MacroEconometric Forecasting

Topic:

Statistical Properties of Times Series Data

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Introduction

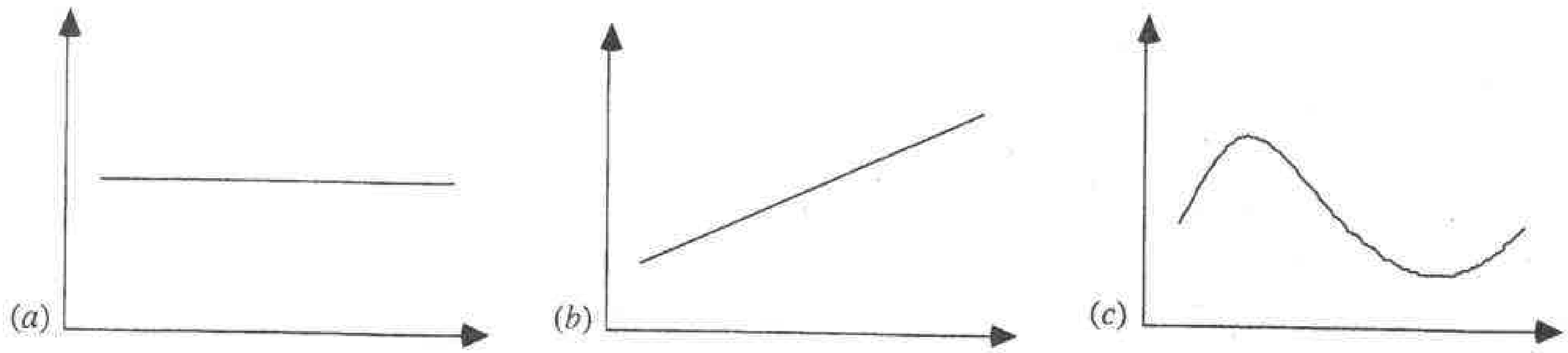
- Regression analysis useful in short-term forecasting, but flawed
- A better approach: base the forecast of a variable on its own history
  - Avoids need to specify a causal relationship and to predict the values of explanatory variables
- Our focus in this chapter is on *time series* methods for forecasting.

Forecasting with Time-Series Models

- Two important features:
  - Uses historical data for the phenomenon we wish to forecast.
  - We seek a routine calculation to apply to a large number of cases and that may be automated, without relying on qualitative information about the underlying phenomena.
- Short-term forecasts are often used in situations that involve forecasting many different variables at frequent intervals.

An Hypothesized Model

- The major components of such a model are usually the following:
  - A base level
  - A trend
  - Cyclic fluctuations



Three Components of Time Series Behavior

The Moving-Average Model

- The n-period **moving average** builds a forecast by averaging the observations in the most recent n periods:

$$A_t = (x_t + x_{t-1} + \ldots + x_{t-n+1}) / n$$

- where x_t represents the observation made in period t, and A_t denotes the moving average calculated after making the observation in period t.

Convention

- We adopt the following convention for the steps in forecasting:
  - Make the observation in period t
  - Carry out the necessary calculations
  - Use the calculations to forecast period $(t + 1)$

	A	B	C	D	E	F	G
1	**Moving Average Example**						
2							
3	*Period*		***4-week Moving Average***				
4	*t*	*Observed*	*A(t)*	*Forecast*	*Error*	*Deviation*	*Percent*
5	1	73					
6	2	106					
7	3	76					
8	4	89	86.00				
9	5	106	94.25	86.00	20.00	20.00	19%
10	6	113	96.00	94.25	18.75	18.75	17%
11	7	96	101.00	96.00	0.00	0.00	0%
12	8	66	95.25	101.00	-35.00	35.00	53%
13	9	104	94.75	95.25	8.75	8.75	8%
14	10	76	85.50	94.75	-18.75	18.75	25%
15	11			85.50			
16							

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Worksheet for Calculating Moving Averages

What Number of Periods to Include in Moving Average?

