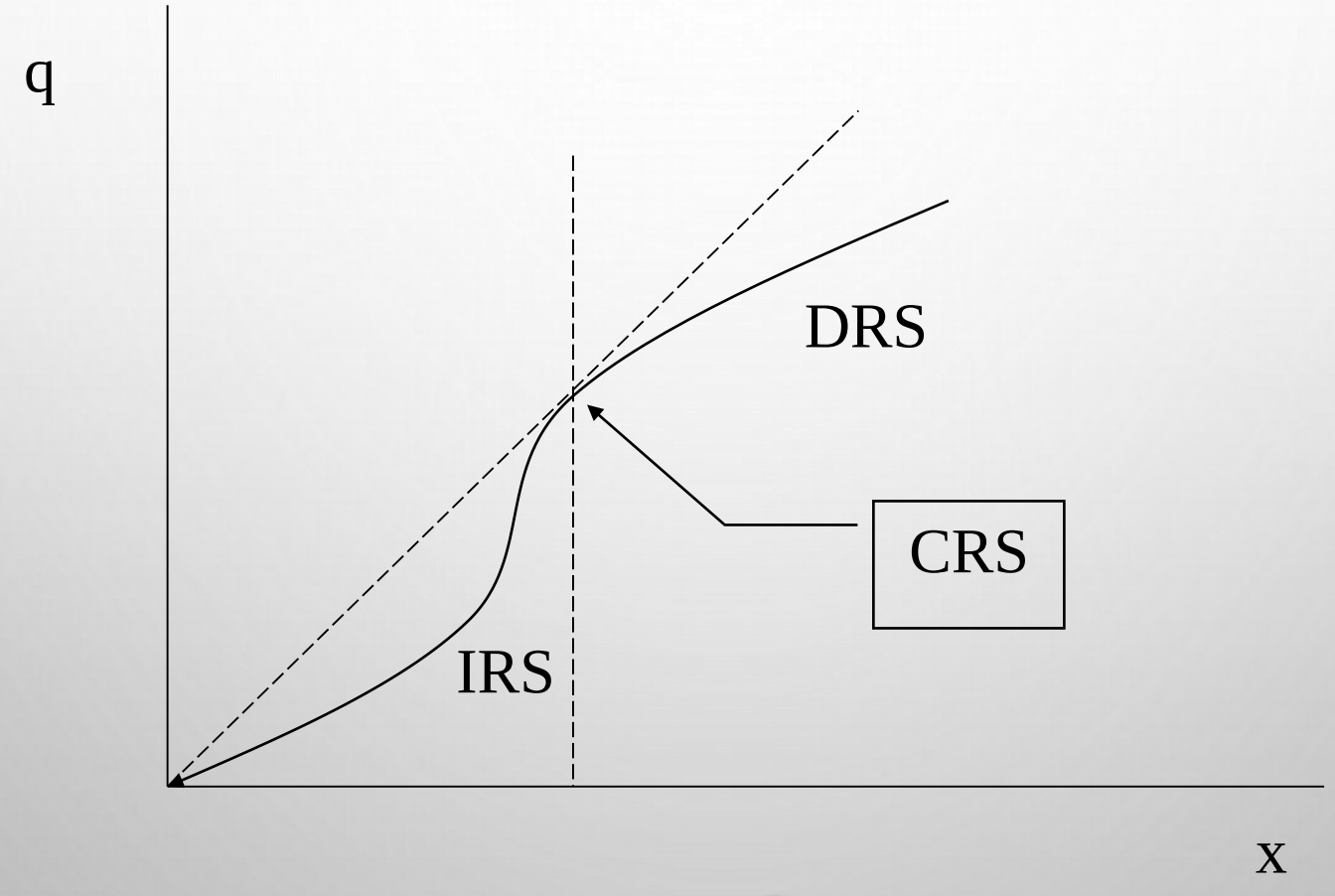
Production elasticity $E_n = \frac{\partial f(\mathbf{x})}{\partial x_n} \frac{x_n}{q}$

