

DEA RESULTS & ANALYSIS

COURSE: INNOVATION DEVELOPMENT IN
COMMERCIAL BANKS

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Data Envelopment Analysis

- It is a non-parametric technique
 - Makes no assumptions about the form of the production technology or function
- It is a non-stochastic approach
 - All the observations are treated as non-stochastic
- The name of the technique is because we try to build a frontier by enveloping all the observed input-output vectors
 - Efficiency of each firm is measured by the distance of its input-output vectors to the frontier
- It fits a piece-wise linear frontier using a linear programming technique
- The method is an extension of the Free-Disposal-Hull technique – it imposes convexity

Data Envelopment Analysis

- **Farrell (1957) suggested a linear convex hull approach to frontier estimation**
- **Boles (1966) and Afriat (1978) suggested the use of mathematical programming approach**
- **Charnes, Cooper and Rhodes (1978) coined the term *data envelopment analysis* (DEA). Proposed an input orientation with CRS**
- **Banker, Charnes and Cooper (1984) proposed VRS model.**
- **Good review of the method in Seiford (1996), Charnes et al. (1995), Lovell (1994) and Seiford and Thrall (1990)**

DEA

- **Need data on input and output quantities of each firm**
- **Can handle multiple inputs and multiple outputs**
- **Linear programming (LP) used to construct a non-parametric piece-wise surface over the data**
- **Need to solve one LP for each DMU involved**
- **TE = distance each firm is below this surface**
- **Input-orientated model looks at the amount by which inputs can be proportionally reduced, with outputs fixed**
- **Output-orientated model looks at the amount by which outputs can be proportionally expanded, with inputs fixed**
- **DEA can be conducted under the assumption of *constant returns to scale (CRS)* or *variable returns to scale (VRS)***

The input-orientated CRS model

$$\min_{\theta, \lambda} \theta,$$

$$\text{st} \quad -\mathbf{q}_i + \mathbf{Q}\lambda \geq \mathbf{0},$$

$$\theta\mathbf{x}_i - \mathbf{X}\lambda \geq \mathbf{0},$$

$$\lambda \geq \mathbf{0},$$

Notation: K inputs, M outputs, I firms

\mathbf{x}_i is $K \times 1$ vector of inputs of i -th firm

\mathbf{q}_i is $M \times 1$ vector of outputs of i -th firm

\mathbf{X} is a $K \times I$ input matrix, \mathbf{Q} is a $M \times I$ output matrix

θ is a scalar (=TE), λ is a $I \times 1$ vector of constants

Intuitive interpretation

- The problem takes the i -th firm and then seeks to radially contract the input vector x_i as much as possible
- The inner-boundary is a piece-wise linear isoquant determined by the observed data points
- The radial contraction of the input vector x_i produces a projected point (X^λ, Q^λ) on the surface of this technology
- The projected point is a linear combination of the observed data points
- The constraints ensure that this projected point cannot lie outside the feasible set
- θ is the technical efficiency score and in the range 0 to 1. A score of 1 implies that the DMU is on the frontier.

The Dual DEA LP

$$\max_{\mu, \nu} (\boldsymbol{\mu}' \mathbf{q}_i),$$

$$\text{st } \boldsymbol{\nu}' \mathbf{x}_i = 1,$$

$$\boldsymbol{\mu}' \mathbf{q}_j - \boldsymbol{\nu}' \mathbf{x}_j \leq \mathbf{0}, \quad j=1,2,\dots,I,$$