- There is no definitive answer, but there is a trade-off to consider.
- Suppose the mean of the underlying process remains stable:
  If we include very few data points, then the moving average exhibits more variability than if we include a larger number of data points. In that sense, we get more stability from including more points.
- Suppose there is an unanticipated change in the mean of the underlying process:
  If we include very few data points, our moving average will tend to track the changed process more closely than if we include a larger number of data points. In that case, we get more responsiveness from including fewer points.

	A	B	C	D	E	F	G	H	I	J	K	L	M
1	**Moving Average Example**												
2													
3	*Period*		***4-week Moving Average***						***6-week Moving Average***				
4	*t*	*Observed*	*A(t)*	*Forecast*	*Error*	*Deviation*	*Percent*		*A(t)*	*Forecast*	*Difference*	*Deviation*	*Percent*
5	1	100											
6	2	100											
7	3	100											
8	4	100	100.00										
9	5	100	100.00	100.00	0.00	0.00	0%						
10	6	100	100.00	100.00	0.00	0.00	0%		100.00				
11	7	100	100.00	100.00	0.00	0.00	0%		100.00	100.00	0.00	0.00	0%
12	8	100	100.00	100.00	0.00	0.00	0%		100.00	100.00	0.00	0.00	0%
13	9	100	100.00	100.00	0.00	0.00	0%		100.00	100.00	0.00	0.00	0%
14	10	180	120.00	100.00	80.00	80.00	44%		113.33	100.00	80.00	80.00	44%
15	11	180	140.00	120.00	60.00	60.00	33%		126.67	113.33	66.67	66.67	37%
16	12	180	160.00	140.00	40.00	40.00	22%		140.00	126.67	53.33	53.33	30%
17	13	180	180.00	160.00	20.00	20.00	11%		153.33	140.00	40.00	40.00	22%
18	14	180	180.00	180.00	0.00	0.00	0%		166.67	153.33	26.67	26.67	15%
19	15	180	180.00	180.00	0.00	0.00	0%		180.00	166.67	13.33	13.33	7%
20	16	180	180.00	180.00	0.00	0.00	0%		180.00	180.00	0.00	0.00	0%
21	17	180	180.00	180.00	0.00	0.00	0%		180.00	180.00	0.00	0.00	0%
22	18	180	180.00	180.00	0.00	0.00	0%		180.00	180.00	0.00	0.00	0%
23	19	180	180.00	180.00	0.00	0.00	0%		180.00	180.00	0.00	0.00	0%
24	20	180	180.00	180.00	0.00	0.00	0%		180.00	180.00	0.00	0.00	0%
25													
26					*MSE*	*MAD*	*MAPE*				*MSE*	*MAD*	*MAPE*
27		*For periods 10-15*			2000.00	33.33	18.5%				2696.30	46.67	25.9%
28													

Moving-Average Calculations in a Stylized Example

Moving Average Example

Period		***4-week Moving Average***					***6-week Moving Average***				
t	*Observed*	*A(t)*	*Forecast*	*Error*	*Deviation*	*Percent*	*A(t)*	*Forecast*	*Difference*	*Deviation*	*Percent*
1	73										
2	106										
3	76										
4	89	86.00									
5	106	94.25	86.00	20.00	20.00	19%					
6	113	96.00	94.25	18.75	18.75	17%	93.83				
7	96	101.00	96.00	0.00	0.00	0%	97.67	93.83	2.17	2.17	2%
8	66	95.25	101.00	-35.00	35.00	53%	91.00	97.67	-31.67	31.67	48%
9	104	94.75	95.25	8.75	8.75	8%	95.67	91.00	13.00	13.00	13%
10	76	85.50	94.75	-18.75	18.75	25%	93.50	95.67	-19.67	19.67	26%
			85.50					93.50			