Scale elasticity $\varepsilon = \sum_{n=1}^{N} E_n$

RETURNS TO SCALE

- **A PRODUCTION TECHNOLOGY EXHIBITS *CONSTANT RETURNS TO SCALE* (CRS) IF A *Z*% INCREASE IN INPUTS RESULTS IN *Z*% INCREASE IN OUTPUTS (E = 1).**
- **A PRODUCTION TECHNOLOGY EXHIBITS *INCREASING RETURNS TO SCALE* (IRS) IF A *Z*% INCREASE IN INPUTS RESULTS IN A MORE THAN *Z*% INCREASE IN OUTPUTS (E > 1).**
- **A PRODUCTION TECHNOLOGY EXHIBITS *DECREASING RETURNS TO SCALE* (DRS) IF A *Z*% INCREASE IN INPUTS RESULTS IN A LESS THAN *Z*% INCREASE IN OUTPUTS (E < 1).**

RETURNS TO SCALE



ECONOMIES OF SCOPE

- **IS IT LESS COSTLY TO PRODUCE *M* DIFFERENT PRODUCTS IN ONE FIRM VERSUS IN *M* FIRMS?**
- **ONE MEASURE OF ECONOMIES OF SCOPE IS:**

$$S = \frac{\sum_{m=1}^{M} c(\mathbf{w}, q_m) - c(\mathbf{w}, \mathbf{q})}{c(\mathbf{w}, \mathbf{q})}$$

- ***S* > 0 IMPLIES ECONOMIES OF SCOPE – IT IS BETTER TO PRODUCE THE *M* OUTPUTS IN ONE FIRM.**
- **OTHER MEASURES:**
  - **PRODUCT SPECIFIC MEASURES**
  - **SECOND DERIVATIVE MEASURES**

