

MALMQUIST RESULTS & ANALYSIS

COURSE: INNOVATION DEVELOPMENT IN
COMMERCIAL BANKS

PRESENTED BY FARKHOD ODILOV

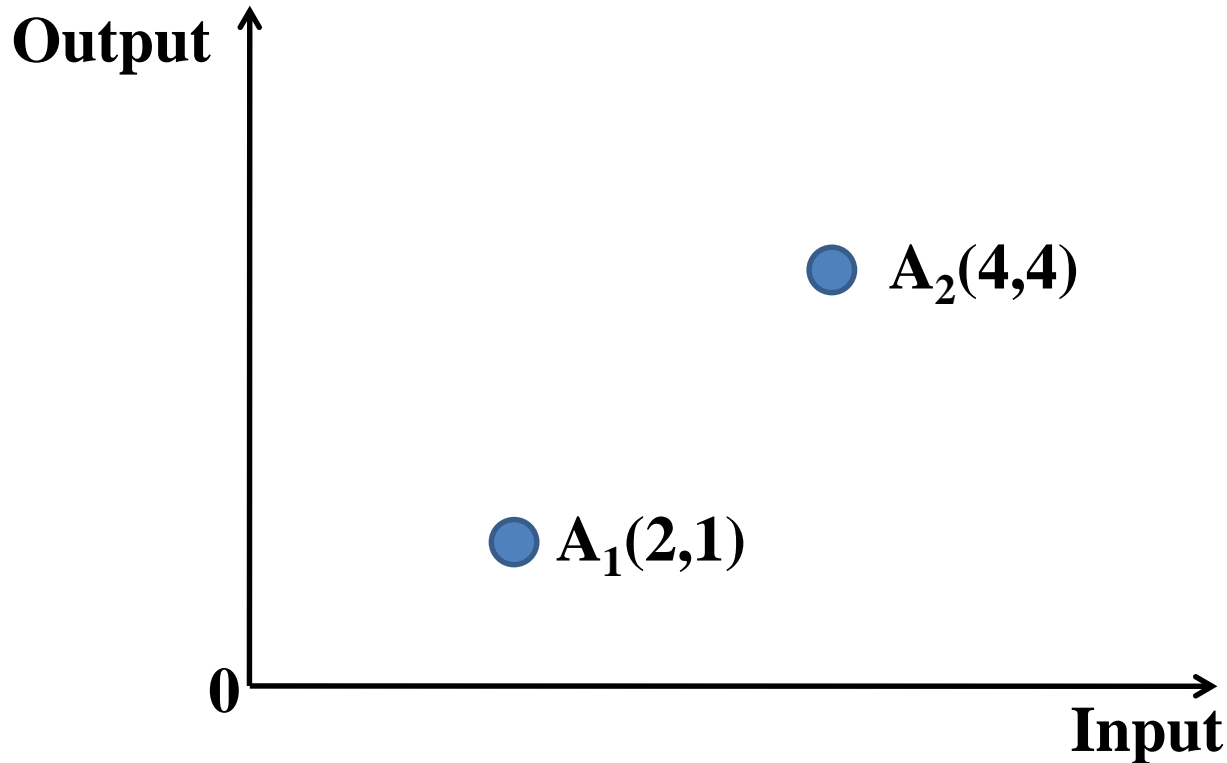
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Malmquist Productivity Index

- Productivity = Output / Input
- Productivity (Growth) Index measures the Productivity changes over Time
- Malmquist (Productivity Growth) Index measures the productivity changes along with time variations and can be decomposed into changes in efficiency and technology.

Malmquist Productivity Index



- Productivity Index = $(4/4)/(1/2) = 2$
 - ☞ Productivity is improved by 100%

Malmquist Productivity Index

- Malmquist Productivity Index

$$M_I^t(x^t, y^t, x^{t+1}, y^{t+1}) = \frac{D_I^t(x^{t+1}, y^{t+1})}{D_I^t(x^t, y^t)}$$

Where Input based distance function at time t is defined by

$$D_I^t(x^t, y^t) = \max \{ \theta \mid (x^t / \theta, y^t) \in P^t(x^t, y^t) \}$$

for Production Possibility Set $P(x^t, y^t)$

Input vector $x = \{x_1, x_2, x_3, \dots, x_m\}$

Output vector $y = \{y_1, y_2, y_3, \dots, y_n\}$

☞ M_I^t is measured by production possibility set P^t at time t.

Malmquist Productivity Index

- Malmquist Productivity Index

And accordingly,

$$D_I^{t+1}(x^t, y^t) = \max \{ \theta \mid (x^t / \theta, y^t) \in P^{t+1}(x^{t+1}, y^{t+1}) \}$$