$$\boldsymbol{\mu}, \boldsymbol{\nu} \geq \mathbf{0},$$

where

$\boldsymbol{\mu} = M \times 1$ vector of output weights

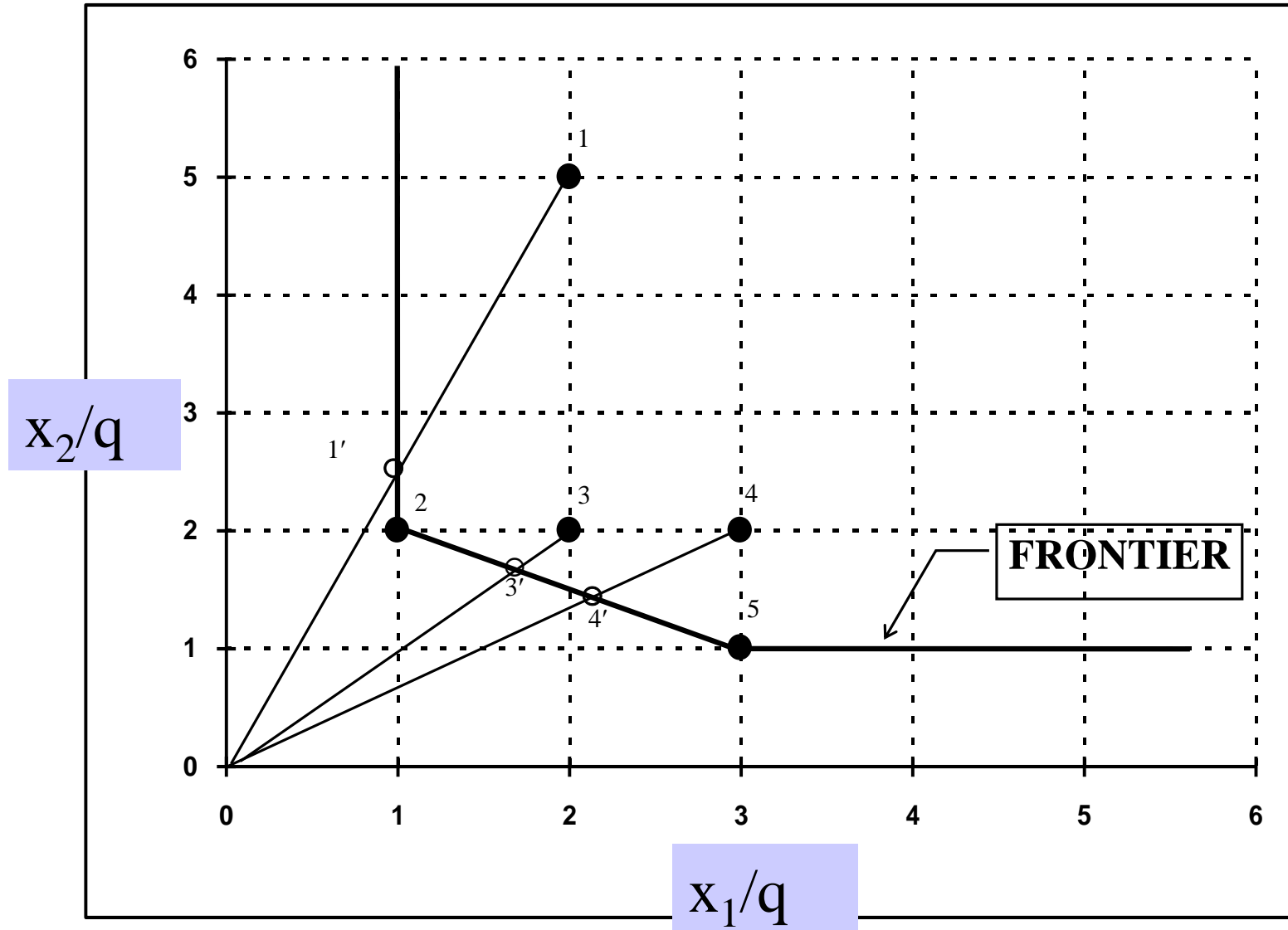
$\boldsymbol{\nu} = K \times 1$ vector of input weights

- Identical TE scores: $TE_i = \boldsymbol{\mu}' \mathbf{q}_i / \boldsymbol{\nu}' \mathbf{x}_i$
- More constraints (I versus $M+K$)
- Shadow price interpretation

A Simple Numerical Example

firm	q	x₁	x₂	x₁/q	x₂/q
1	1	2	5	2	5
2	2	2	4	1	2
3	3	6	6	2	2
4	1	3	2	3	2
5	2	6	2	3	1

A Simple Numerical Example



CRS – DEA Results

firm	θ	λ_1	λ_2	λ_3	λ_4	λ_5	IS ₁	IS ₂	OS
1	0.5	-	0.5	-	-	-	-	0.5	-
2	1.0	-	1.0	-	-	-	-	-	-
3	0.833	-	1.0	-	-	0.5	-	-	-
4	0.714	-	0.214	-	-	0.286	-	-	-
5	1.0	-	-	-	-	1.0	-	-	-

- **The first firm has an efficiency score of 0.5. The technically efficient frontier has a slack on input 2 – input two could be reduced by 0.5 without affecting the output.**
- **Firm 1 has only one peer, firm 1.**
- **Firm 2 is a peer for firms 1, 3 and 4 where as firm 5 is a peer for firms 3 and 4.**
- **There are no input slacks for firms 3 and 4.**

LP for firm number 3

$$\min_{\theta, \lambda} \theta$$

st

$$-q_3 + (q_1\lambda_1 + q_2\lambda_2 + q_3\lambda_3 + q_4\lambda_4 + q_5\lambda_5) \geq 0$$

$$\theta x_{13} - (x_{11}\lambda_1 + x_{12}\lambda_2 + x_{13}\lambda_3 + x_{14}\lambda_4 + x_{15}\lambda_5) \geq 0$$

$$\theta x_{23} - (x_{21}\lambda_1 + x_{22}\lambda_2 + x_{23}\lambda_3 + x_{24}\lambda_4 + x_{25}\lambda_5) \geq 0$$