Comparison of 4-week and 6-week Moving Averages

Measures of Forecast Accuracy

- MSE: the Mean Squared Error between forecast and actual
- MAD: the Mean Absolute Deviation between forecast and actual
- MAPE: the Mean Absolute Percent Error between forecast and actual

$$\text{MSE} = \frac{1}{(u-v+1)}\sum_{t=u}^{v}(F_t - x_t)^2$$

$$\text{MAD} = \frac{1}{(u-v+1)}\sum_{t=u}^{v}\left|F_t - x_t\right|$$

$$\text{MAPE} = \frac{1}{(u-v+1)}\sum_{t=u}^{v}\left|\frac{F_t - x_t}{x_t}\right|$$

Comparison of Measures of Forecast Accuracy

- The MAD calculation and the MAPE calculation are similar: one is absolute, the other is relative.
- MAPE is usually reserved for comparisons in which the magnitudes of two cases are different.

Excel Tip: Moving Average Calculations

- Excel's Data Analysis tool (Data▸Analysis▸Data Analysis▸Moving Average) contains an option for calculating moving averages.
- Excel assumes that the data appear in a single column, and the tool provides an option of recognizing a title for this column, if it is included in the data range.
- Other options include a graphical display of the actual and forecast data and a calculation of the standard error after each forecast.

The Exponential Smoothing Model

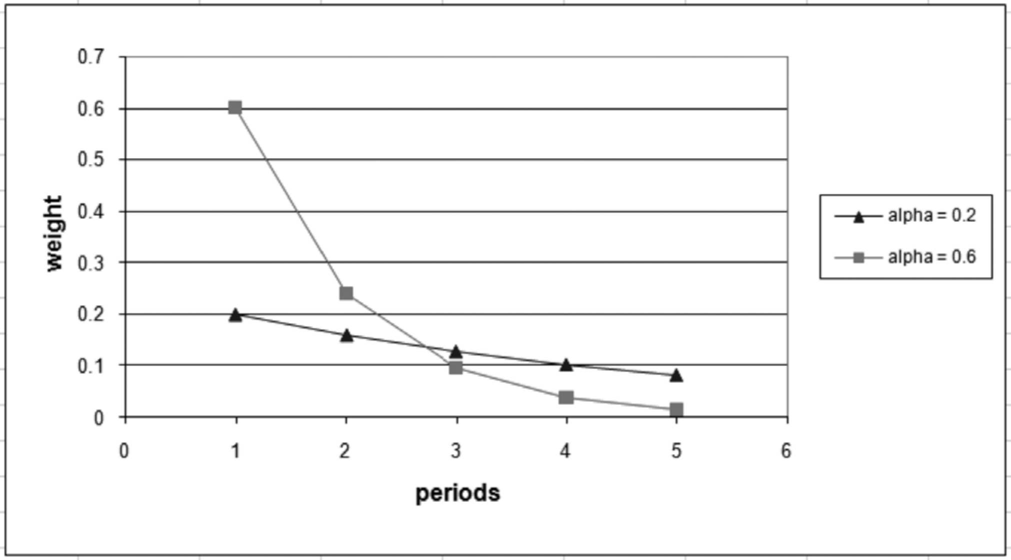
- **Exponential smoothing** weighs recent observations more than older ones.

$$S_t = \alpha x_t + (1 - \alpha) S_{t-1}$$

- Where α (the **smoothing constant)** is some number between zero and one.
- S_t is the **smoothed value** of the observations (our "best guess" as to the value of the mean)
- Our forecasting procedure sets the forecast $F_{t+1} = S_t$.

Smoothing Coefficients

Alpha	*k =*	*1*	*2*	*3*	*4*	*5*	*Total*
0.2	*weight*	0.2	0.16	0.128	0.1024	0.08192	0.672
0.6	*weight*	0.6	0.24	0.096	0.0384	0.01536	0.990