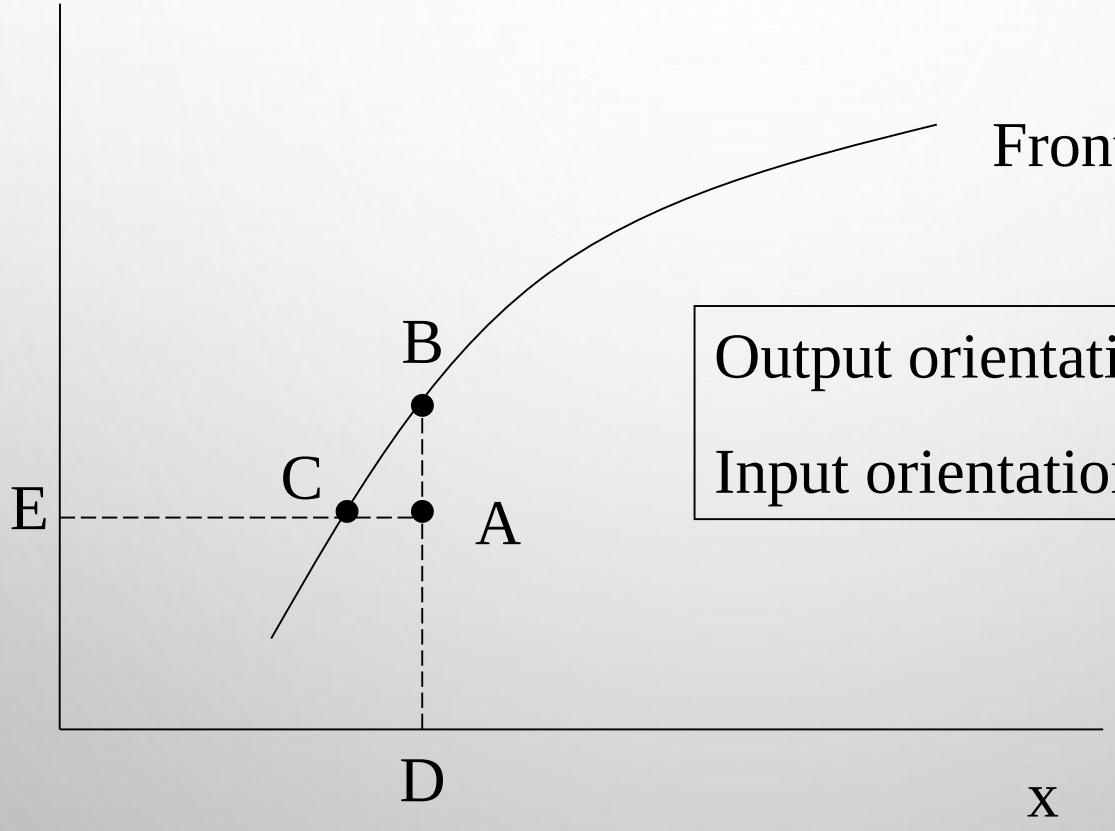
EFFICIENCY MEASURES

- **USING THE DISTANCE FUNCTIONS DEFINED**
- **SO FAR, WE CAN DEFINE:**
  - ***TECHNICAL EFFICIENCY***
  - ***ALLOCATIVE EFFICIENCY***
  - ***ECONOMIC EFFICIENCY***
- **A FIRM IS SAID TO BE *TECHNICALLY EFFICIENT***
- **IF IT OPERATES ON THE FRONTIER OF THE PRODUCTION TECHNOLOGY**
- **A FIRM IS SAID TO BE *ALLOCATIVELY EFFICIENT***
- **IF IT MAKES EFFICIENT ALLOCATION IN TERMS OF**
- **CHOOSING OPTIMAL INPUT AND OUTPUT COMBINATIONS.**
- **A FIRM IS SAID TO BE *ECONOMICALLY EFFICIENT* IF**
- **IT IS BOTH TECHNICALLY AND ALLOCATIVELY EFFICIENT.**

PRODUCTIVITY AND EFFICIENCY CONCEPTS

- **CONCEPTS**
  - **TECHNICAL EFFICIENCY**
  - **SCALE EFFICIENCY**
  - **ALLOCATIVE EFFICIENCY**
  - **COST EFFICIENCY**
  - **REVENUE EFFICIENCY**
  - **TOTAL FACTOR PRODUCTIVITY (TFP)**
- **BRIEF OVERVIEW OF EMPIRICAL METHODS**