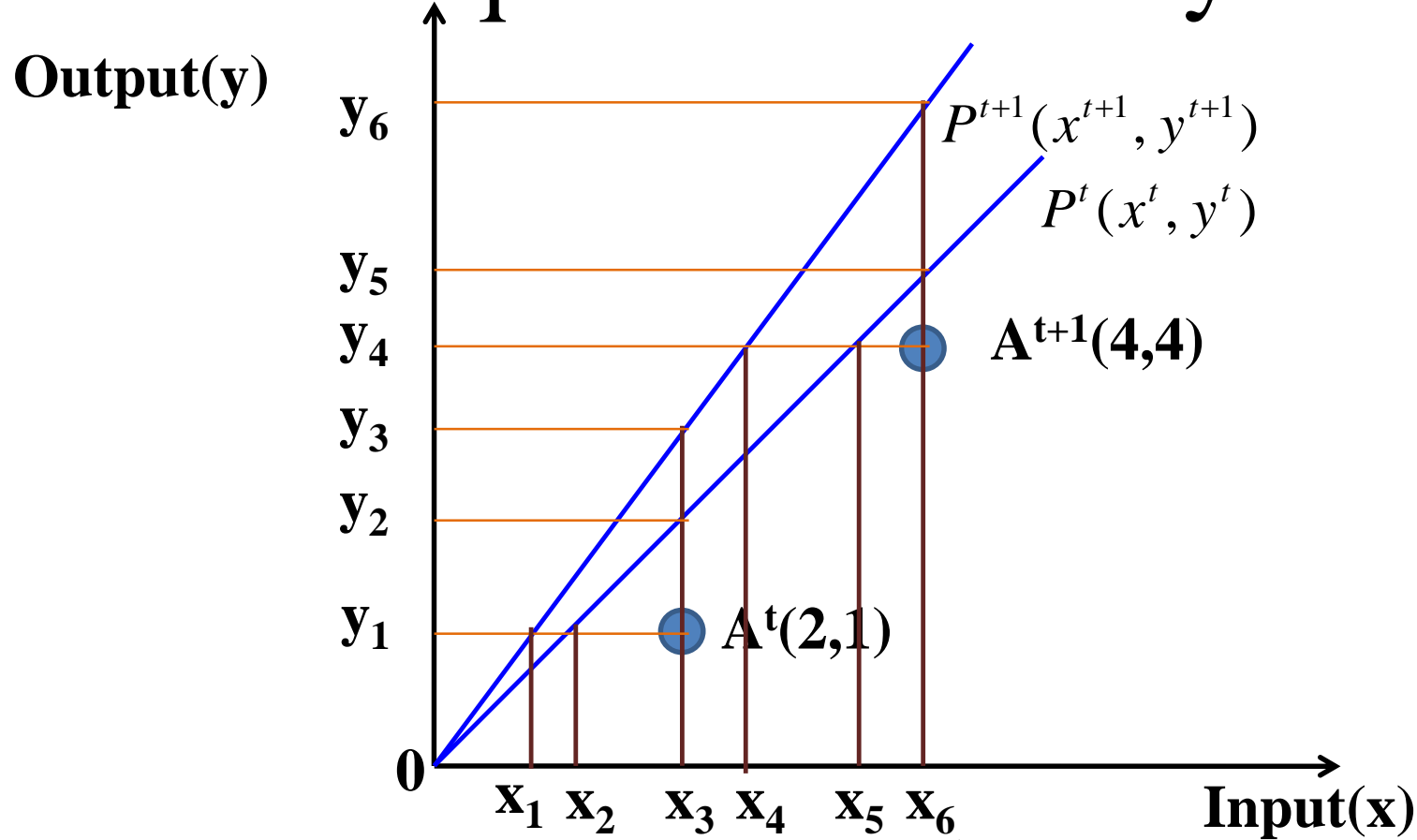
$$D_I^t(x^{t+1}, y^{t+1}) = \max \{ \theta \mid (x^{t+1} / \theta, y^{t+1}) \in P^t(x^t, y^t) \}$$

for cross period distance function.

Further, M_I^{t+1} can be defined as

$$M_I^{t+1}(x^t, y^t, x^{t+1}, y^{t+1}) = \frac{D_I^{t+1}(x^{t+1}, y^{t+1})}{D_I^{t+1}(x^t, y^t)}$$

Malmquist Productivity Index



- Productivity Change =
$$\frac{\frac{oy_4}{ox_6}}{\frac{oy_1}{ox_3}} = \frac{\overline{oy_4}}{\overline{oy_1}} \frac{\overline{ox_3}}{\overline{ox_6}} = (4/1) * (2/4) = 2$$

