

Fiber Optics Communications

Week 4

Optical Signal Dispersion

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Topics of Previous Lecture (Week-3)

- Optical Fiber Loss and attenuation Coefficient
- Origins of Fiber Optics Loss
- Absorption Loss
- Scattering Loss
- Fiber Bending Loss

Week-4: Lecture Learning Outcomes

1. Define optical signal dispersion
2. Describe the impact of dispersion on the performance of fiber optics communications
3. Identify and classify the main types of dispersion in optical fibers
4. Compare the physical origins of chromatic, polarization-mode, and modal dispersion
5. Differentiate between phase velocity and group velocity mathematically and conceptually
6. Analyze how variations in group velocity lead to pulse spreading in optical fibers
7. Analyze pulse broadening due to chromatic dispersion
8. Distinguish between material dispersion and waveguide dispersion
9. Explain the concept of birefringence and its role in PMD
10. Define modal dispersion and explain its origin in multimode fibers

Week-4: Optical Signal Dispersion

Outline

- Introduction to Optical Signal Dispersion
- Types of Optical Signal Dispersion
- Phase Velocity and Group Velocity
- Chromatic Dispersion
- Polarization-Modes Dispersion
- Modal Dispersion

Introduction to Optical Signal Dispersion

- **Optical Signal Dispersion** is broadening of optical pulse while the Lightwave propagating in optical fiber
- Signal dispersion arises due to the dependency of the velocity of propagation of Lightwave on the wavelength of the signal.
- Optical Signals with different wavelength have different propagation velocities within the optical fiber
- Dispersion of signal will cause overlapping of neighboring optical pulses
- When adjacent pulses overlap beyond a certain limit, the receiver cannot distinguish them, leading to signal interpretation errors
- The combined effect of optical signal dispersion and attenuation significantly degrade the fiber optic communication system performance