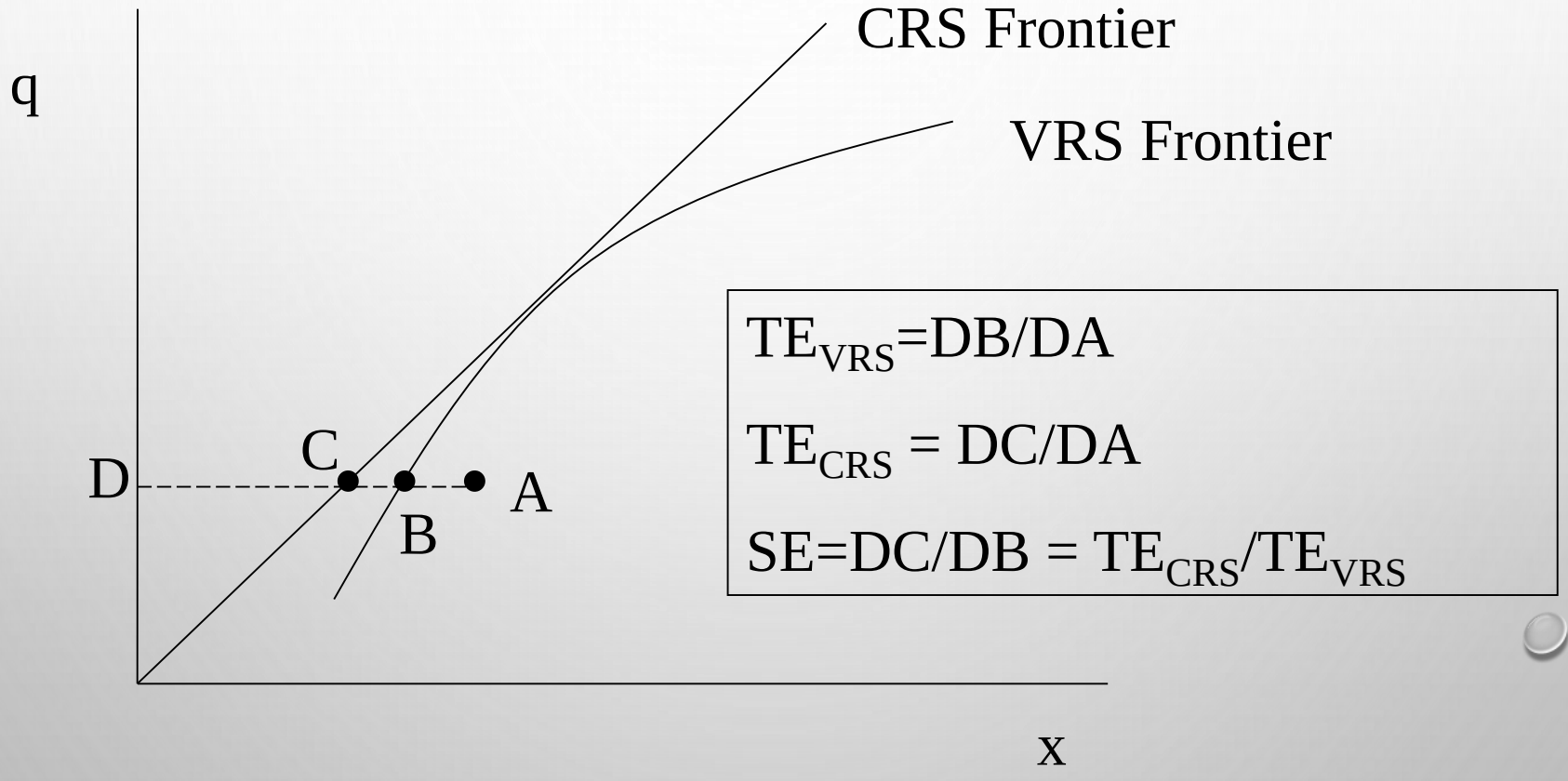
TECHNICAL EFFICIENCY



Output orientation: $TE_O = DA/DB$

Input orientation: $TE_I = EC/EA$

SCALE EFFICIENCY

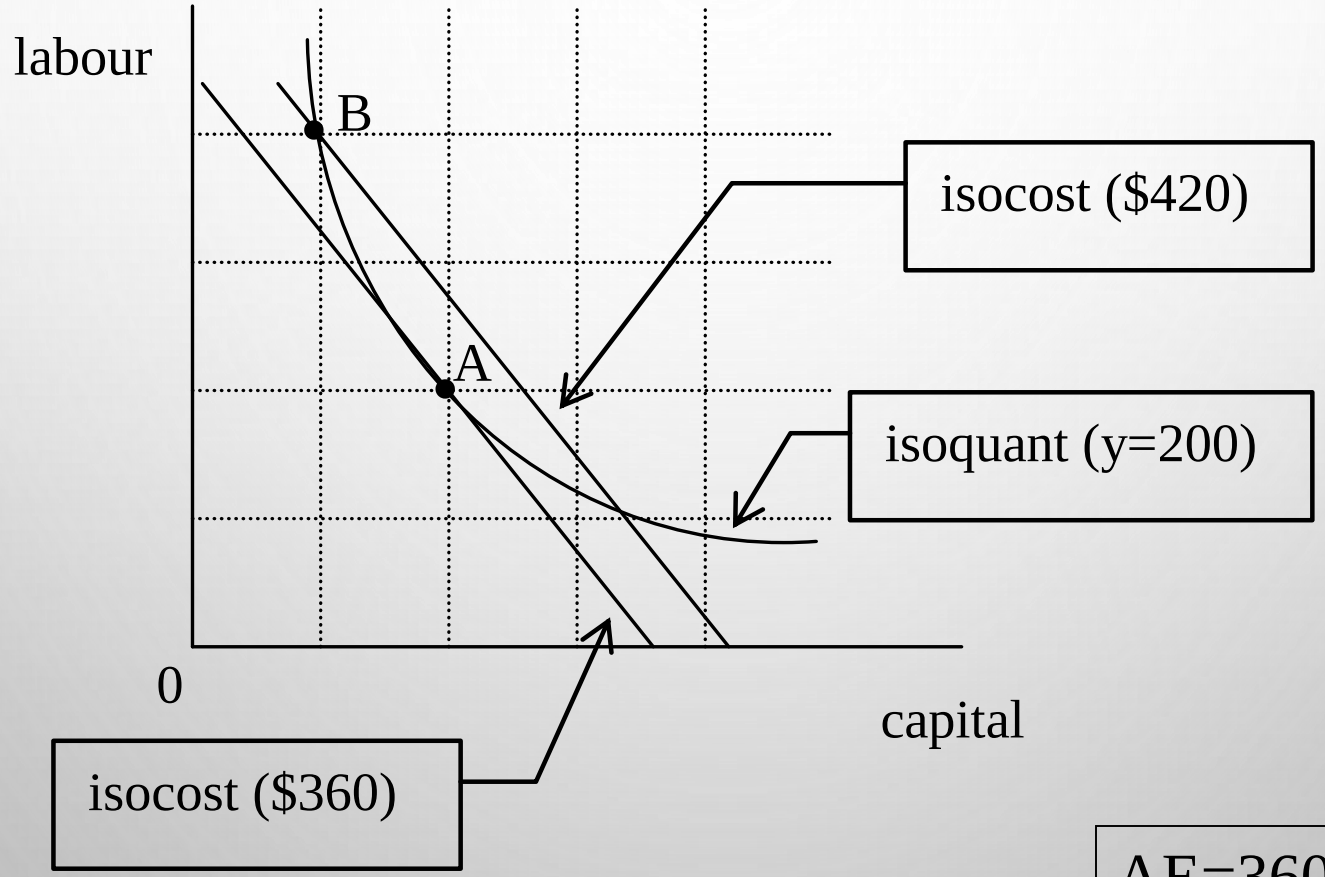