$$\lambda \geq 0$$

where $\lambda = (\lambda_1, \lambda_2, \lambda_3, \lambda_4, \lambda_5)'$

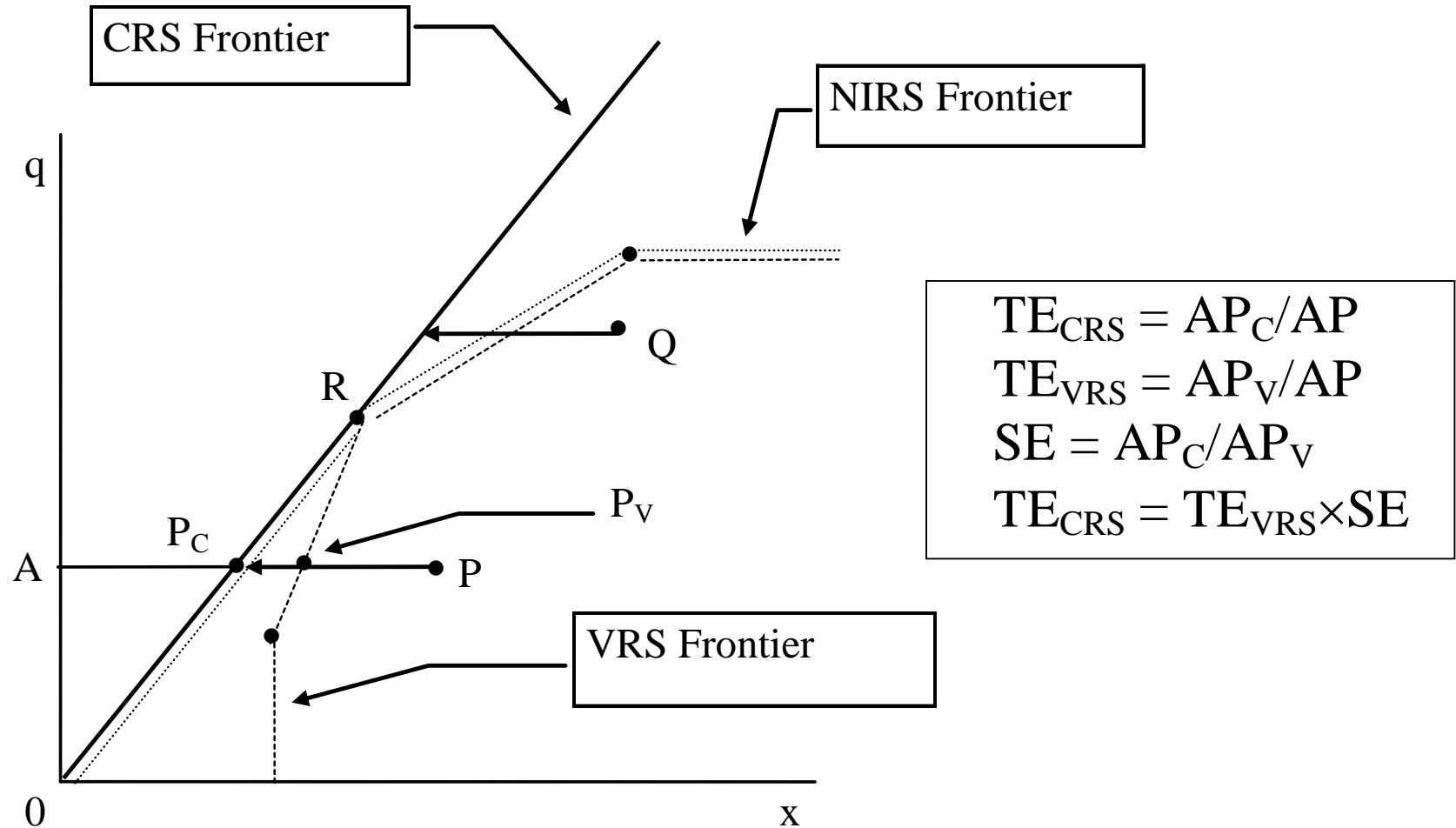
The Variable Returns to Scale (VRS) DEA Model

- **The CRS assumption is only appropriate when all firms are operating at an optimal scale**
- **The use of the CRS specification when all firms are not operating at the optimal scale results in measures of TE which are confounded by scale efficiencies (SE)**
- **The use of the VRS specification permits the calculation of TE devoid of these SE effects**
- **SE can be calculated by estimating both the CRS and VRS models and looking at the difference in scores**
- **VRS model is essentially the CRS with an additional constraint added to the LP problem**

Measuring Scale Efficiency

- Along with the information that a firm is technically inefficient, we would like to know if the firm is too large or too small.
- This information can be obtained by examining scale efficiency.
- Scale efficiency is measured by running DEA under two different scenarios:
 - Run DEA with constant returns to scale (CRS) – run the LP problems listed before.
 - Run DEA with variable returns to scale (VRS) – run the same LP problems with an additional constraint: $\mathbf{1}'\lambda = 1$. That is sum of λ 's is equal to 1.
 - Ratios of TE scores under the two LPs above provide a measure of scale efficiency.
- In order to know if the firm is too large or small, we need to run another VRS DEA with the constraint $\mathbf{1}'\lambda \leq 1$. This problem is known as DEA with *non-increasing returns to scale*.