Malmquist Productivity Index

- Malmquist Productivity Index

$$D_I^t(x^t, y^t) = \overline{ox_2} / \overline{ox_3}$$

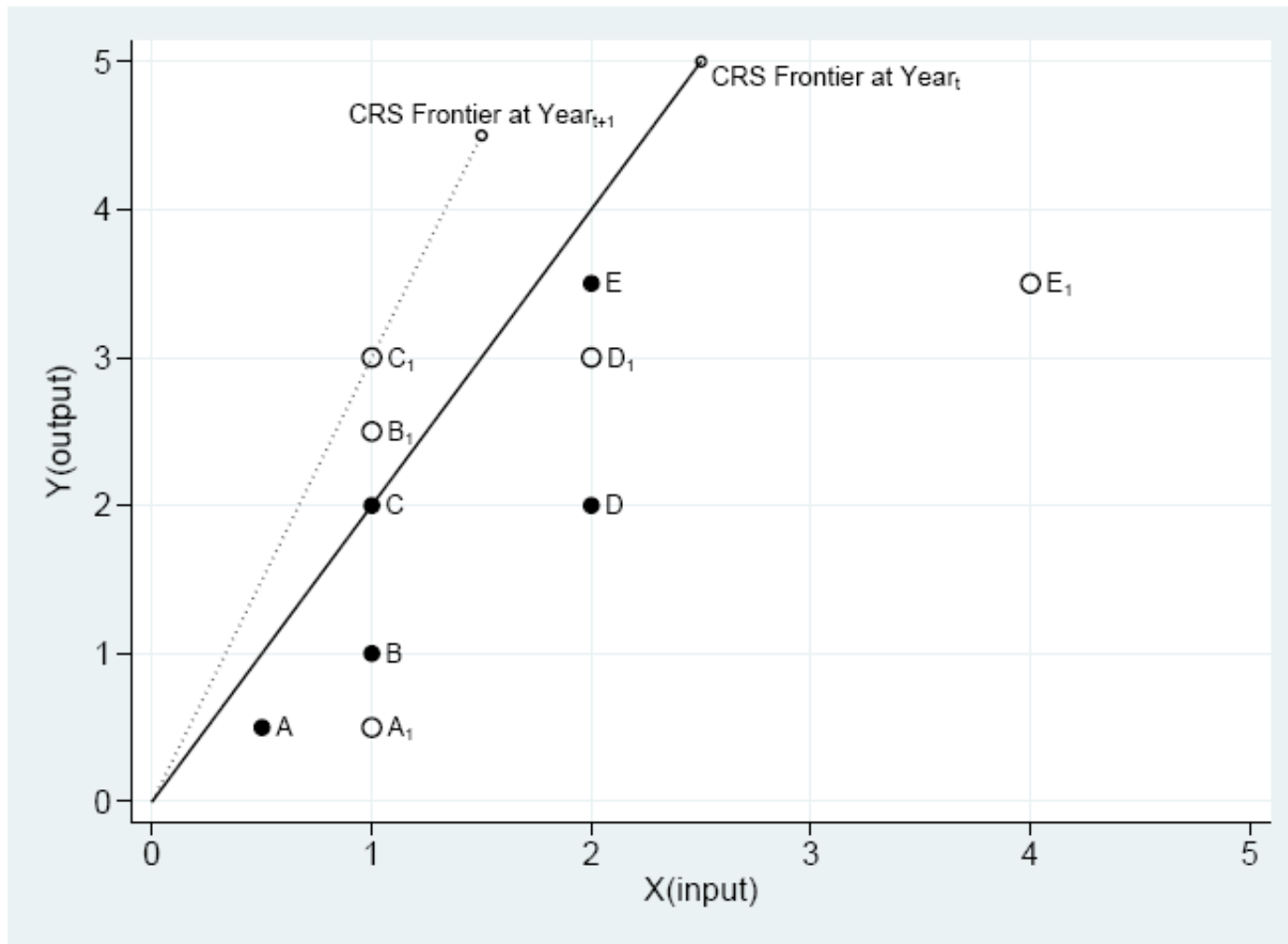
$$D_I^t(x^{t+1}, y^{t+1}) = \overline{ox_5} / \overline{ox_6}$$

$$M_I^t(x^t, y^t, x^{t+1}, y^{t+1}) = \frac{D_I^t(x^{t+1}, y^{t+1})}{D_I^t(x^t, y^t)} = \frac{\overline{ox_5} / \overline{ox_6}}{\overline{ox_2} / \overline{ox_3}}$$

$$= \frac{\overline{ox_3} \overline{ox_5}}{\overline{ox_6} \overline{ox_2}} = \frac{\overline{ox_3} \overline{oy_4}}{\overline{ox_6} \overline{oy_1}} = \text{Productivity Change}$$

Malmquist Index using DEA Frontier

- Concepts of Malmquist Index using CRS Frontier



Malmquist Index using DEA Frontier

The input oriented CRS Malmquist Index using the observations at time t and $t+1$.

$$M_I^t = \frac{E_I^t(x^{t+1}, y^{t+1})}{E_I^t(x^t, y^t)} \quad (1)$$

$$M_I^{t+1} = \frac{E_I^{t+1}(x^{t+1}, y^{t+1})}{E_I^{t+1}(x^t, y^t)} \quad (2)$$

where I denotes the orientation of DEA model.

The Geometric mean of two input oriented CRS Malmquist Indices

$$M_I^G = (M_I^t M_I^{t+1})^{1/2} = \left[\left(\frac{E_I^t(x^{t+1}, y^{t+1})}{E_I^t(x^t, y^t)} \right) \cdot \left(\frac{E_I^{t+1}(x^{t+1}, y^{t+1})}{E_I^{t+1}(x^t, y^t)} \right) \right]^{1/2} \quad (3)$$

Malmquist Index using DEA Frontier

Decomposition of the input oriented geometric mean of Malmquist index using the concept of input oriented efficiency change and input oriented technical change

$$M_I^G = (ECH_I \cdot TCH_I^G) = \left(\frac{E_I^{t+1}(x^{t+1}, y^{t+1})}{E_I^t(x^t, y^t)} \right) \cdot \left[\left(\frac{E_I^t(x^t, y^t)}{E_I^{t+1}(x^t, y^t)} \right) \cdot \left(\frac{E_I^t(x^{t+1}, y^{t+1})}{E_I^{t+1}(x^{t+1}, y^{t+1})} \right) \right]^{1/2} \quad (4)$$

☞ Malmquist Index can be obtained from the DEA measure

The User written command “malmq”

- Program Syntax

`malmq ivars = ovars [if] [in] [, ort(in | out)
period(varname) trace saving(filename)]`

- `ort(in | out)` specifies the orientation. The default is `ort(in)`, meaning input-oriented DEA.
- `period(varname)` identifies the time variable.
- `trace` specifies to save all the sequences displayed in the Results window in the `malmq.log` file. The default is to save the final results in the `malmq.log` file.
- `saving(filename)` specifies that the results be saved in `filename.dta`.

