RESERVOIR OPERATION AND RESERVOIR SIZING USING LINEAR PROGRAMMING (LP)

INTRODUCTION

In the previous lectures, we discussed applications of LP in deciding the optimal irrigation allocation and water quality management. In this lecture we will discuss about the applications of LP in modeling reservoir operation and reservoir sizing.

RESERVOIR OPERATION

Reservoir operation policies are developed to enable the operator to take appropriate decision. The reservoir operation policy indicates the amount of water to be released based on the state of the reservoir, demands and the likely inflow to the reservoir. The release from a single purpose reservoir can be done with the objective of maximizing the benefits. For multi-purpose reservoirs, there is a need to optimally allocate the releases among purposes. The simplest of the operation policies is the standard operation policy (SOP). According to SOP, if the water available (storage, S_t+ inflow, I_t) at a particular period is less than the demand D_t, then all the available water is released. If the available water is more than the demand but less than demand + storage capacity K, then release is equal to the demand. If after releasing the demands, there is no space for extra water, then the excess water is also released. This is shown graphically in figure 1.

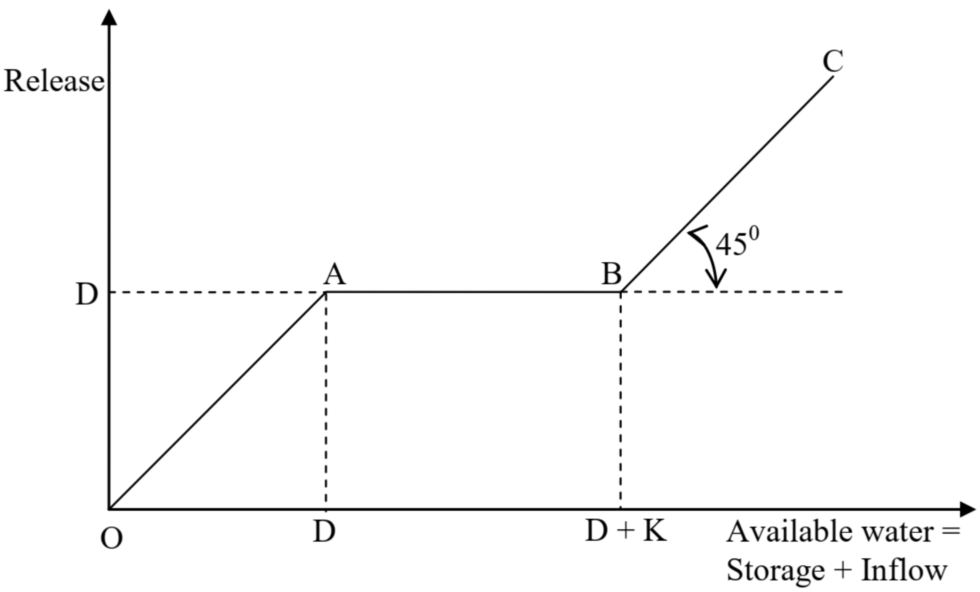

Fig. 1 Standard Operating Policy

Along *OA:* Release = water available; reservoir will be empty after release.

Along *AB:* Release = demand; excess water is stored in the reservoir (filling phase).

At A: Reservoir is empty after release.

At *B:* Reservoir is full after release.

Along *BC:* Release = demand + excess of availability over the capacity (spill)

The releases according to the SOP need not be optimum. The optimization of reservoir operation is done often by linear programming (LP) and dynamic programming (DP). DP will be explained in the next module.

Derivation of optimal operating policy using LP

Consider a reservoir of capacity *K*. The optimization problem is to determine the releases R_t that optimize an objective function satisfying all the constraints. The objective function can be a function of storage volume or release. The typical constraints in a reservoir optimization model include conservation of mass and other hydrological and hydraulic constraints, minimum and maximum storage and release, hydropower and water requirements as well as hydropower generation limitations.