Comparison of Weights Placed on *k*-year-old Data

Exponential Smoothing Example

	Alpha	**0.2**									
Period							***4-week Moving Average***				
t	*Observed*	*S(t)*	*Forecast*	*Error*	*Deviation*	*Percent*	*A(t)*	*Forecast*	*Difference*	*Deviation*	*Percent*
1	73	73.00									
2	106	79.60	73.00								
3	76	78.88	79.60								
4	89	80.90	78.88				86.00				
5	106	85.92	80.90	25.10	25.10	24%	94.25	86.00	20.00		19%
6	113	91.34	85.92	27.08	27.08	24%	96.00	94.25	18.75	18.75	17%
7	96	92.27	91.34				101.00				
8	66	87.02	92.27	-26.27	26.27	40%	95.25	101.00	-35.00	35.00	53%
9	104	90.41	87.02	16.98	16.98	16%	94.75	95.25	8.75	8.75	8%
10	73	86.93	90.41	-17.41	17.41	24%	84.75	94.75	-21.75	21.75	30%
11	77	84.94	86.93	-9.93	9.93	13%	80.00	84.75	-7.75	7.75	10%
12	119	91.76	84.94	34.06	34.06	29%	93.25	80.00	39.00	39.00	33%
13	84	90.20	91.76	-7.76	7.76	9%	88.25	93.25	-9.25	9.25	11%
14	71	86.36	90.20	-19.20	19.20	27%	87.75	88.25	-17.25	17.25	24%
15	88	86.69	86.36	1.64	1.64	2%	90.50	87.75	0.25	0.25	0%
16	97	88.75	86.69	10.31	10.31	11%	85.00	90.50	6.50	6.50	7%
17	69	84.80	88.75	-19.75	19.75	29%	81.25	85.00	-16.00	16.00	23%
18	117	91.24	84.80	32.20	32.20	28%	92.75	81.25	35.75	35.75	31%
19	80	88.99	91.24	-11.24	11.24	14%	90.75	92.75	-12.75	12.75	16%
20	73	85.79	88.99	-15.99	15.99	22%	84.75	90.75	-17.75	17.75	24%
				MSE	*MAD*	*MAPE*			*MSE*	*MAD*	*MAPE*
	For periods 5-20			391.99	17.47	19.7%			409.02	16.66	19.1%

Initial value S(1) = X(1)

S(t) = αx(t) + (1-α)S(t-1)