Introduction to Optical Signal Dispersion

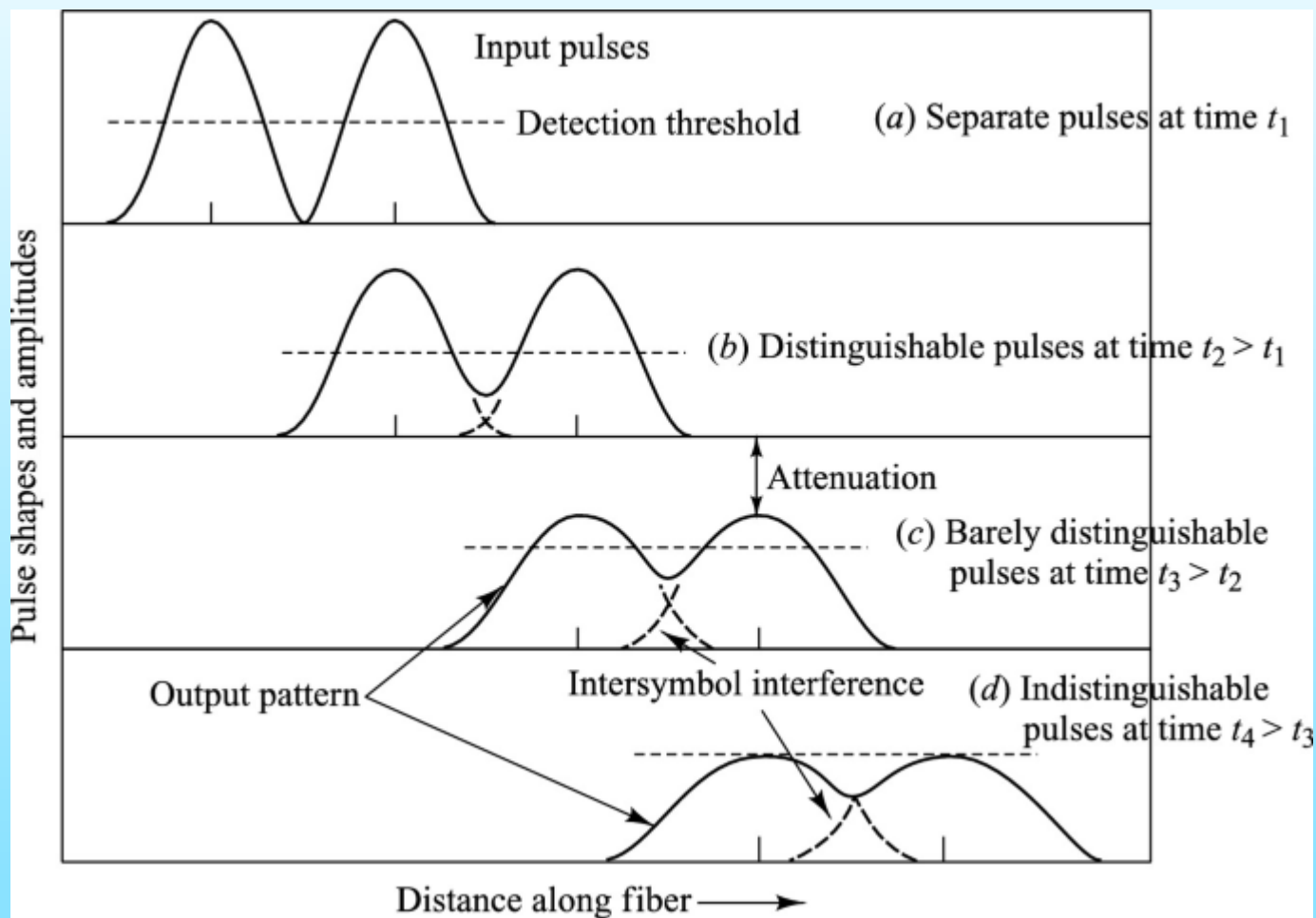


Figure 1: Effect of optical signal dispersion

Source: G. Keiser, "Optical Signal Attenuation and Dispersion," in Fiber Optic Communications, Singapore: Springer, 2021, Ch. 3. https://media.springernature.com/lw685/springer-static/image/chp%3A10.1007%2F978-981-33-4665-9_3/MediaObjects/495048_1_En_3_Fig6_HTML.png

Types of Optical Signal Dispersion

- Different types of signal dispersion occur on the optical signal while propagating down the fiber:

1. Chromatic Dispersion

- Pulse spreading within a single mode
- Also called **group velocity dispersion**

2. Modal Dispersion

- Appear only in multimode fibers
- Each mode possesses different group velocity at a single frequency

3. Polarization-Mode Dispersion

- Arises from slight differences in propagation speeds of two orthogonal polarization modes in a single mode fiber.
- Related to the nonuniformity of the refractive index of the fiber along two orthogonal directions

Phase Velocity

- Phase velocity: is the velocity of constant phase plane of the Lightwave
- A monochromatic wave propagating in z-direction and the field oscillating in x-direction is given by [1]:

$$\psi(x, t) = B \cos(\omega t - \beta z) = B \cos(\varphi(t)) \quad (1)$$

Where:

B Amplitude of the field

β The propagation constant along the z-direction

ω The radial or angular frequency

$\varphi(t)$ The phase of the signal at given time t

- Phase velocity can be calculated by finding the velocity of the following constant phase:

$$\varphi(t) = \omega t - \beta z = A \quad (2)$$

Where:

A Constant phase value

Phase Velocity

- Phase velocity V_p can be obtained from Eq (2) as:

$$V_p = \frac{\partial z}{\partial t} = \frac{\partial}{\partial t} \left(\frac{1}{\beta} (\omega t - A) \right) = \frac{\omega}{\beta} \quad (3)$$