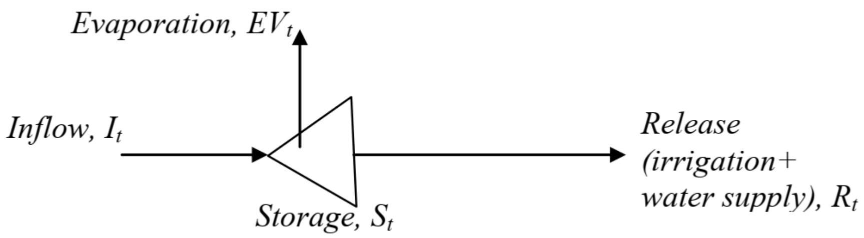

Fig. 2 Single reservoir operation

Consider the objective of meeting the demands to the extent possible i.e., maximizing the releases. The optimization model can be formulated as:

$$\text{Maximize} \sum_t R_t$$

Subject to

(i) Hydraulic constraints as defined by the reservoir continuity equation

$$S_{t+1} = S_t + I_t - EV_t - R_t - O_t \quad \text{for all } t$$

where O_t is the outflow. The constraints for outflow are

$O_t = 0$ if $S_t + I_t - EV_t - R_t \leq K$

$= K - [S_t + I_t - EV_t - R_t]$ if $S_t + I_t - EV_t - R_t > K$

(ii) Reservoir capacity

$S_t \leq K - K_d$ for all t, where K_d is the dead storage

or simply $S_t \leq K$

$S_t \geq 0$ for all t.

(iii) Target demand

$R_t \leq D_t$ for all t.

$R_t \geq 0$ for all t.

Large LP problems can be solved very efficiently using LINGO - Language for INteractive General Optimization, LINDO Systems Inc, USA

Example

Derive an optimal operating policy for a reservoir to meet a long-term objective. Single reservoir operation with deterministic inflows. K = 400.

Table 1. Inflow, evaporation and demand values of the reservoir

t	Inflows	Evaporation	Demand
1	90.7	10	71.5
2	450.6	8	140.5
3	380.4	8	140.5
4	153.2	8	80.6
5	120	6	30.6
6	55	6	240.6
7	29.06	5	241.7
8	24.27	6	190.5
9	30.87	6	98.1
10	15.9	8	0
11	12.8	9	0
12	15.9	10	0

Solution

Objective function Maximize $\sum_t R_t$

Subject to

$S_{t+1} = S_t + I_t - EV_t - R_t - O_t$ for all t where O_t is the outflow

$$O_t = 0 \qquad \text{if} \quad S_t + I_t - EV_t - R_t \leq K$$

$$= K - [S_t + I_t - EV_t - R_t] \qquad \text{if} \quad S_t + I_t - EV_t - R_t > K$$

$$S_t \leq 400\ ;\ S_t \geq 0;\ R_t \leq D_t;\ R_t \geq 0 \qquad \text{for all t.}$$

The problem is solved using LINGO and the solution is given in table 2.

Table2. LP solution

t	S_t	I_t	D_t	R_t	EV_t	S_{t+1}	O_t
1	17.6	90.7	71.5	71.5	10	26.8	0
2	26.8	450.6	140.5	140.5	8	328.9	0
3	328.9	380.4	140.5	140.5	8	400	160.8
4	400	153.2	80.6	80.6	8	400	64.6
5	400	120	30.6	30.6	6	400	83.4
6	400	55	240.6	240.6	6	208.4	0
7	208.4	29.06	241.7	232.21	5	0.25	0
8	0.25	24.27	190.5	18.27	6	0.25	0
9	0.25	30.87	98.1	25.12	6	0	0
10	0	15.9	0	0	8	7.9	0
11	7.9	12.8	0	0	9	11.7	0
12	11.7	15.9	0	0	10	17.6	0

The rule curve derived is shown in figure 3.