Calculation of Scale Efficiency in DEA



Calculation of Scale Efficiency in DEA

VRS

$$\min_{\theta, \lambda} \theta,$$

$$\text{st} \quad -\mathbf{q}_i + \mathbf{Q}\lambda \geq \mathbf{0},$$

$$\theta \mathbf{x}_i - \mathbf{X}\lambda \geq \mathbf{0},$$

$$\mathbf{N}\mathbf{1}'\lambda = 1$$

$$\lambda \geq \mathbf{0}$$

NIRS

$$\min_{\theta, \lambda} \theta,$$

$$\text{st} \quad -\mathbf{q}_i + \mathbf{Q}\lambda \geq \mathbf{0},$$

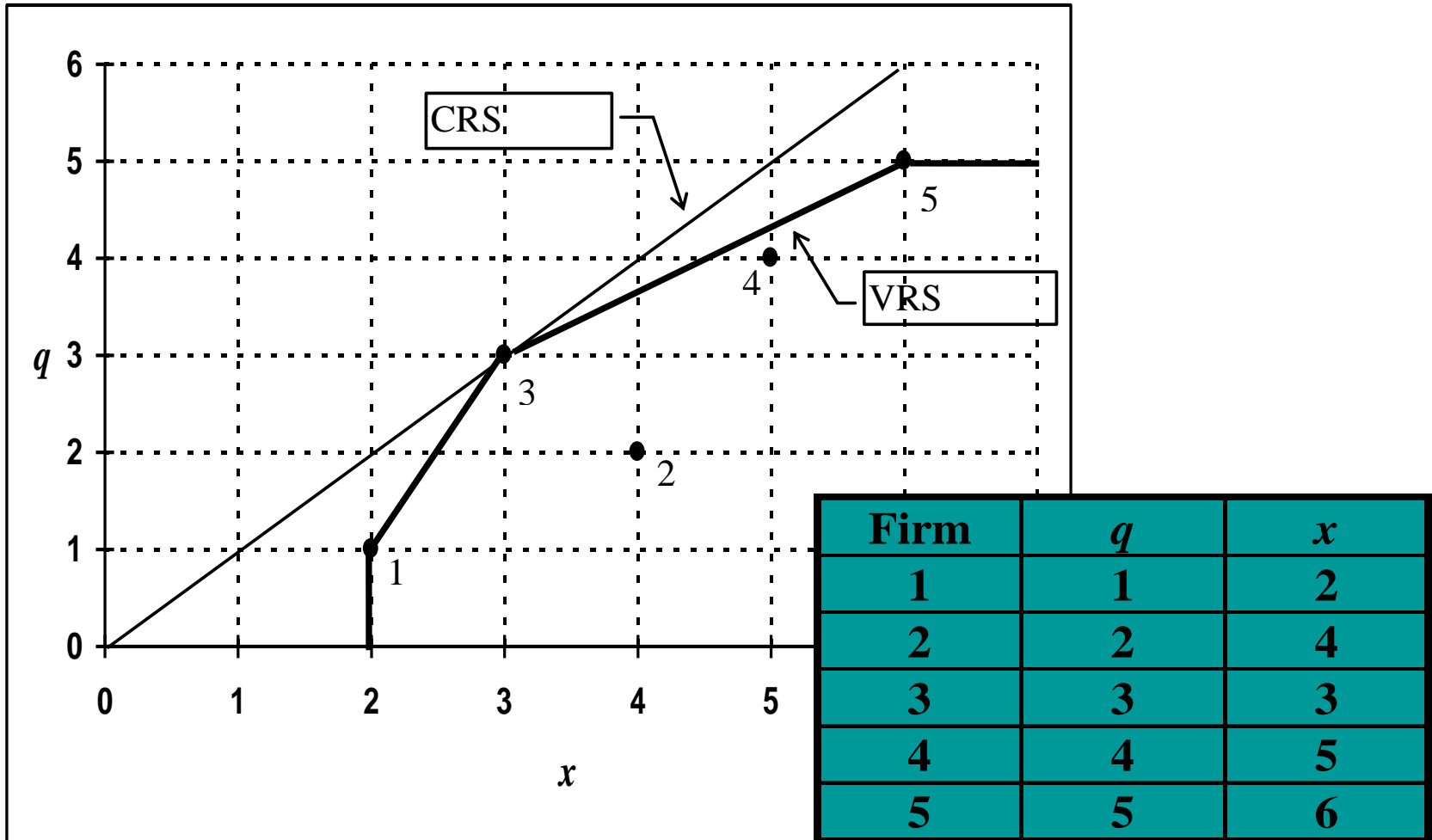
$$\theta \mathbf{x}_i - \mathbf{X}\lambda \geq \mathbf{0},$$

$$\mathbf{N}\mathbf{1}'\lambda \leq 1$$

$$\lambda \geq \mathbf{0}$$

[$\mathbf{N}\mathbf{1}$ is an $I \times 1$ vector of ones]

Scale efficiency example



Scale efficiency results

Firm	CRS TE	VRS TE	Scale	
1	0.500	1.000	0.500	irs
2	0.500	0.625	0.800	irs
3	1.000	1.000	1.000	-
4	0.800	0.900	0.889	drs
5	0.833	1.000	0.833	drs
mean	0.727	0.905	0.804	

Note: $SE = TE_{CRS} / TE_{VRS}$

Output orientated DEA models

- Proportionally expand outputs, with inputs held fixed**
- Produces same frontier as input orientated model**
- The TE scores are identical under CRS - but can differ under VRS**
- Selection of orientation depends on which set (outputs or inputs) the firm has most control over**