- See “`malmq.ado`” file for the details

Notes and Examples

- Notes
 - Updated “*dea.ado*”, “*malm.ado*” files
 - In terms of accuracy and computational efficiency?
Current version is more focused on ‘accuracy’
 - Tested for 365DMU data set for *dea.ado* command and compared with other DEA programs.

Notes and Examples

- Example

- Data : see “365dmu.dta” for *dea* command and “panel_data_for_malmquist_dea.dta” for *malmq* command.

- Try the following commands

- **dea i_total = o_licnese o_sic o_nsic o_dpatent o_fpatent, rts(crs) ort(i)**

- **malmq i_AC = O_SPI O_CPI, ort(i) period(period)**

Notes and Examples

– Result

- For dea: Results including the messages “No Solution(LOOP grather than maxiter):[DMUi=119][LOOP=16001]CRS-IN-SI-PII”.
 - ✓ See “dea.log” file for details
 - ✓ Compare with results by other programs
- For malmq
 - ✓ see “malmquist.log” file for details
 - ✓ Compare with results by other programs

Measuring Productivity Growth

- Now we consider the case we have data on firms over time – output and input quantities for each firm over t .
- The problem is one of measuring productivity growth – total/multi-factor productivity (TFP/MFP) growth.
- TFP index for two periods s and t would depend upon output and input quantities in the two periods. Let us denote this by $F(\mathbf{x}_t, \mathbf{q}_t, \mathbf{x}_s, \mathbf{q}_s)$.
- It should satisfy the property:

$$F(\lambda \mathbf{x}_s, \mu \mathbf{q}_s, \mathbf{x}_s, \mathbf{q}_s) = \mu / \lambda \text{ for all } \mu, \lambda > 0$$

In general, the TFP index should be homogeneous of degree +1 \mathbf{q} and -1 in \mathbf{x} .

Four approaches to TFP Measurement

1. Hicks-Moorsteen Index

$$\text{HM TFP Index} = \frac{\text{Growth in output}}{\text{Growth in input}} = \frac{\text{Output quantity index}}{\text{Input quantity index}}$$

We may use any formula of our choice in computing this index – as long as the formula selected is properly selected.

2. TFP based on Profitability ratio

$$\text{TFP index} = \frac{R_t^* / R_s^*}{C_t^* / C_s^*} = \frac{(R_t / R_s) / \text{output price index}}{(C_t / C_s) / \text{input price index}}$$

- **This can be viewed as a special case of Hicks-Moorsteen index.**
- **There is considerable interest in the decomposition of revenue and cost changes (leading to profit change). Such decompositions can be undertaken using Bennett index numbers (see Diewert, 1998 and 2000).**

Malmquist Productivity Index

- **Malmquist Productivity index makes use of distance functions to measure productivity change.**
- **It can be defined using input or output orientated distance functions.**
- **This approach was first proposed in Caves, Christensen and Diewert (1982).**
- **We just look at the Output-orientated Malquist productivity Index (MPI).**
- **Using period s-technology:**

$$m_o^s(\mathbf{q}_s, \mathbf{q}_t, \mathbf{x}_s, \mathbf{x}_t) = \frac{d_o^s(\mathbf{q}_t, \mathbf{x}_t)}{d_o^s(\mathbf{q}_s, \mathbf{x}_s)}$$

Malmquist Productivity Index

- Using period t-technology:

$$m_o^t(\mathbf{q}_s, \mathbf{q}_t, \mathbf{x}_s, \mathbf{x}_t) = \frac{d_o^t(\mathbf{q}_t, \mathbf{x}_t)}{d_o^t(\mathbf{q}_s, \mathbf{x}_s)}$$

- Since there are two possible MFP measures, based on period s and period t technology, the MFP is defined as the geometric average of the two:

$$\begin{aligned} m_o(\mathbf{q}_s, \mathbf{q}_t, \mathbf{x}_s, \mathbf{x}_t) &= \left[m_o^s(\mathbf{q}_s, \mathbf{q}_t, \mathbf{x}_s, \mathbf{x}_t) \times m_o^t(\mathbf{q}_s, \mathbf{q}_t, \mathbf{x}_s, \mathbf{x}_t) \right]^{0.5} \\ &= \left[\frac{d_o^s(\mathbf{x}_t, \mathbf{q}_t)}{d_o^s(\mathbf{x}_s, \mathbf{q}_s)} \times \frac{d_o^t(\mathbf{x}_t, \mathbf{q}_t)}{d_o^t(\mathbf{x}_s, \mathbf{q}_s)} \right]^{0.5} \end{aligned}$$

Malmquist Productivity Index - Properties

Properties of Malmquist Productivity Index:

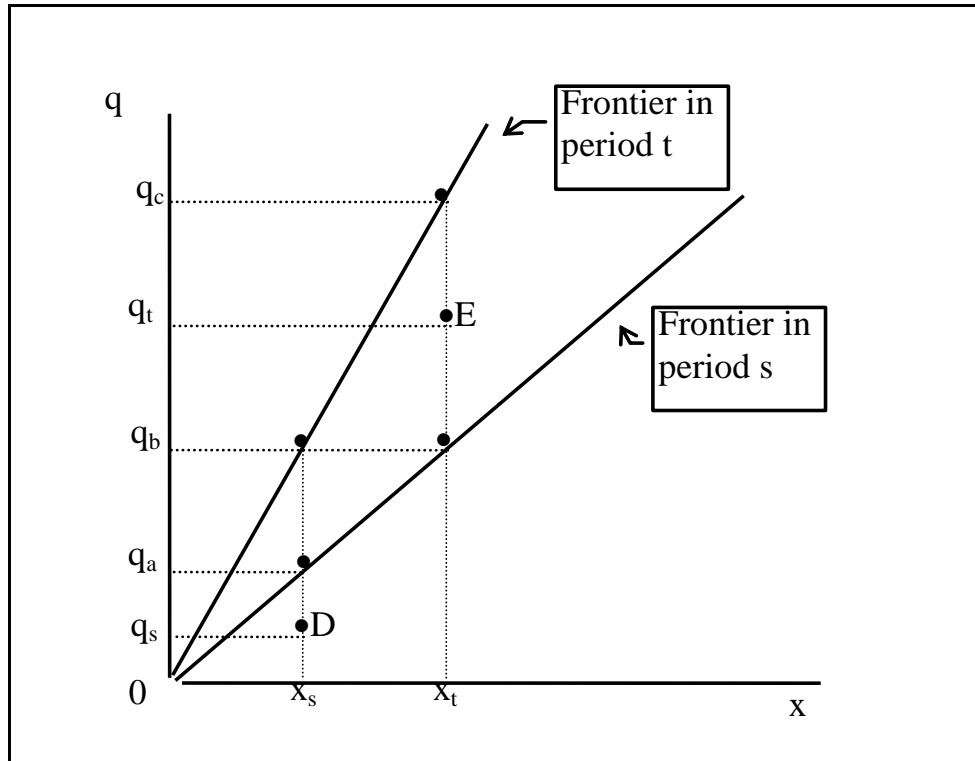
1. It can be decomposed into efficiency change and technical change:

$$m_o(\mathbf{q}_s, \mathbf{q}_t, \mathbf{x}_s, \mathbf{x}_t) = \frac{d_o^t(\mathbf{x}_t, \mathbf{q}_t)}{d_o^s(\mathbf{x}_s, \mathbf{q}_s)} \left[\frac{d_o^s(\mathbf{x}_t, \mathbf{q}_t)}{d_o^t(\mathbf{x}_t, \mathbf{q}_t)} \times \frac{d_o^s(\mathbf{x}_s, \mathbf{q}_s)}{d_o^t(\mathbf{x}_s, \mathbf{q}_s)} \right]^{0.5}$$

Efficiency change Technical change

2. Malmquist productivity index is the same as the Hicks-Moorsteen index if the technology exhibits global constant returns to scale and inverse homotheticity.

Malmquist Productivity Index



$$\text{Efficiency change} = \frac{q_t / q_c}{q_s / q_a} \quad \text{Technical change} = \left[\frac{q_t / q_b}{q_t / q_c} \times \frac{q_s / q_a}{q_s / q_b} \right]^{0.5}$$

Malmquist Productivity Index - Properties

3. The input-orientated Malmquist productivity index is given by:

$$TFPC = \left[\frac{d_s(q_s, x_t)}{d_s(q_t, x_t)} \frac{d_t(q_s, x_s)}{d_t(q_t, x_t)} \right]^{0.5}$$

5. Output-orientated and input-orientated Malmquist indexes coincide if the technology exhibits constant returns to scale.

6. The Malmquist Productivity index does not adequately account for scale change.

6. The Malmquist productivity index does not satisfy transitivity property. So we need to use the EKS method to make them transitive.

Malmquist Productivity Index - Properties

- 7. If panel data on input and output quantities are available then there is no need for price data.**
- 8. If only two data points are available, then we need to use index number approach – may require some behavioural assumptions**

Productivity Index – components approach

The last approach is to measure productivity change by identifying various sources of productivity growth:

1. Efficiency change
2. Technical change
3. Scale efficiency change
4. Output and input mix effect

Then Productivity change is measured as the product of the four changes above. The resulting index is:

$$TFPC^{s,t}(\mathbf{x}_s, \mathbf{x}_t, \mathbf{q}_s, \mathbf{q}_t) = \left[\frac{d_o^{*s}(\mathbf{x}_t, \mathbf{q}_t)}{d_o^{*s}(\mathbf{x}_s, \mathbf{q}_s)} \times \frac{d_o^{*t}(\mathbf{x}_t, \mathbf{q}_t)}{d_o^{*t}(\mathbf{x}_s, \mathbf{q}_s)} \right]^{0.5}$$

where (*) denotes “cone technology” – the smallest CRS technology that encompasses the technology – in periods t and s.