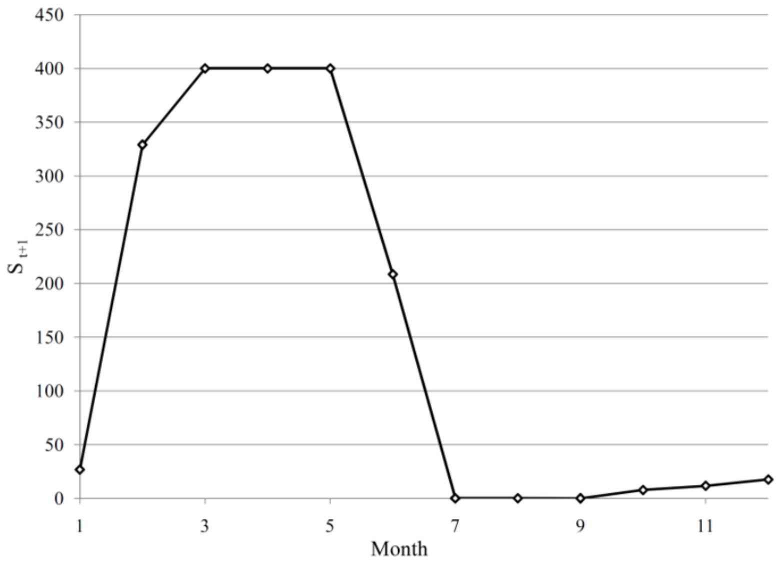

Fig. 3 Rule curve

The optimal operation of a multipurpose single and multiple reservoir systems are discussed in module 5.

RESERVOIR SIZING

In many situations, annual demand may be less than the total inflow to a particular site. However, the time distribution of demand and inflows may not match, which in turn result in surplus in some periods and deficit in some other periods. Hence, there is a need of storage structure i.e., reservoir to store water in periods of excess flow and make it available when there is a deficit. In order to enable regulation of the storage to best meet the specified demands, the reservoir storage capacity should be enough. The problem of reservoir sizing involves determination of the required storage capacity of the reservoir when inflows and demands in a sequence of periods are given. Reservoir capacity can be determined using two methods: Mass curve method and Sequent peak algorithm method.

Mass diagram method

It was developed by W. Rippl (1883). A mass curve is a plot of the cumulative flow volumes as a function of time. Mass curve analysis is done using a graphical method called Ripple's method. It involves finding the maximum positive cumulative difference between a sequence of pre-specified (desired) reservoir releases R_t and known inflows Q_t (as shown in figure 4). One can visualize this as starting with a full reservoir, and going through a sequence of simulations in which the inflows and releases are added and subtracted from that initial storage volume value. Doing this over two cycles of the record of inflows will identify the maximum deficit volume associated with those inflows and releases. This is the required reservoir storage.

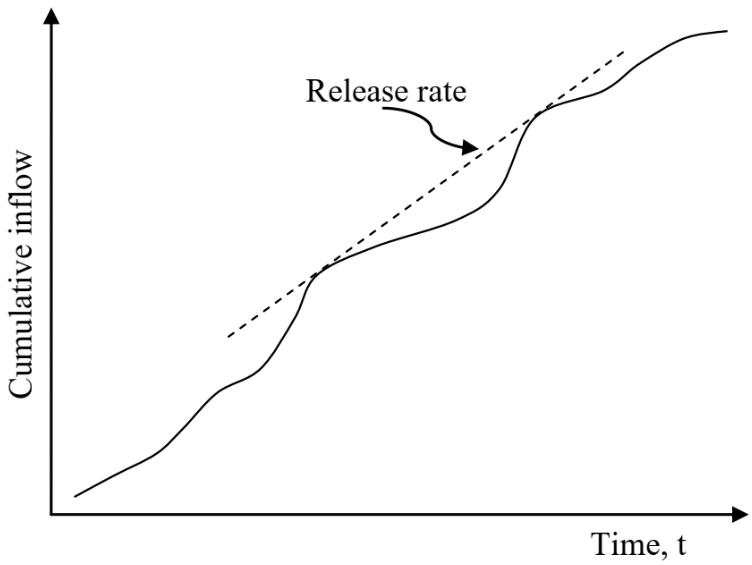

Fig. 4 Typical mass curve

Sequent Peak Algorithm

This algorithm computes the cumulative sum of differences between the inflows and reservoir releases for all periods t over the time interval [0, T]. Let K_t be the maximum total storage requirement needed for periods 1 through period t and R_t be the required release in period t, and Q_t be the inflow in that period. Setting K_0 equal to 0, the procedure involves calculating K_t using equation below for upto twice the total length of record. Algebraically,

$$K_t = \begin{cases} R_t - Q_t + K_{t-1} & \text{if positive} \\ 0 & \text{otherwise} \end{cases}$$

The maximum of all K_t is the required storage capacity for the specified releases R_t and inflows, Q_t.