Output orientation

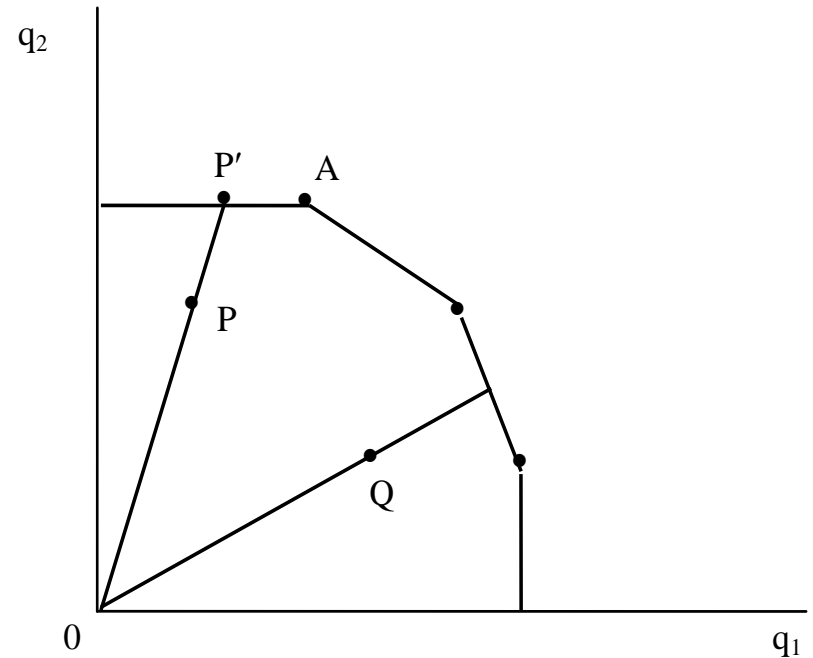
$$\max_{\phi, \lambda} \phi,$$

$$\text{st } -\phi \mathbf{q}_i + \mathbf{Q}\lambda \geq \mathbf{0},$$

$$\mathbf{x}_i - \mathbf{X}\lambda \geq \mathbf{0},$$

$$\mathbf{N}\mathbf{1}'\lambda = 1$$

$$\lambda \geq \mathbf{0}$$



where ϕ is a scalar: $1 \leq \phi < \infty$, and $TE=1/\phi$.

DEA – Computer Packages

- **There are many computer packages available in the market**
 - **Basically any programmes with LP options can be used for solving DEA LP problems**
 - **SAS, SHAZAM and other econometric packages can also be used**
 - **Specialist DEA packages are also available:**
 - **ONFront; IDEAS; Frontier Analysit; Warwick DEA and DEAP**
- **DEAP – Data Envelopment Analysis Program**
 - **Prepared by Prof. Tim Coelli of CEPA**
 - **Package available free of cost from CEPA website (www.uq.edu.au/economics/cepa)**
 - **Easy to use – many illustrations in the textbook**

Example:

Data: 1 2
2 4
3 3
4 5
5 6

outputs

inputs

Instructions:

eg1-dta.txt	DATA FILE NAME
eg1-out.txt	OUTPUT FILE NAME
5	NUMBER OF FIRMS
1	NUMBER OF TIME PERIODS
1	NUMBER OF OUTPUTS
1	NUMBER OF INPUTS
0	0=INPUT AND 1=OUTPUT ORIENTATED
1	0=CRS AND 1=VRS
0	0=DEA(MULTI-STAGE), 1=COST-DEA, 2=MALMQUIST-DEA, 3=DEA(1-STAGE), 4=DEA(2-STAGE)

Some of the instructions are useful when we have information on prices or when we have data on several time periods.

DEAP Output – Some Components

Input orientated DEA

Scale assumption: VRS

Slacks calculated using multi-stage method

EFFICIENCY SUMMARY:

firm crste vrste scale

1 0.500 1.000 0.500 irs

2 0.500 0.625 0.800 irs

3 1.000 1.000 1.000 -

4 0.800 0.900 0.889 drs

5 0.833 1.000 0.833 drs

mean 0.727 0.905 0.804

Note: crste = technical efficiency from CRS DEA

vrste = technical efficiency from VRS DEA

scale = scale efficiency = crste/vrste

Note also that all subsequent tables refer to VRS results

In this example we have one peer under CRS and three peers under VRS.

DEAP Output – Some Components

SUMMARY OF PEERS:

firm peers:

1	1
2	1 3
3	3
4	3 5
5	5

SUMMARY OF PEER WEIGHTS:

(in same order as above)

firm peer weights:

1	1.000
2	0.500 0.500
3	1.000
4	0.500 0.500
5	1.000

PEER COUNT SUMMARY:

(i.e., no. times each firm is a peer for another)

firm peer count:

1	1
2	0
3	2
4	0
5	1

Data Envelopment Analysis – some extensions

- **Allocative efficiency**
- **Super efficiency**
- **Peeling the frontiers**
- **Restrictions on weights**
- **Treatment of environmental variables**
 - **Tobit Regressions – 2nd stage**
- **Constraints on input reductions**
- **Measures of variability**
 - **Jackknife methods**
 - **Bootstrap methods**