A(t) = [X(t)+X(t-1)+X(t-2)+X(t-3)] / 4

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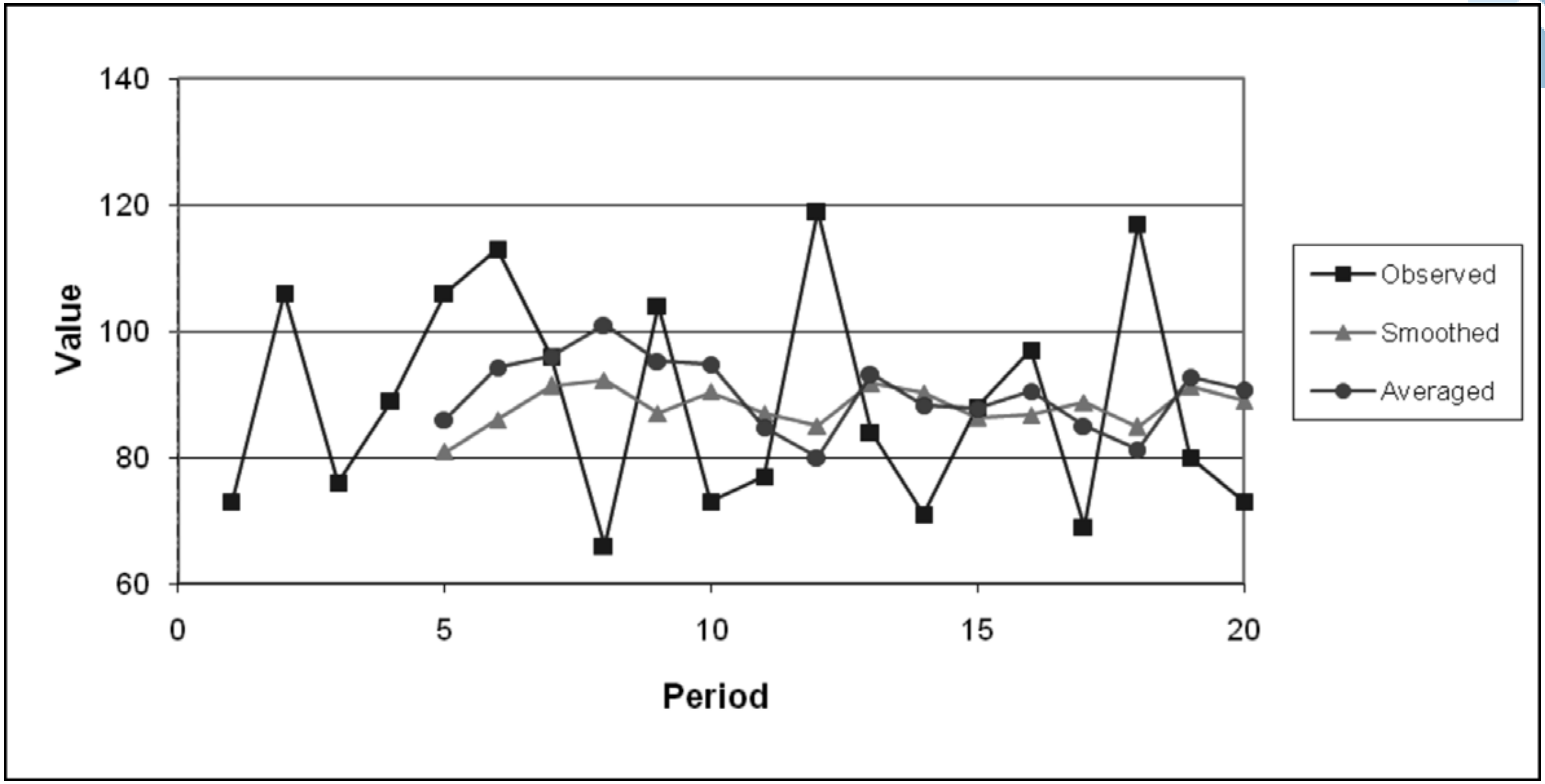
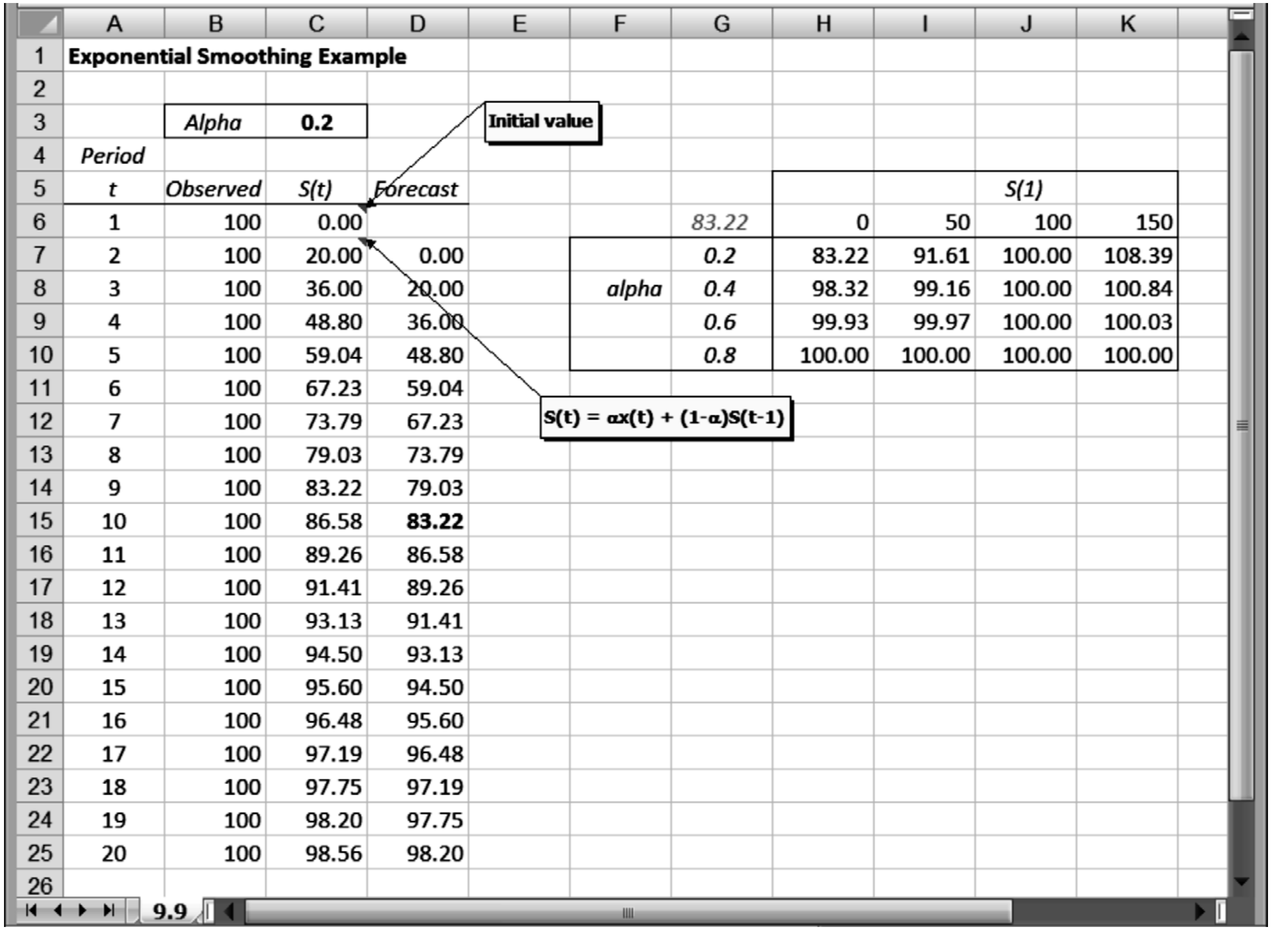
Worksheet for Exponential Smoothing Calculations