Formulation of reservoir sizing using LP

Linear Programming can be used to obtain reservoir capacity more elegantly by considering variable demands and evaporation rates. The optimization problem is

Minimize K_a

where K_a is the active storage capacity

Subject to

(i) Hydraulic constraints as defined by the reservoir continuity equation

$$S_{t+1} = S_t + I_t - EV_t - R_t - O_t \quad \text{for all } t$$

(ii) Reservoir capacity

$$S_t \leq K_a \qquad \text{for all } t$$

$$S_{T+1} = S_t \qquad \text{where T is the last period.}$$

(iii) Target demands

$$R_t \geq D_t \qquad \text{for all t.}$$

STORAGE YIELD

A complementary problem to reservoir capacity estimation can be done by maximizing the yield. Firm yield is the constant (or largest) quantity of flow that can be released at all times. It is the flow magnitude that is equaled or exceeded 100% of time for a historical sequence of flows. Linear Programming can be used to maximize the yield, R (per period) from a reservoir of given capacity, K. The optimization problem can be stated as:

Maximize R

Subject to

(i) Storage continuity equation

$$S_{t+1} = S_t + I_t - EV_t - R_t - O_t \qquad \text{for all } t$$

(ii) Reservoir capacity

$$S_t \leq K_a \qquad \text{for all } t$$

$$S_{T+1} = S_t \qquad \text{where T is the last period.}$$

GROUNDWATER SYSTEMS

INTRODUCTION

Groundwater management deals with planning, implementation, development and operation of water resources containing groundwater. Numerical-simulation models have been used extensively for understanding the flow characteristics of aquifers and evaluate the groundwater resource. While simulation models attempt various scenarios to find the best objective, optimization models directly consider the management objective taking care of all the constraints. In this lecture, we will discuss about the governing equations in these modes and various management models.

GROUNDWATER HYDROLOGY

Subsurface water is stored underneath in subsurface formations called aquifers. In an unconfined aquifer, the upper surface is the water table itself. On the other hand, a confined aquifer is confined under pressure greater than the atmospheric. A confined aquifer may be confined between two impermeable layers. An aquifer serves two functions: storage and transmission.

Storage function is exhibited through porosity Φ, specific yield S_y and storage coefficient *S*. Transmission function is exhibited through the permeability property (coefficient of permeability *K*). Porosity is the measure of the amount of water an aquifer can hold. Specific yield is the water drained from a saturated sample of unit volume. Specific retention is the water retained in the unit volume. Porosity is the sum of specific yield and specific retention. Storage coefficient is the volume of water an aquifer releases or stores per unit surface area per unit decline of head.

Permeability is a measure of the ease of movement of water through aquifers. The coefficient of permeability or hydraulic conductivity is the rate of flow of water through a unit cross-sectional area under a unit hydraulic gradient. Transmissivity, *T* is the rate of flow of water through a vertical strip of unit width extending the saturated thickness of the aquifer under a unit hydraulic gradient. Therefore,

$T = K\,b$ for a confined aquifer where *b* is the saturated thickness of aquifer

$T = K\,h$ for a confined aquifer where *h* is the head (saturated thickness)

Darcy's law

The flow thorough an aquifer is expressed by Darcy's law which states that flow rate through a porous media is proportional to the head loss and inversely proportional to the length of flow path. It can be expressed as

$$v = -K\frac{\partial h}{\partial l} \tag{1}$$

where *v* is the velocity or specific discharge, *l* is the length of flow along the average direction and $\frac{\partial h}{\partial l}$ is the rate of headloss per unit length. Then, the total discharge, *q* is

$$q = Av = -KA\frac{\partial h}{\partial l} \tag{2}$$

SIMULATION OF GROUNDWATER SYSTEMS

Governing equations:

Darcy's law in terms of transmissivity is

$$v = -\frac{T}{b}\frac{\partial h}{\partial l} \quad \text{for confined aquifers} \qquad (3)$$

$$v = -\frac{T}{h}\frac{\partial h}{\partial l} \quad \text{for unconfined aquifers} \qquad (4)$$

Considering a two-dimensional horizontal flow as shown by a rectangular control volume element in Figure 1, the general equations of flow can be expressed as:

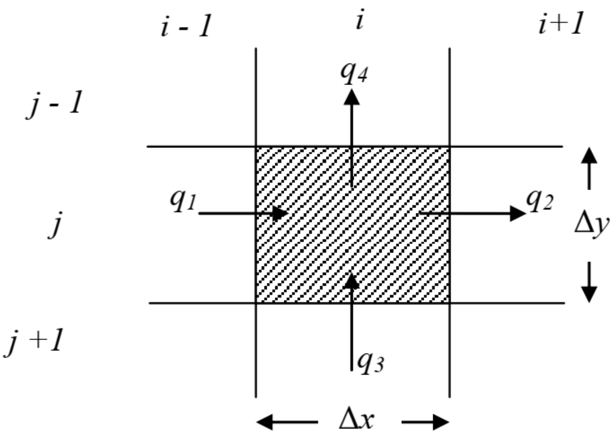

Fig. 1

The flow discharge $q = Av$ for four sides

$$q_1 = -T_x \Delta y \left(\frac{\partial h}{\partial x}\right)_1 \qquad (5)$$

$$q_2 = -T_x \Delta y \left(\frac{\partial h}{\partial x}\right)_2 \qquad (6)$$

$$q_3 = -T_y \Delta x \left(\frac{\partial h}{\partial x}\right)_3 \qquad (7)$$

$$q_4 = -T_y \Delta x \left(\frac{\partial h}{\partial x}\right)_4 \qquad (8)$$

where $A = \Delta x.\ h$ for unconfined case or $A = \Delta x.\ b$ for confined case and assuming constant transmissivities along the x and y directions. $\left(\frac{\partial h}{\partial x}\right)_1, \left(\frac{\partial h}{\partial x}\right)_2 ...$ are the hydraulic gradients at sides 1,2 ,… respectively.

The rate at which water is stored or released in the element is

$$q_5 = S\,\Delta x\,\Delta y\left(\frac{\partial h}{\partial t}\right)_4 \tag{9}$$

where S is the storage coefficient of the element.

The flow rate for constant net withdrawal or recharge for time Δt

$$q_6 = q_t \tag{10}$$

Applying continuity law,

$$q_1 - q_2 + q_3 + q_4 = q_5 + q_6 \tag{11}$$

Substituting eqns. 5 – 10 in above eqn, and dividing by Δx Δy and simplifying, the final form of eqn. 11 will be

$$T_x \frac{\partial^2 h}{\partial x^2} + T_y \frac{\partial^2 h}{\partial y^2} = S\frac{\partial h}{\partial t} + W \tag{12}$$

where $W = q / \Delta x\ \Delta y$.

These equations can be written in finite difference form and solved for each rectangular element. The partial derivatives in eqns. 5-9 can be expressed in finite difference form as,

$$\begin{aligned}
\left(\frac{\partial h}{\partial x}\right)_1 &= \left(\frac{h_{i-1,j,t} - h_{i,j,t}}{\Delta x_i}\right)\\
\left(\frac{\partial h}{\partial x}\right)_2 &= \left(\frac{h_{i,j,t} - h_{i+1,j,t}}{\Delta x_i}\right)\\
\left(\frac{\partial h}{\partial y}\right)_3 &= \left(\frac{h_{i,j,t} - h_{i,j,t}}{\Delta y_j}\right)\\
\left(\frac{\partial h}{\partial y}\right)_4 &= \left(\frac{h_{i,j,t} - h_{i,j-1,t}}{\Delta y_j}\right)\\
\left(\frac{\partial h}{\partial t}\right) &= \left(\frac{h_{i,j,t} - h_{i,j,t-1}}{\Delta t}\right)
\end{aligned} \tag{13}$$

These can be substituted in eqn. 12 and solved using finite element methods.

OPTIMIZATION MODEL

Optimization models for hydraulic management for groundwater have been developed based on three approaches: embedding approach, optimal control approach and unit response matrix approach. In embedding approach, the equations from a simulation model are incorporated into an optimization model directly. In optimal control approach, the simulation model solves the governing equations, for each iteration of the optimization. It works on the concept of optimal control theory. In response matrix approach, a unit response matrix is generated by running the simulation model several times with unit pumpage at a single node. Total drawdowns are then determined by superpositions.

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