TE_{VRS}=DB/DA

TE_{CRS} = DC/DA

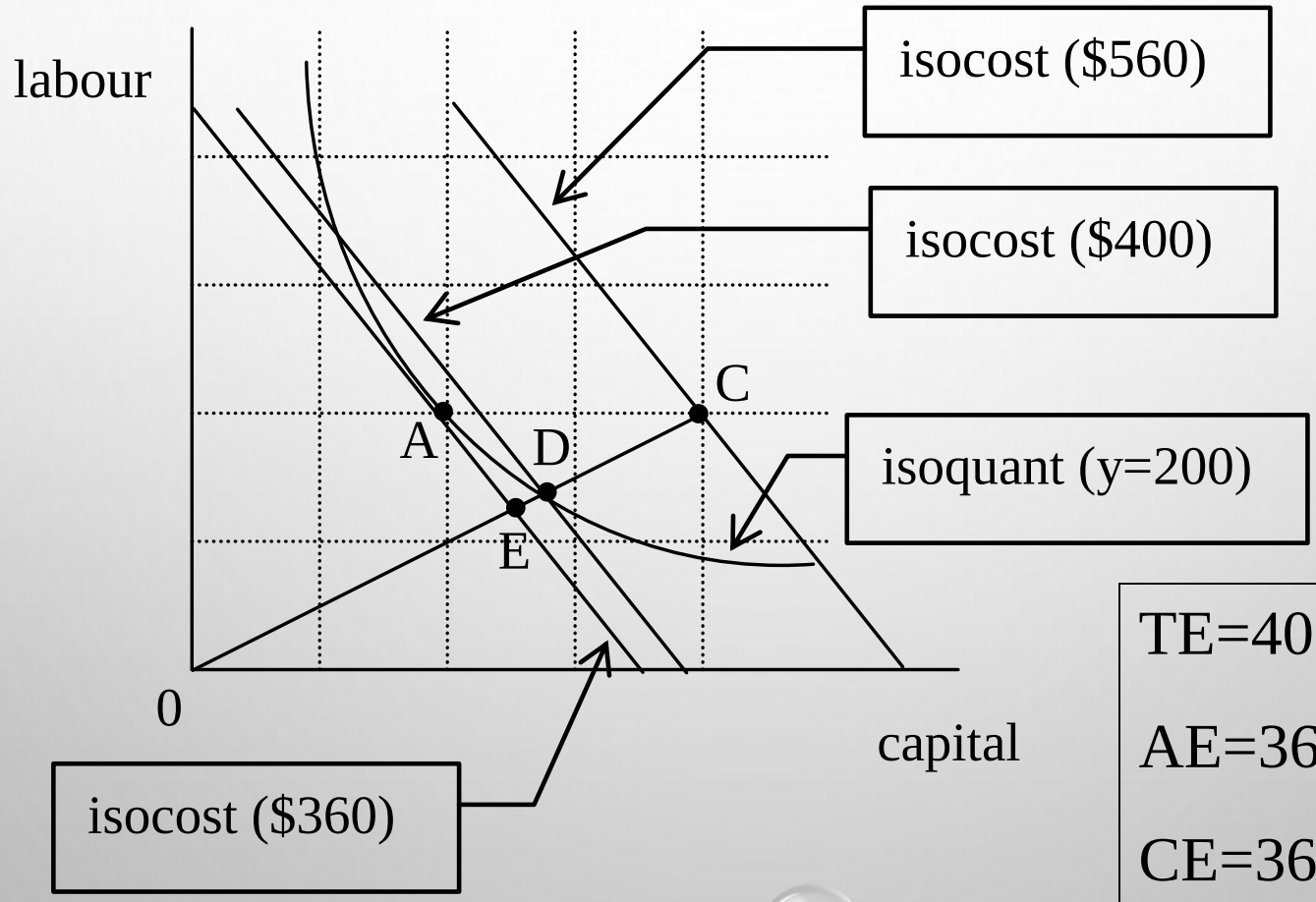
SE=DC/DB = TE_{CRS}/TE_{VRS}

ALLOCATIVE EFFICIENCY



AE=360/420=0.86

ALLOCATIVE EFFICIENCY (2)