Calculation of (input-mix) allocative efficiency

- Here we discuss the second efficiency concept that deals with optimal input mix or output mix
- Input price data also required
- Must solve 2 DEA models:
 - standard TE model – DEA model to determine the production frontier
 - cost efficiency (CE) model – minimum cost solution subject to feasibility
- Allocative efficiency (AE) then calculated as:
$$AE = CE / TE$$
- Revenue and profit efficiency solutions can be derived through suitable modifications of the DEA model (pp. 184-185)

Cost Minimisation DEA

$$\min_{\lambda, \mathbf{x}_i^*} \mathbf{w}_i' \mathbf{x}_i^*$$

$$\text{st } -\mathbf{q}_i + \mathbf{Q}\lambda \geq \mathbf{0}$$

$$\mathbf{x}_i^* - \mathbf{X}\lambda \geq \mathbf{0}$$

$$\mathbf{1}'\lambda = 1$$

$$\lambda \geq \mathbf{0}$$

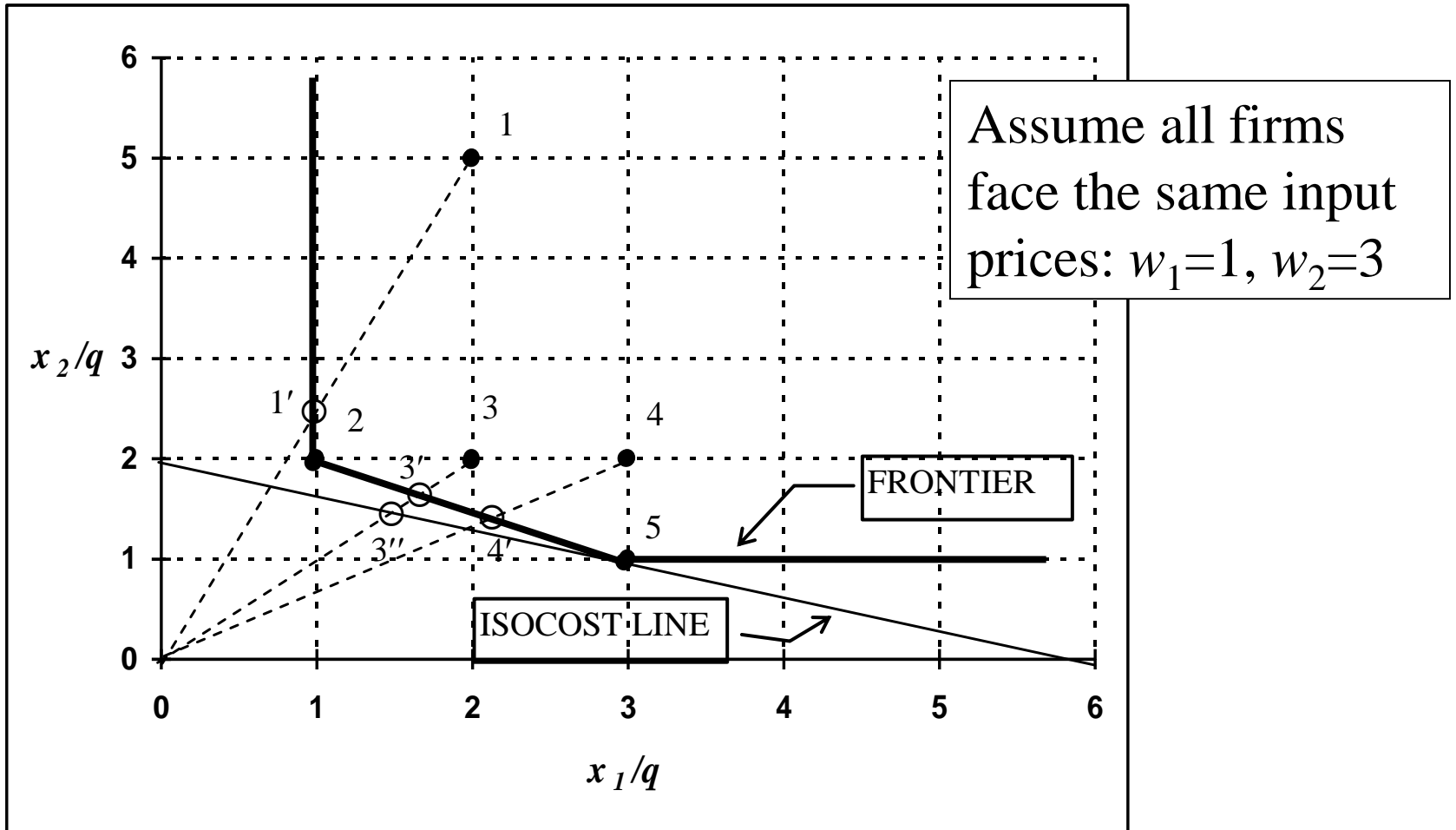
$\mathbf{w}_i = K \times 1$ input price vector

$\mathbf{x}_i^* = K \times 1$ vector of cost-minimising input quantities

$\text{CE} = \mathbf{w}_i' \mathbf{x}_i^* / \mathbf{w}_i' \mathbf{x}_i = \text{Economic Efficiency}$

