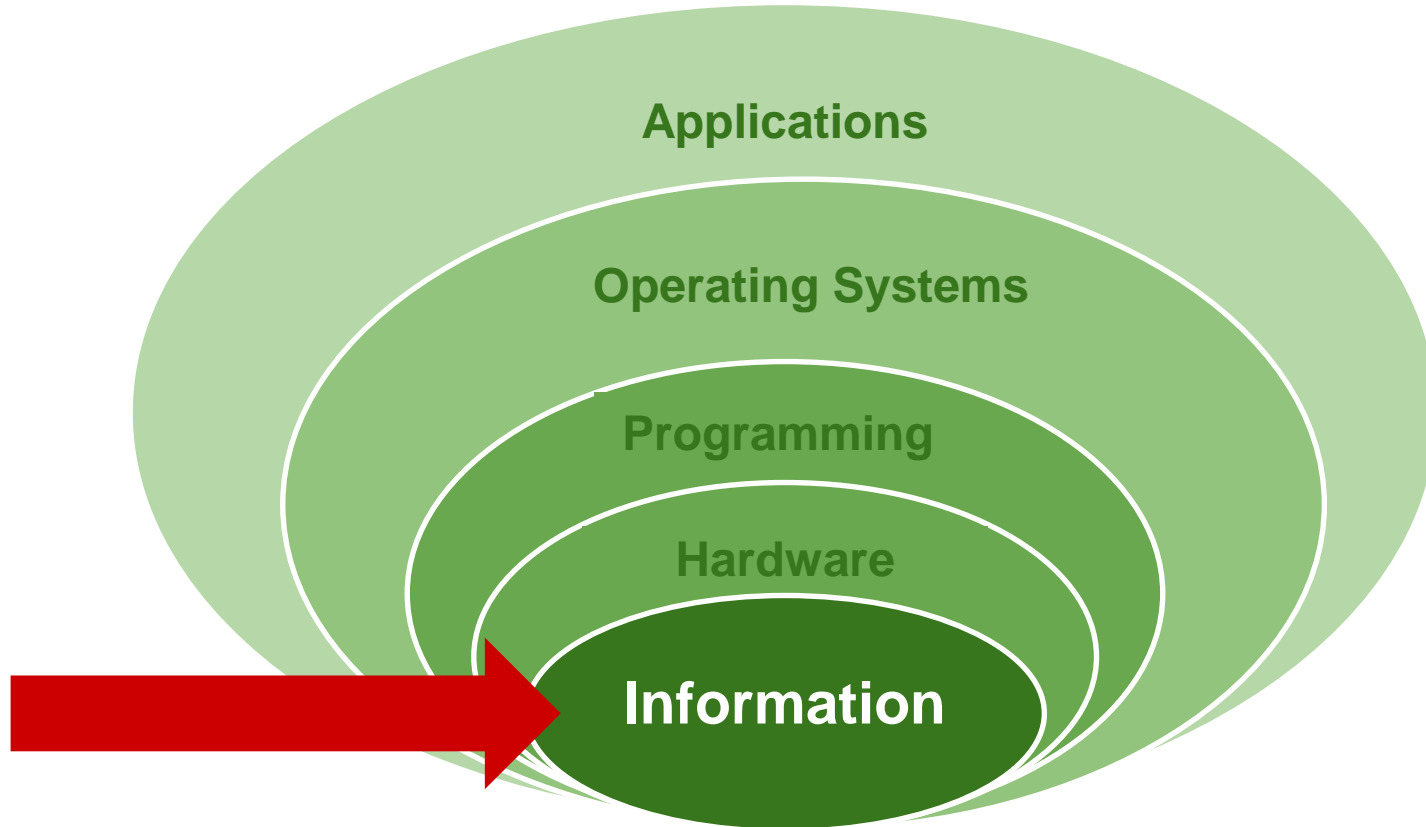


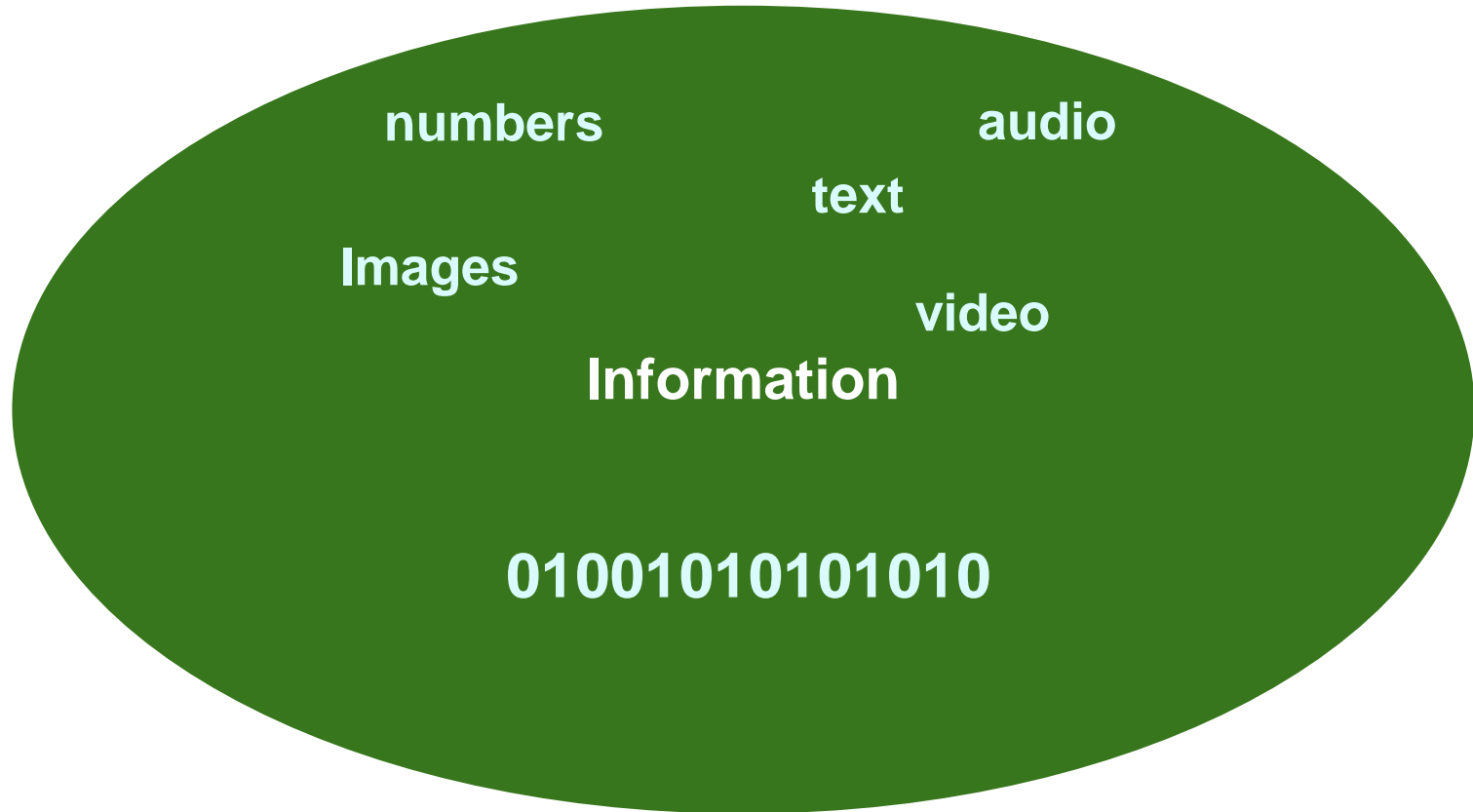
Computer Science Fundamentals

Lecture 3: Binary values and number systems
Lecturer: Olga Yugay

Today's focus



Today's focus



Agenda

- Numbers and Computing
- Positional notation
 - Binary, Octal and Hexadecimal
 - Conversion

Numbers and computing

Why learn binary, hex and etc?

- To understand the nature of a computer (data storage, memory, TCP/IP, debugging, cryptography, bit-planes, compression, etc.) and the "how's" and "why's", an understanding of binary, octal and hex is required.
- Virtually everything dealing with computers can be traced back to bits, bytes, memory, and CPU register addressing.

Use of hex (some examples)

Hex can be used to represent large binary numbers in just a few digits.

- E.g. 11111111 = FF

Hexadecimal numbers are often **used** to represent colours within HTML and CSS

- **orange** is #FFA500, which is (255 red, 165 green, 0 blue).
- 6 digit hex colour code represents **intensities** of **red, green and blue**.

In many programming languages (e.g. Python), **prefix 0x** is used to signify hex number

- 0xFF

Q&A

- Does computer understand “this phrase”?
- Or does it understand numbers?