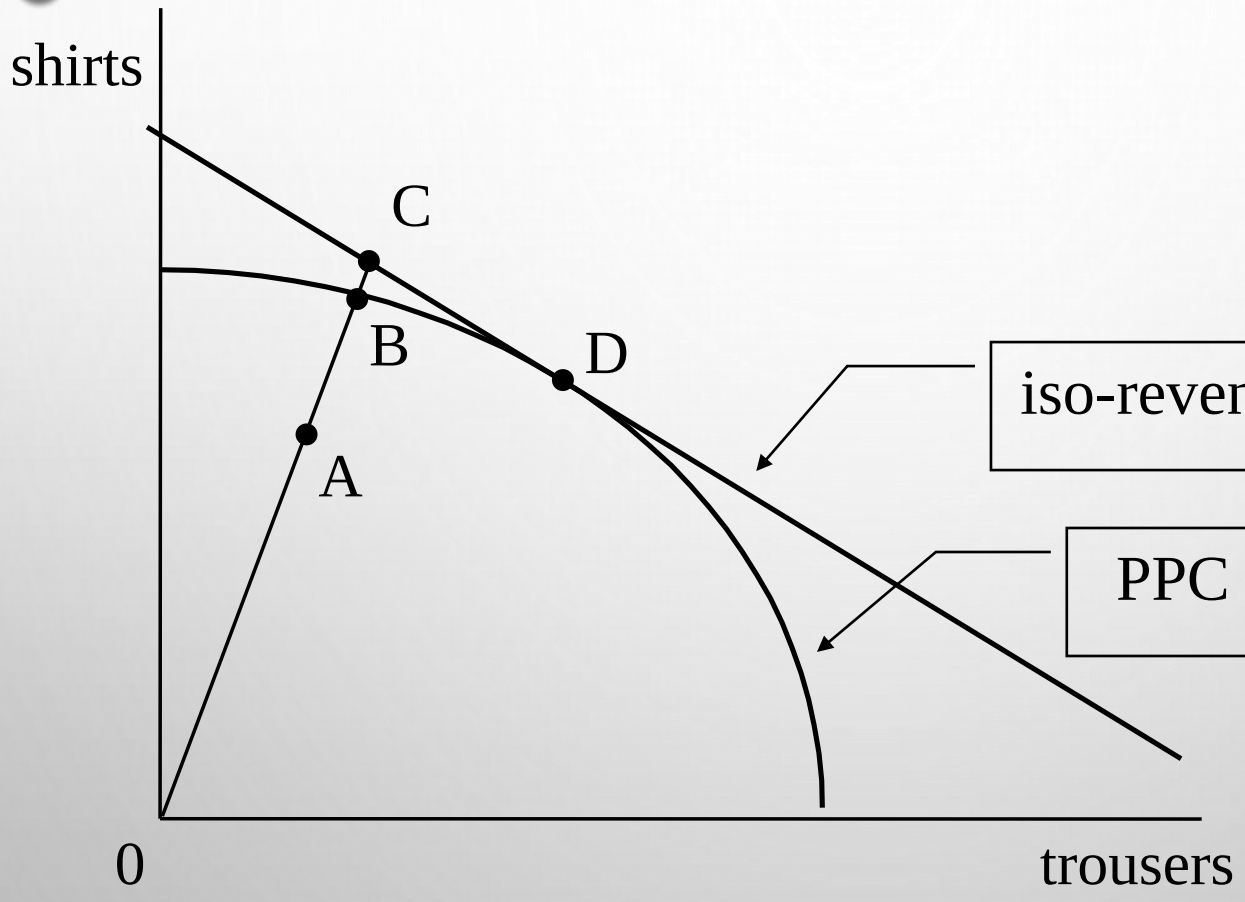


TE=400/560=0.71

AE=360/400=0.9

CE=360/560=0.64

OUTPUT ORIENTATED EFFICIENCY



$TE_O = 0A/0B$

$AE_O = 0B/0C$

$RE = 0A/0C$

$= TE_O \times AE_O$

PRODUCTIVITY?