MPI using Index Numbers

- **This is the case where we have only two observed points (q_t, x_t) and (q_s, x_s) .**
- **In this case we can compute MPI provided we have additional information on prices, (p_t, w_t) and (p_s, w_s) and assume that that the firms are technically and allocatively efficient.**
- **In this case, we have the following result from Caves, Christensen and Diewert which makes it possible to compute the Malmquist productivity index using Tornqvist index numbers.**
- **The result states that:**

MPI -Index Number approach

- If the output distance functions in periods s and t are represented by translog functional form with identical second order terms and then under the assumption of technical and allocative efficiency, we can use the Tornqvist output and input quantity index numbers to compute the Malmquist productivity index.

$$m_o(q_s, q_t, x_s, x_t) = \left[m_o^t(q_s, q_t, x_s, x_t) \times m_o^s(q_s, q_t, x_s, x_t) \right]^{0.5}$$

$$= \frac{\text{Tornqvist output index}}{\text{Tornqvist input index}} \times \prod_{k=1}^K \left(\frac{x_{kt}}{x_{ks}} \right)^{s_k^*/2}$$

where $s_k^* = s_{kt}(1 - \varepsilon_t) + s_{kis}(1 - \varepsilon_s)$ ε_t and ε_s are the local returns-to-scale values in periods t and s ,

MPI -Index Number approach

- **If in both periods there is constant returns to scale, then the MPI is simply given by the ratio of the output and input indexes computed using the Tornqvist formula.**

$$= \frac{1}{2} \sum_{i=1}^M (r_{is} + r_{it}) (\ln q_{it} - \ln q_{is}) - \frac{1}{2} \sum_{j=1}^K (s_{js} + s_{jt}) (\ln x_{jt} - \ln x_{js})$$

- **We note that the MPI based on Tornqvist index is not transitive. We can use EKS method to generate transitive Malmquist Productivity indexes.**
- **Another useful result: If the distance functions are quadratic and under the assumption of technical and allocative efficiency, the Malmquist index is given by the ratio of Fisher output and input quantity index numbers.**

Transitive Tornqvist TFP Index

If we apply the EKS method and generate transitive index numbers, we can show that

$$\begin{aligned} \ln TFP_{st}^{transitive} = & \left[\frac{1}{2} \sum_{i=1}^M (r_{it} + \bar{r}_i) (\ln q_{it} - \overline{\ln q_i}) \right. \\ & \left. - \frac{1}{2} \sum_{i=1}^M (r_{is} + \bar{r}_i) (\ln q_{is} - \overline{\ln q_i}) \right] \\ & - \left[\frac{1}{2} \sum_{j=1}^K (s_{jt} + \bar{s}_j) (\ln x_{jt} - \overline{\ln x_j}) \right. \\ & \left. - \frac{1}{2} \sum_{j=1}^K (s_{js} + \bar{s}_j) (\ln x_{js} - \overline{\ln x_j}) \right] \end{aligned}$$

Example

- **Recall our example in session 2**
- **Two firms producing t-shirts using labour and capital (machines)**
- **Let us now assume that they face different input prices**

firm	labour		capital		cost	output
	x_1	w_1	x_2	w_2		q
A	2	80	2	100	360	200
B	4	90	1	120	480	200

- **In this example we compare productivity across 2 firms (instead of 2 periods)**
- **First we calculate the input cost shares**
- **Labour share for firm A**
$$= (2 \times 80) / (2 \times 80 + 2 \times 100) = 0.44$$
- **Labour share for firm B**
$$= (4 \times 90) / (4 \times 90 + 1 \times 120) = 0.75$$
- **Thus the capital shares are $(1 - 0.44) = 0.56$ and $(1 - 0.75) = 0.25$, respectively**

$$\begin{aligned}\text{Ln Output index} &= \ln(200) - \ln(200) \\ &= 0.0\end{aligned}$$

$$\begin{aligned}\text{Ln Input index} &= [0.5(0.44+0.75)(\ln(2)-\ln(4)) \\ &\quad + 0.5(0.56+0.25)(\ln(2)-\ln(1))] \\ &= -0.13\end{aligned}$$

$$\begin{aligned}\text{Ln TFP Index} &= 0.0 - (-0.13) \\ &= 0.13\end{aligned}$$

$$\text{TFP Index} = \exp(0.13) = 1.139$$

ie. firm A is 14% more productive than firm B

Malmquist Productivity Index Using DEA

- **We recall that the Malmquist productivity index depends upon four different distance functions.**

$$\begin{aligned} m_o(\mathbf{q}_s, \mathbf{q}_t, \mathbf{x}_s, \mathbf{x}_t) &= \left[m_o^s(\mathbf{q}_s, \mathbf{q}_t, \mathbf{x}_s, \mathbf{x}_t) \times m_o^t(\mathbf{q}_s, \mathbf{q}_t, \mathbf{x}_s, \mathbf{x}_t) \right]^{0.5} \\ &= \left[\frac{d_o^s(\mathbf{x}_t, \mathbf{q}_t)}{d_o^s(\mathbf{x}_s, \mathbf{q}_s)} \times \frac{d_o^t(\mathbf{x}_t, \mathbf{q}_t)}{d_o^t(\mathbf{x}_s, \mathbf{q}_s)} \right]^{0.5} \end{aligned}$$

- **If we have observed output and input quantity data for a cross-section of firms in periods s and t we can identify the production frontier using DEA and use them in computing the distance needed. In general we need to solve the following four linear programming problems:**