Q&A

- On the lowest level the computer understands only “on” and “off”
 - “on” - high voltage
 - “off” – low voltage

Intro

- Modern computers are **binary** machines
- All information is represented in **binary** form

Numbers and Computing

- Computer executes **numeric computations**
- All types of information that is stored on a computer is stored as **numbers (0s and 1s)**¹
 - **Bit**, storage unit can be either 0 or 1

¹ Dale, N., & Lewis, J. (2020). Computer Science Illuminated (7th ed.). Jones & Bartlett Learning, Ch1

Positional notation

Positional notation

- **Base** of a number system specifies the **number of digits used in the system**.
 - Digits always begin with **0** and continue through **one less than the base**
 - Eg: Base 10 has digits from 0 to 9
- Bases significant in computing
 - Base 2 - binary
 - Base 8 - octal
 - Base 16 – hexadecimal ²

² Dale, N., & Lewis, J. (2020). Computer Science Illuminated (7th ed.). Jones & Bartlett Learning, Ch1

Base 10 (decimal)	Base 2 (binary)	Base 8 (octal)	Base 16 (hexadecimal)
0	0	0	0
1	1	1	1
2	10	2	2
3	11	3	3
4	100	4	4
5	101	5	5
6	110	6	6
7	111	7	7
8	1000	10	8
9	1001	11	9
10	1010	12	A
11	1011	13	B
12	1100	14	C
13	1101	15	D
14	1110	16	E

Binary and Data

[Khan Academy and Code.org | Binary & Data](#) ³

³ Youtube.com. 2022. *Khan Academy and Code.org | Binary & Data*. [online] Available at: <<https://www.youtube.com/watch?v=ewokFOSxabs>> [Accessed 19 April 2022].

Positional notation

Positional notation is a system of expressing numbers in which

- the digits are **arranged in succession** (0, 1, 2, 3...)
- the position of each digit has a **place value**
- the number is equal to the **sum of the products of each digit by its place value.**⁴

9	4	3
$9 * 10^2$	$4 * 10^1$	$3 * 10^0$
900	40	3
$900 + 40 + 3 = 943$		

Place value

⁴ Dale, N., & Lewis, J. (2020).
Computer Science Illuminated (7th ed.).
Jones & Bartlett Learning, Ch1

Positional notation

- The **rightmost** digit represents its value multiplied by the **base** to the zeroth power, the digit **to the left** represents its value multiplied by the base to the first power
- Number 943 demystified: $9 \cdot x^2 + 4 \cdot x^1 + 3 \cdot x^0$

9	4	3
$9 \cdot 10^2$	$4 \cdot 10^1$	$3 \cdot 10^0$
900	40	3
$900 + 40 + 3 = 943$		

Base

Positional notation and conversion

Binary 101 corresponds to decimal 5

16,8,4,2,1

1	0	1
$1 * 2^2$	$0 * 2^1$	$1 * 2^0$
4	0	1
$4 + 0 + 1 = 5$		

Positional notation and conversion

Hex A1 corresponds to decimal 161

	A	1
	$10 * 16^1$	$1 * 16^0$
	160	1
$160 + 1 = 161$		

Binary to decimal

To find the decimal equivalent for binary
100101

2^5	2^4	2^3	2^2	2^1	2^0					
1	0	0	1	0	1					
(1×2^5)	(0×2^4)	(0×2^3)	(1×2^2)	(0×2^1)	(1×2^0)					
32	+	0	+	0	+	4	+	0	+	1

Place
value



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Binary to octal $2^3=8$

- To convert from binary to octal, you start at the rightmost binary digit and mark the digits in groups of threes.
- Then you convert each group of **three** to its **octal** value.⁵
- E.g. 1010110

$$\begin{array}{ccc} \underline{1} & \underline{010} & \underline{110} \\ 1 & 2 & 6 \end{array}$$

$$\mathbf{110} = 1*2^2+1*2^1+\mathbf{0*2^0}=6$$

$$\mathbf{010} = 0*2^2+1*2^1+0*2^0=2$$

$$\mathbf{1} = 1*2^0$$

Binary to hexadecimal

- mark digits from right to left in groups of **four**.
- each group of **four** to its hex value.

e.g.

1010110

101 0110
5 6

Hexadecimal value

How:

$$1*2^2+0*2^1+1*2^0 = 4+1=5$$

$$0*2^3+1*2^2+1*2^1+0*2^0 = 4+2=6$$

Decimal to different base

The algorithm:

While the quotient is not zero

Divide the decimal number by the base

Make the remainder the next digit to the left in the answer

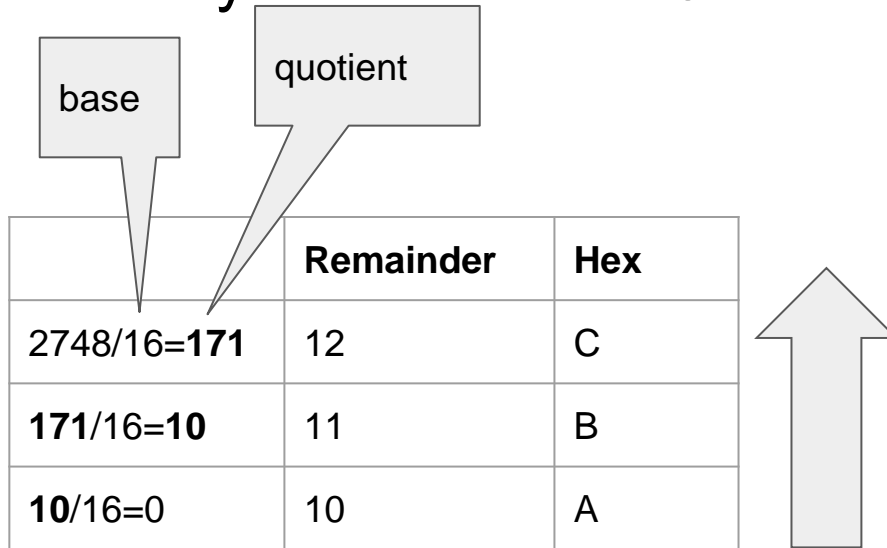
Replace the decimal number with the quotient ⁶

⁶ Dale, N., & Lewis, J. (2020). Computer Science Illuminated (7th ed.). Jones & Bartlett Learning, Ch1

Example: Decimal to hex

Let's try to convert **2748** in **decimal** to its **hex equivalent**⁷

	Remainder	Hex
2748/16=171	12	C
171/16=10	11	B
10/16=0	10	A



The number is ABC

⁷ Dale, N., & Lewis, J. (2020). Computer Science Illuminated (7th ed.). Jones & Bartlett Learning, Ch1

Octal to binary

- look up/calculate each octal digit to obtain the equivalent group of three binary digits.⁸

Octal:	0	1	2	3	4	5	6	7
Binary:	000	001	010	011	100	101	110	111

Octal = 3 4 5

Binary = 011 100 101

⁸ Dale, N., & Lewis, J. (2020). Computer Science Illuminated (7th ed.). Jones & Bartlett Learning, Ch1

Octal to hex

Think logically:

1. Convert octal to binary
2. ...your ideas about the next step

2^5 2^4 2^3 2^2 2^1 2^0

Hex to binary

- Convert each digit to 4 bit binary

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- Use “two to the power of position value”..

4 = 0100

2 = 0010

01000010 in hex

Hex to octal

Think logically, how do you think it happens?

Convert from decimal to other number systems

The algorithm:

While the quotient is not zero

Divide the decimal number by the new base

Make the remainder the next digit to the left in the answer

Replace the decimal number with the quotient⁹

⁹ Dale, N., & Lewis, J. (2020). Computer Science Illuminated (7th ed.). Jones & Bartlett Learning, Ch1

Video on number systems

[Representing Numbers and Letters with Binary: Crash Course Computer Science](https://thecrashcourse.com/courses/representing-numbers-and-letters-with-binary-crash-course-computer-science-4/)⁹

⁹ Crash Course. 2022. *Representing Numbers and Letters with Binary: Crash Course Computer Science #4 - Crash Course*. [online] Available at: <<https://thecrashcourse.com/courses/representing-numbers-and-letters-with-binary-crash-course-computer-science-4/>> [Accessed 19 April 2022].

Homework

- Dale, N., & Lewis, J. (2020). Computer Science Illuminated (7th ed.). Jones & Bartlett Learning, Chapter 2
 - End of the chapter questions and exercises
- Why Do Computers Use 1s and 0s? Binary and Transistors Explained.
<https://www.youtube.com/watch?v=Xpk67YzOn5w>

References

- Dale, N., & Lewis, J. (2020). *Computer Science Illuminated* (7th ed.). Jones & Bartlett Learning, Chapter 3
- Crash Course. 2022. *Representing Numbers and Letters with Binary: Crash Course Computer Science #4 - Crash Course*. [online] Available at: <<https://thecrashcourse.com/courses/representing-numbers-and-letters-with-binary-crash-course-computer-science-4/>> [Accessed 19 April 2022].
- Youtube.com. 2022. Basics explained, Why Do Computers Use 1s and 0s? Binary and Transistors Explained. [online] Available at: <<https://www.youtube.com/watch?v=Xpk67YzOn5w>> [Accessed 19 April 2022].
- Youtube.com. 2022. *Khan Academy and Code.org | Binary & Data*. [online] Available at: <<https://www.youtube.com/watch?v=ewokFOSxabs>> [Accessed 19 April 2022].