$\text{AE} = \text{EE} / \text{TE}$

A Simple Cost Efficiency Example



CRS Cost Efficiency DEA Results

firm	technical efficiency	allocative efficiency	cost efficiency
1	0.500	0.706	0.353
2	1.000	0.857	0.857
3	0.833	0.900	0.750
4	0.714	0.933	0.667
5	1.000	1.000	1.000
mean	0.810	0.879	0.725

Super efficiency

- **DEA identifies a number of peers that are used in benchmarking**
- **Each of the peers has TE score equal to one**
- **There is no way of ranking the firms which have a TE score equal to 1**
- **Super efficiency is a concept developed to address this issue – then all the firms on the frontier can also be ranked.**
 - **For firm i which is a peer, run a DEA after dropping the firm in the benchmarking firms and compute a TE score – this TE score can be greater than 1.**
 - **Continue this for all the firms which are peers, each will have TE score different from 1 (greater than 1).**
 - **These scores are known as “super efficiency” scores**
 - **Rank all the peer firms using their “super efficiency scores”.**

Koopman's efficiency

- **DEA can project an efficient observation to the flat portion of the frontier**
- **Due to linear nature of DEA, there will be input and output slacks**
- **Koopman's efficiency points refer to the *states* to which inefficient DMUs should strive to get to**
- **Two options:**
 - **Run an LP to maximise the sum of slacks required to move the first stage point to a Koopman's efficient point (Ali and Selfod, 1993) – does not necessarily lead to a point with minimum disruption**
 - **Multi-stage radial DEA models (Coelli, 1997) – leads to a Koopman's efficient point which is similar**

Peeling the frontier

- **We recall that DEA is a non-stochastic technique**
 - This means all data are treated as if there is no noise or measurement error
 - In the presence of measurement errors and noise, DEA can produce “strange” results
 - This can happen when firms with errors end up as peers
- **A way to check if DEA scores of firms are affected by noise, it is a common practice to check the sensitivity of the TE scores after dropping all the peers and re-running DEA or after dropping some suspect firms from the DEA.**
- **This procedure is known as “peeling”. It is a procedure highly recommended provided there are enough observations.**

Non-discretionary variables

For example:

$$\begin{aligned} \min_{\theta, \lambda} \quad & \theta, \\ \text{st} \quad & -\mathbf{q}_i + \mathbf{Q}\boldsymbol{\lambda} \geq \mathbf{0}, \\ & \theta \mathbf{x}_i^D - \mathbf{X}^D \boldsymbol{\lambda} \geq \mathbf{0}, \\ & \mathbf{x}_i^{ND} - \mathbf{X}^{ND} \boldsymbol{\lambda} \geq \mathbf{0}, \\ & \mathbf{1}'\boldsymbol{\lambda} = 1 \\ & \boldsymbol{\lambda} \geq \mathbf{0}, \end{aligned}$$

\mathbf{X}^D = discretionary inputs and
 \mathbf{X}^{ND} = non-discretionary inputs

How to account for environment?

- **Environment: All factors which could influence the efficiency of a firm**
 - **Public versus privately owned firms**
 - **Locational characteristics**
 - **Power distribution networks influenced by size and density**
 - **Socio-economic characteristics of a suburb**
 - **Institutional factors – Regulation; Unions**
- **Possible approaches:**
 - **Second stage regression analysis of efficiency scores**
 - **Tobit Model as the scores are between 0 and 1**
 - **Can include dummy or categorical variables**
 - **Testing hypothesis on the effect of specific variables**

How to account for environment?

- **Include some of the variables in the LP of DEA**
 - **Impose equality constraints**
 - **Impose restrictions on linear combinations**
 - **Divide the firms into groups according to a given environmental variable**
 - **Ownership**
 - **Location**
 - **Conduct DEA separately and then use metafrontiers (we will deal with this in the last session)**

How to account for environment?

- **Include some of the variables in the LP of DEA**
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How to account for environment?

- **Some caution in using environmental variables**
 - **Reduced degrees of freedom**
 - **Must decide the direction of the effect *a priori***
 - **Cannot test for statistical significance**
 - **Cannot include categorical variables**
 - **Cannot include variables with negative values**

DEA - some comments

- ***Dimensionality problem:*** Too many variables and data on limited number of firms
 - We end up with many firms on the frontier
 - Problem similar to the one of degrees of freedom in regression models
 - Number of observations should be adequate to estimate a translog model (greater than the number of parameters in a translong model)
- ***Problems of measurement and noise***
 - DEA treats all data as observations as it is non-stochastic
 - Observations with noise may end up as technically efficient firms
 - Outliers can seriously affect the production frontier
 - It is a good idea to examine basic input-output ratios to eliminate outliers in data

Benchmarking Australian Universities

Carrington, Coelli and Rao (2005) Economic Papers

- **Number of universities: 36**
- **Study period: 1996 -2000**
- **A conceptual framework:**
 - **What are the main functions of a university?**
 - **Teaching, research and community service**
 - **How do we measure them?**
 - **Measuring research performance**
 - **Publications**
 - **Research grants**
 - **Impact**