Malmquist Productivity Index Using DEA

- These four LP's are solved under CRS assumption

$$\begin{aligned} [d_o^t(\mathbf{q}_t, \mathbf{x}_t)]^{-1} &= \max_{\phi, \lambda} \phi, \\ \text{st} \quad & -\phi \mathbf{q}_{it} + \mathbf{Q}_t \boldsymbol{\lambda} \geq \mathbf{0}, \\ & \mathbf{x}_{it} - \mathbf{X}_t \boldsymbol{\lambda} \geq \mathbf{0}, \\ & \boldsymbol{\lambda} \geq \mathbf{0}, \end{aligned}$$

$$\begin{aligned} [d_o^s(\mathbf{q}_s, \mathbf{x}_s)]^{-1} &= \max_{\phi, \lambda} \phi, \\ \text{st} \quad & -\phi \mathbf{q}_{is} + \mathbf{Q}_s \boldsymbol{\lambda} \geq \mathbf{0}, \\ & \mathbf{x}_{is} - \mathbf{X}_s \boldsymbol{\lambda} \geq \mathbf{0}, \\ & \boldsymbol{\lambda} \geq \mathbf{0}, \end{aligned}$$

$$\begin{aligned} [d_o^t(\mathbf{q}_s, \mathbf{x}_t)]^{-1} &= \max_{\phi, \lambda} \phi, \\ \text{st} \quad & -\phi \mathbf{q}_{is} + \mathbf{Q}_t \boldsymbol{\lambda} \geq \mathbf{0}, \\ & \mathbf{x}_{is} - \mathbf{X}_t \boldsymbol{\lambda} \geq \mathbf{0}, \\ & \boldsymbol{\lambda} \geq \mathbf{0}, \end{aligned}$$

$$\begin{aligned} [d_o^s(\mathbf{q}_t, \mathbf{x}_t)]^{-1} &= \max_{\phi, \lambda} \phi, \\ \text{st} \quad & -\phi \mathbf{q}_{it} + \mathbf{Q}_s \boldsymbol{\lambda} \geq \mathbf{0}, \\ & \mathbf{x}_{it} - \mathbf{X}_s \boldsymbol{\lambda} \geq \mathbf{0}, \\ & \boldsymbol{\lambda} \geq \mathbf{0}, \end{aligned}$$

Calculation using DEA

- **TFP growth can be computed using DEA and we need to run 6 different DEA LPs:**
 - **VRS observation s versus frontier s**
 - **VRS observation t versus frontier l**
 - **CRS observation s versus frontier s**
 - **CRS observation t versus frontier t**
 - **CRS observation s versus frontier t**
 - **CRS observation t versus frontier s**
- **Repeat for each observation between each pair of adjacent periods**
- **We note here that some VRS LPs may not have a solution – but always guaranteed for CRS**

Calculation using DEA

Listing of Data File, EG4-DTA.TXT

1 2 Listing of Instruction File, EG4-INS.TXT
2 4
3 3

4 5	eg4-dta.txt	DATA FILE NAME
5 6	eg4-out.txt	OUTPUT FILE NAME
1 2	5	NUMBER OF FIRMS
3 4	3	NUMBER OF TIME PERIODS
1 2	1	NUMBER OF OUTPUTS
4 3	1	NUMBER OF INPUTS
3 5	1	0=INPUT AND 1=OUTPUT ORIENTATED
3 5	0	0=CRS AND 1=VRS
5 5	2	0=DEA (MULTI-STAGE), 1=COST-DEA, 2=MALMQUIST-DEA, 3=DEA (1-STAGE), 4=DEA (2-STAGE)

1 2
3 4

4 3
3 5
5 5

DEAP output

MALMQUIST INDEX SUMMARY OF ANNUAL MEANS

year	effch	techch	pech	sech	tfpch
2	0.844	1.333	0.955	0.883	1.125
3	1.000	1.000	1.000	1.000	1.000
mean	0.918	1.155	0.977	0.940	1.061

MALMQUIST INDEX SUMMARY OF FIRM MEANS

firm	effch	techch	pech	sech	tfpch
1	0.866	1.155	1.000	0.866	1.000
2	1.061	1.155	1.106	0.959	1.225
3	1.000	1.155	1.000	1.000	1.155
4	0.750	1.155	0.806	0.930	0.866
5	0.949	1.155	1.000	0.949	1.095
mean	0.918	1.155	0.977	0.940	1.061

[Note that all Malmquist index averages are geometric means]

TFP decomposition with an SFA production function

Suppose we estimate a translog production frontier of the following form using panel data under the standard distributional assumptions

$$\begin{aligned} \ln q_{it} = & \beta_0 + \sum_{n=1}^N \beta_n \ln x_{nit} + \frac{1}{2} \sum_{n=1}^N \sum_{j=1}^N \beta_{nj} \ln x_{nit} \ln x_{nit} \\ & + \sum_{n=1}^N \beta_{tn} t \ln x_{nit} + \beta_t t + \frac{1}{2} \beta_{tt} t^2 + v_{it} - u_{it}, \\ & i=1,2,\dots,I, \quad t=1,2,\dots,T, \end{aligned}$$

Efficiency change = TE_{it}/TE_{is} .

where $TE_{it} = E(\exp(-u_{it})|e_{it})$,

$$\text{Technical change} = \exp \left\{ \frac{1}{2} \left[\frac{\partial \ln q_{is}}{\partial s} + \frac{\partial \ln q_{it}}{\partial t} \right] \right\}.$$

$$\text{Scale change} = \exp \left\{ \frac{1}{2} \sum_{n=1}^N [\varepsilon_{nis} SF_{is} + \varepsilon_{nit} SF_{it}] \ln(x_{nit} / x_{nis}) \right\},$$