Comparison of Smoothed and Averaged Forecasts



Exponential Smoothing Example

Alpha	**0.2**

Period t	*Observed*	*S(t)*	*Forecast*
1	100	0.00	
2	100	20.00	0.00
3	100	36.00	20.00
4	100	48.80	36.00
5	100	59.04	48.80
6	100	67.23	59.04
7	100	73.79	67.23
8	100	79.03	73.79
9	100	83.22	79.03
10	100	86.58	**83.22**
11	100	89.26	86.58
12	100	91.41	89.26
13	100	93.13	91.41
14	100	94.50	93.13
15	100	95.60	94.50
16	100	96.48	95.60
17	100	97.19	96.48
18	100	97.75	97.19
19	100	98.20	97.75
20	100	98.56	98.20

		S(1)			
	83.22	0	50	100	150
	0.2	83.22	91.61	100.00	108.39
alpha	*0.4*	98.32	99.16	100.00	100.84
	0.6	99.93	99.97	100.00	100.03
	0.8	100.00	100.00	100.00	100.00

Exponential Smoothing Calculations in a Stylized Example

Excel Tip: Implementing Exponential Smoothing

- Excel's Data Analysis tool contains an option for calculating forecasts using exponential smoothing.
- The Exponential Smoothing module resembles the Moving Average module, but instead of asking for the number of periods, it asks for the **damping factor**, which is the complement of the smoothing factor, or $(1 - \alpha)$.
- Options exist for chart output and for a calculation of the standard error.