- **PRODUCTIVITY = OUTPUT/INPUT**
- **WHAT TO DO IF WE HAVE MORE THAN ONE INPUT AND/OR OUTPUT?**
  - **PARTIAL PRODUCTIVITY MEASURES**
  - **AGGREGATION**

EXAMPLE

- **TWO FIRMS PRODUCING T-SHIRTS USING LABOUR AND CAPITAL (MACHINES).**
- **THE PARTIAL PRODUCTIVITY RATIOS CONFLICT.**

firm	labour (x_1)	capital (x_2)	output (q)	q/x_1	q/x_2
A	2	2	200	100	100
B	4	1	200	50	200

TOTAL FACTOR PRODUCTIVITY (TFP)

- Use an aggregate measure of input:

  $TFP = y/(a_1x_1+a_2x_2)$

- What should we use as the weights? – prices?
- Data: Labour wage = \$80 per day and

  Rental price of the machines = \$100 per day
- Calculation:

  $TFP_A = 200/(80\times2+100\times2) = 200/360 = 0.56$

  $TFP_B = 200/(80\times4+100\times1) = 200/420 = 0.48$

  =>A is more productive using this measure.

TFP DECOMPOSITION

- **CAN DECOMPOSE TFP DIFFERENCE BETWEEN 2 FIRMS (AT ONE POINT IN TIME) INTO 3 TYPES OF EFFICIENCY:**
  - **TECHNICAL EFFICIENCY;**
  - **ALLOCATIVE EFFICIENCY; AND**
  - **SCALE EFFICIENCY.**
- **NEED TO KNOW THE TECHNOLOGY**

TFP GROWTH COMPONENTS

- **TECHNICAL CHANGE (TC)**
- **TECHNICAL EFFICIENCY CHANGE (TEC)**
- **SCALE EFFICIENCY CHANGE (SEC)**
- **ALLOCATIVE EFFICIENCY CHANGE (AEC)**

HOW DO WE MEASURE EFFICIENCY?

- **DEPENDS UPON THE TYPE OF DATA AVAILABLE**
- **FOR THE MEASUREMENT PURPOSE.**
- **THREE TYPES:**
  - **OBSERVED INPUT AND OUTPUT DATA FOR A GIVEN**
  - **FIRM OVER TWO PERIODS OR DATA FOR A FEW FIRMS**
  - **AT A GIVEN POINT OF TIME;**
  - **OBSERVED INPUT AND OUTPUT DATA FOR A LARGE**
  - **SAMPLE OF FIRMS FROM A GIVEN INDUSTRY (CROSS-SECTIONAL DATA)**
  - **PANEL DATA ON A CROSS-SECTION OF FIRMS OVER TIME**
- **IN THE FIRST CASE MEASUREMENT IS LIMITED TO**
- **PRODUCTIVITY MEASUREMENT BASED ON RESTRICTIVE ASSUMPTIONS.**

OVERVIEW OF METHODS

- **INDEX NUMBERS (IN)**
  - **PRICE AND QUANTITY INDEX NUMBERS USED IN AGGREGATION (EG. TORNQVIST, FISHER)**
- **DATA ENVELOPMENT ANALYSIS (DEA)**
  - **NON-PARAMETRIC, LINEAR PROGRAMMING**
- **STOCHASTIC FRONTIER ANALYSIS (SFA)**
  - **PARAMETRIC, ECONOMETRIC**

RELATIVE MERITS OF INDEX NUMBERS

- **ADVANTAGES:**
  - **ONLY NEED 2 OBSERVATIONS**
  - **TRANSPARENT AND REPRODUCIBLE**
  - **EASY TO CALCULATE**
- **DISADVANTAGES:**
  - **NEED PRICE INFORMATION**
  - **CANNOT DECOMPOSE**

RELATIVE MERITS OF FRONTIER METHODS

- **DEA ADVANTAGES:**
  - **NO NEED TO SPECIFY FUNCTIONAL FORM OR DISTRIBUTIONAL FORMS FOR ERRORS**
  - **EASY TO ACCOMMODATE MULTIPLE OUTPUTS**
  - **EASY TO CALCULATE**
- **SFA ADVANTAGES:**
  - **ATTEMPTS TO ACCOUNT FOR DATA NOISE**
  - **CAN CONDUCT HYPOTHESIS TESTS**

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