Benchmarking Australian Universities

Output and Input measures	Quality Measures
<i>Teaching Output</i>	<i>Output Quality</i>
Student load (EFTSU)	Students broadly overall satisfied with course (%)
Science student load (EFTSU)	Average graduate starting salary (\$)
Non-science student load (EFTSU)	Graduate full-time employment (%)
Student load (WEFTSU)	
Research higher degree student load (WEFTSU)	
Non-research higher degree student load (WEFTSU)	
Completions (EFTSU)	

Benchmarking Australian Universities

<i>Research Output</i>	<i>Environment</i>
Weighted publications (number)	Proportion of Indigenous Australian students
Research Quantum (\$)	Proportion of students from a low socioeconomic background
	Proportion of students from rural and remote regions
<i>Input measure</i>	Proportion of part-time and external students
Operating costs (\$'000m)	Average tertiary entrant ranking (%)
<i>Input Quality</i>	Location (metropolitan or not)
Proportion of academics Associate Professor and above	Science student load (%)
	Research student load (%)

Benchmarking Australian Universities

SUMMARY RESULTS: EFFICIENCY OF UNIVERITIES, 2000

Institution	CRS efficiency	VRS efficiency	Scale efficiency	Nature of scale inefficiency
Charles Sturt University	1.00	1.00	1.00	-
Macquarie University	1.00	1.00	1.00	-
Southern Cross University	0.84	0.98	0.85	irs
The University of New England	0.67	0.69	0.97	irs
University of New South Wales	0.88	0.93	0.95	drs
The University of Newcastle	0.89	0.91	0.97	drs
The University of Sydney	0.84	1.00	0.84	drs
University of Technology, Sydney	0.88	0.93	0.94	drs
University of Western Sydney	0.91	1.00	0.91	drs
University of Wollongong	0.89	0.89	1.00	-
Deakin University	0.75	0.81	0.92	drs
La Trobe University	0.77	0.84	0.92	drs
Monash University	0.84	1.00	0.84	drs
RMIT University	0.86	1.00	0.86	drs
Swinburne University of Technology	0.94	0.98	0.95	irs
The University of Melbourne	0.94	1.00	0.94	drs
University of Ballarat	0.76	1.00	0.76	irs
Victoria University of Technology	0.91	0.92	0.99	irs
Central Queensland University	0.81	0.83	0.97	irs
Griffith University	0.79	0.87	0.90	drs
James Cook University	0.82	0.85	0.96	irs
Queensland University of Technology	0.89	0.99	0.90	drs
The University of Queensland	0.94	1.00	0.94	drs

Benchmarking Australian Universities

SUMMARY RESULTS: EFFICIENCY OF UNIVERITIES, 2000

University of Southern Queensland	0.84	0.90	0.93	irs
Curtin University of Technology	0.73	0.83	0.89	drs
Edith Cowan University	0.96	0.97	0.99	irs
Murdoch University	0.94	0.98	0.96	irs
The University of Western Australia	1.00	1.00	1.00	-
The Flinders University of South Australia	1.00	1.00	1.00	-
The University of Adelaide	0.98	0.99	1.00	drs
University of South Australia	0.87	0.94	0.93	drs
University of Tasmania	0.89	0.89	0.99	drs
Northern Territory University	0.60	1.00	0.60	irs
University of Canberra	0.77	0.85	0.91	irs
Australian Catholic University	0.84	0.98	0.86	irs
Mean efficiency	0.86	0.94	0.92	
Minimum	0.60	0.69	0.60	
Maximum	1.00	1.00	1.00	
Efficient universities	4	12	6	

PEER AND PEER WEIGHTS FOR LESS EFFICIENT UNIVERSITIES

Institution	Peers and peer weights		
Southern Cross University	Macquarie University (0.133)	Charles Sturt University (0.032)	University of Ballarat (0.835)
The University of New England	Macquarie University (0.313)	Charles Sturt University (0.364)	University of Ballarat (0.463)
University of New South Wales	Uni of Melbourne (0.875)	Uni of Western Australia (0.125)	
The University of Newcastle	Uni of Melbourne (0.099)	Uni of Western Sydney (0.031)	Macquarie University (0.870)
University of Technology, Sydney	Macquarie University (0.463)	Uni of Melbourne (0.012)	Uni of Western Sydney (0.525)
University of Wollongong	Flinders University (0.314)	Macquarie University (0.649)	University of Ballarat (0.037)
Deakin University	Uni of Melbourne (0.104)	Uni of Western Sydney (0.505)	Macquarie University (0.391)
La Trobe University	Uni of Melbourne (0.151)	Uni of Western Sydney (0.273)	Macquarie University (0.576)
Swinburne University of Technology	Macquarie University (0.237)	Charles Sturt University (0.334)	University of Ballarat (0.429)

Other results

Sensitivity Analysis:

- Choice of alternative measures of output and inputs
- Corrected Ordinary Least Squares (COLS)
- Results are relatively robust

Productivity Growth:

- TFP growth of 1.8 percent per annum over the period 1996-2000
- Technical change accounted for 2.1 percent per annum; efficiency decline 0.7 percent per annum; and scale efficiency improvements 0.4 per annum.
- Productivity growth in university sector comparable to other sectors.

Quality issues:

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