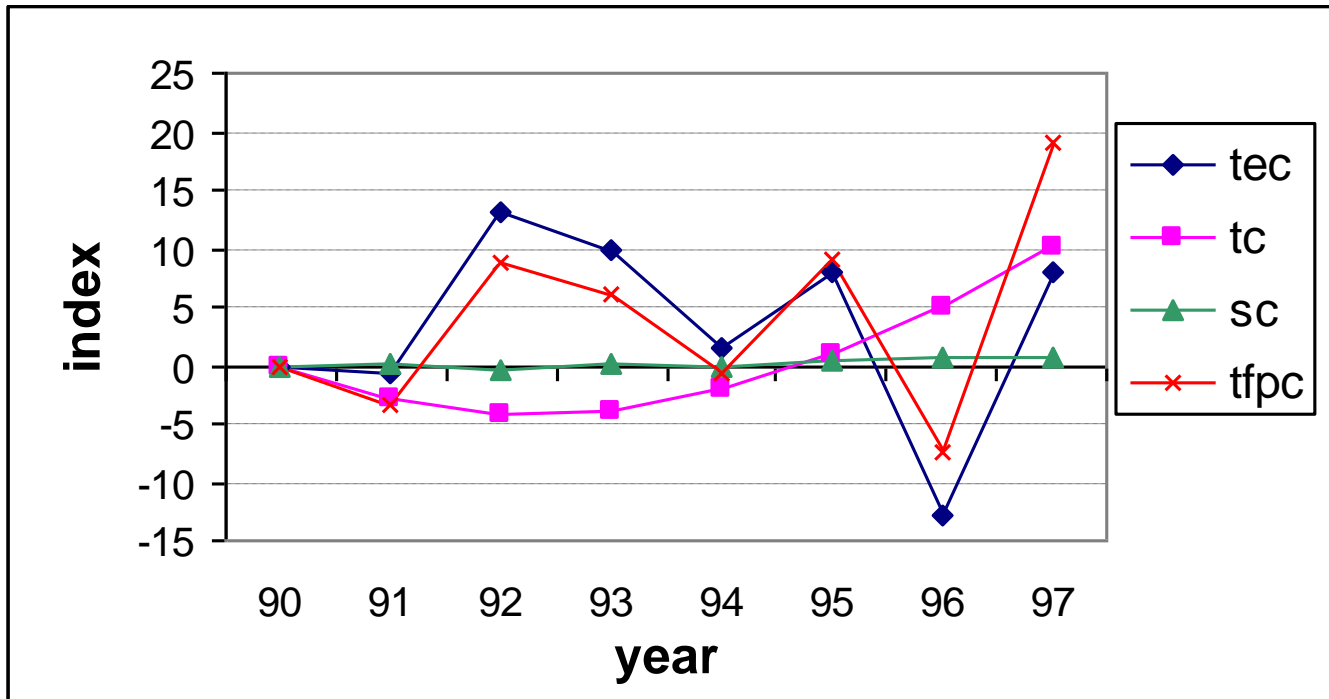
$$\text{where } SF_{is} = (\varepsilon_{is} - 1) / \varepsilon_{is}, \quad \varepsilon_{is} = \sum_{n=1}^N \varepsilon_{nis} \quad \text{and} \quad \varepsilon_{nis} = \frac{\partial \ln q_{is}}{\partial \ln x_{nis}}.$$

TFP decomposition with an SFA production function

Maximum-Likelihood Estimates of the Stochastic Frontier Model

Coefficient	Estimate	Standard Error	t-ratio
β_0	0.342	0.033	10.230
β_1	0.453	0.063	7.223
β_2	0.286	0.062	4.623
β_3	0.232	0.036	6.391
β_t	0.015	0.007	2.108
β_{11}	-0.509	0.225	-2.263
β_{12}	0.613	0.169	3.622
β_{13}	0.068	0.144	0.475
β_{1t}	0.005	0.024	0.215
β_{22}	-0.539	0.264	-2.047
β_{23}	-0.159	0.148	-1.073
β_{2t}	0.024	0.026	0.942
β_{33}	0.021	0.093	0.230
β_{3t}	-0.034	0.018	-1.893
β_{tt}	0.015	0.007	2.176
σ_s^2	0.223	0.025	9.033
γ	0.896	0.033	27.237
Log-likelihood	-70.592		

TFP decomposition – numerical example



References:

- Merton, R.C. (1992). Financial innovation and economic performance, *Journal of Applied Corporate Finance*, 4(4), 12-22.
- Van Horne, J.C. (1980). Of financial innovations and excesses, *Journal of Finance*, 40(3), 621-36.
- Scott Frame and Lawrence White (2002), Technological Change, Financial Innovation and diffusion in Banking, Working Paper 2002, Federal Reserve Bank of Atlanta.
- Solomon Tadesse (2005), Financial Development and Technology, Working Paper No. 749, University of Michigan.
- Solow, Robert M., (1957). Technical Change and the Aggregate Production Function, *Review of Economics and Statistics*, 39 (August), 312-320.
- Van Horne, James. Of Financial Innovations and Excesses. *Journal of Finance*. Volume 40(3), July 1985. pp 621-636.
- Miller, Merton H. "Financial Innovation: Achievements and Prospects." *Journal of Applied Corporate Finance*. Volume 4, Winter 1992. pp 4-12.
- Josh Lerner (2006), The new financial thing: The origins of financial innovations, *Journal of Financial Economics* 79 ,223–255
- Schumpeter, Joseph A., (1950), *Capitalism, Socialism, and Democracy*, 3rd ed.: Harper & Brothers, New York (1) (PDF) Financial Innovation and Development of Commercial Banks in Sri Lanka. Available