Basic Model with Trend in Data | *Trend*: 10

Alpha: **0.2**

Period *t*	*Observed*	*S(t)*	*Forecast*	*Error*
1	100	100.00		
2	100	100.00		
3	100	100.00	100.00	0.00
4	110	102.00	100.00	-10.00
5	120	105.60	[illegible]	[illegible]
6	130	110.48	[illegible]	[illegible]
7	140	116.38	110.48	-29.52
8	150	123.11	116.38	-33.62
9	160	130.49	123.11	-36.89
10	170	138.39	130.49	-39.51
11	180	146.71	138.39	-41.61
12	190	155.37	146.71	-43.29
13	200	164.29	155.37	-44.63
14	210	173.44	164.29	-45.71
24	310	270.37	260.46	-49.54
25	320	280.30	270.37	-49.63

Initial value

S(t) = αx(t) + (1-α)S(t-1)

Sensitivity of Error to α at t=25

α	Error
	-49.63
0.1	-90.15
0.2	-49.63
0.3	-33.32
0.4	-25.00
0.5	-20.00
0.6	-16.67
0.7	-14.29
0.8	-12.50
0.9	-11.11

7.7 7.8 7.9 7.10

Trend Model Calculations with a Trend in the Data

Holt's Method

- This more flexible procedure uses two smoothing constants, as shown in the following formulas:

$$S_t = \alpha x_t + (1 - \alpha)(S_{t-1} + T_{t-1})$$

$$T_t = \beta(S_t - S_{t-1}) + (1 - \beta)T_{t-1}$$

$$F_{t+1} = S_t + T_t$$

Stylized Example for Holt's Method

Alpha	0.2	Beta	0.5

Period t	Observed	S(t)	T(t)	Forecast	Error
1	100	100.0	0.0		
2	100	100.0	0.0		
3	100	100.0	0.0	100.0	0.0
4	110	102.0	1.0	100.0	10.0
5	120	106.4	2.7	103.0	17.0
6	130	113.3	4.8	109.1	20.9
7	140	122.5	7.0	118.1	21.9
8	150	133.6	9.0	129.4	[illegible]
9	160	146.1	10.8	142.6	17.4
10	170	159.5	12.1	156.0	13.1
11	180	173.3	12.9	[illegible]	[illegible]
12	190	187.0	13.3	186.2	3.8
13	200	200.2	13.3	200.3	-0.3
14	210	212.8	12.9	213.5	-3.5
15	220	224.6	12.4	225.7	-5.7
24	310	309.0	9.0	308.8	1.2
25	320	318.4	9.2	318.1	1.9

$F(t+1) = S(t)+T(t)$

$T(t)=\beta(S(t)-S(t-1))+(1-\beta)T(t-1)$

$S(t) = \alpha x(t) + (1-\alpha)(S(t-1)+T(t-1))$

Sensitivity of Error at t=25

		Beta			
	1.9	*0.2*	*0.4*	*0.6*	*0.8*
Alpha	*0.2*	-3.9	-0.6	2.8	0.9
	0.4	-0.2	0.1	-0.1	0.0
	0.6	0.1	0.0	0.0	0.0
	0.8	0.1	0.0	0.0	0.0

9.9 | 9.10 | 9.11 | 9.12

Holt's Method with a Trend in the Data

Exponential Smoothing with Trend and Cyclical Factors

- We can take the exponential smoothing model further and include a *cyclical* (or seasonal) factor.
- For a cyclical effect, there are two types of models: an *additive model* and a *multiplicative model*.
- See text for formulas.

Summary

- Moving averages and exponential smoothing are widely used for routine short-term forecasting.

Summary

- By making projections from past data, these methods assume that the future will resemble the past.

Summary

- However, the exponential smoothing procedure is sophisticated enough to permit representations of a linear trend and a cyclical factor in its calculations.
- Exponential smoothing procedures are adaptive.

Summary

- Implementing an exponential smoothing procedure requires that initial values be specified and a smoothing factor be chosen.

Summary

- The smoothing factor should be chosen to trade off stability and responsiveness in an appropriate manner.

Summary

- Although Excel contains a Data Analysis tool for calculating moving-average forecasts and exponentially-smoothed forecasts, the tool does not accommodate the most powerful version of exponential smoothing, which includes trend and cyclical components.

Reference and source

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