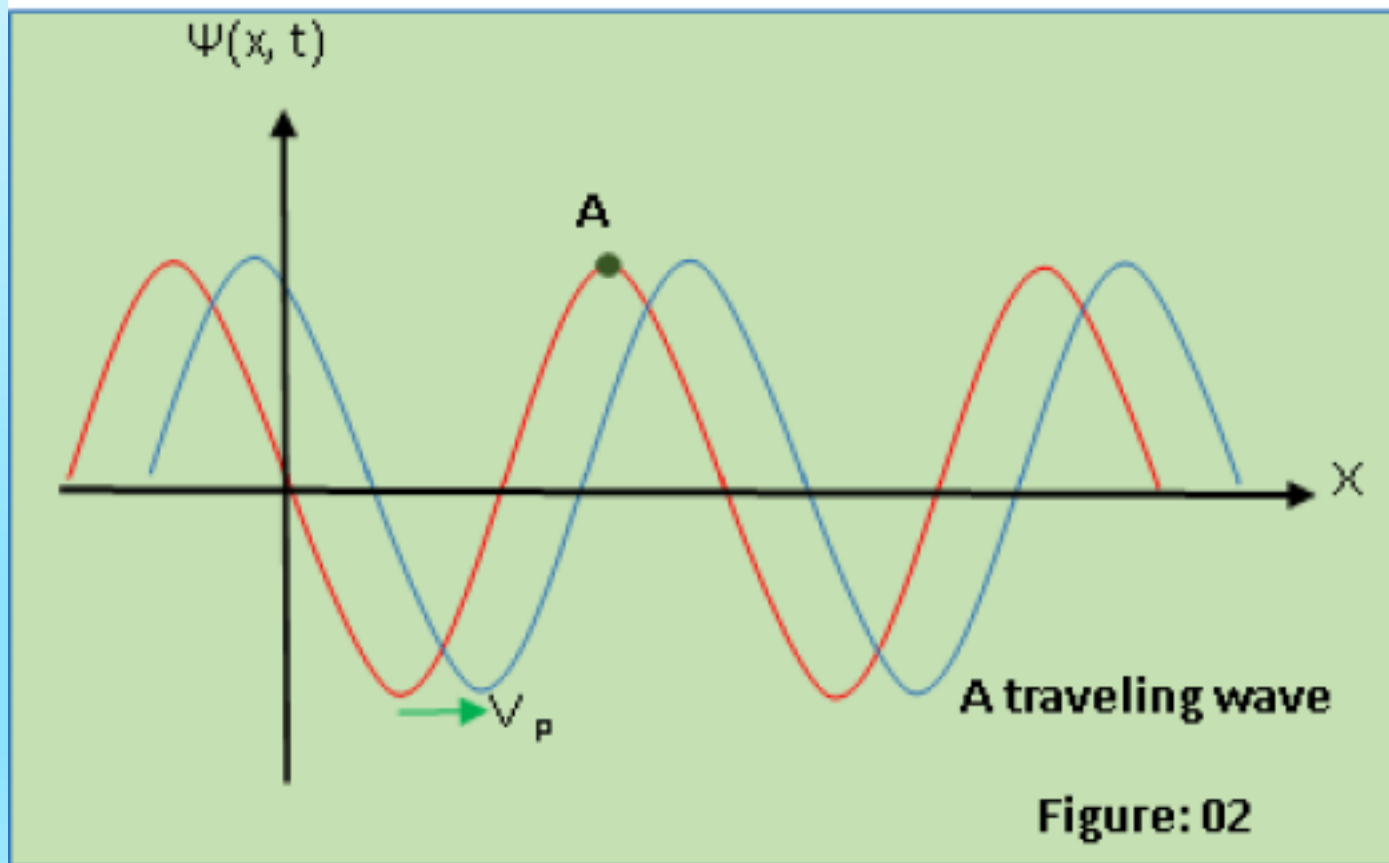


Figure 2: Phase Velocity

Source: "Difference between phase velocity and group velocity," Pediaa, April 15, 2016. <https://i0.wp.com/pediaa.com/wp-content/uploads/2016/04/Difference-Between-Phase-Velocity-and-Group-Velocity-image-2.png?resize=551%2C335>

Group Velocity

- Group Velocity is the actual velocity at which the signal information & energy is traveling down the fiber:
- It is always less than the speed of light in the optical medium
- The equation of two co-propagating waves consisting of two monochromatic fields of frequencies $\omega + \partial\omega$ and $\omega - \partial\omega$ is given by:

$$\begin{aligned}\psi(x,t) &= \psi_1(x,t) + \psi_2(x,t) \\ &= B \cos((\omega + \partial\omega)t - (\beta + \partial\beta)z) + B \cos((\omega - \partial\omega)t - (\beta - \partial\beta)z) \\ &= 2B \cos(\omega t - \beta z) \cos(\partial\omega t - \partial\beta z)\end{aligned}\quad (4)$$

- For $\omega \gg \partial\omega$, the variation in $\cos(\omega t - \beta z)$ is much faster than that in $\cos(\partial\omega t - \partial\beta z)$

Group Velocity

- Hence, we can set the term $(\partial \omega t - \partial \beta z)$ as constant compared to the 1st term
- The equation given in Eq(4) has the nature of amplitude modulation where:

$\cos(\omega t - \beta z)$ Is the carrier signal

$\cos(\partial \omega t - \partial \beta z)$ Is the envelope

- The group velocity V_g and phase velocity V_p can be obtained by setting the term $(\partial \omega t - \partial \beta z)$ and $(\omega t - \beta z)$ as a constant respectively as:

$$V_g = \frac{\partial \omega}{\partial \beta} \quad (5)$$

$$V_p = \frac{\omega}{\beta} \quad (6)$$

Group Velocity

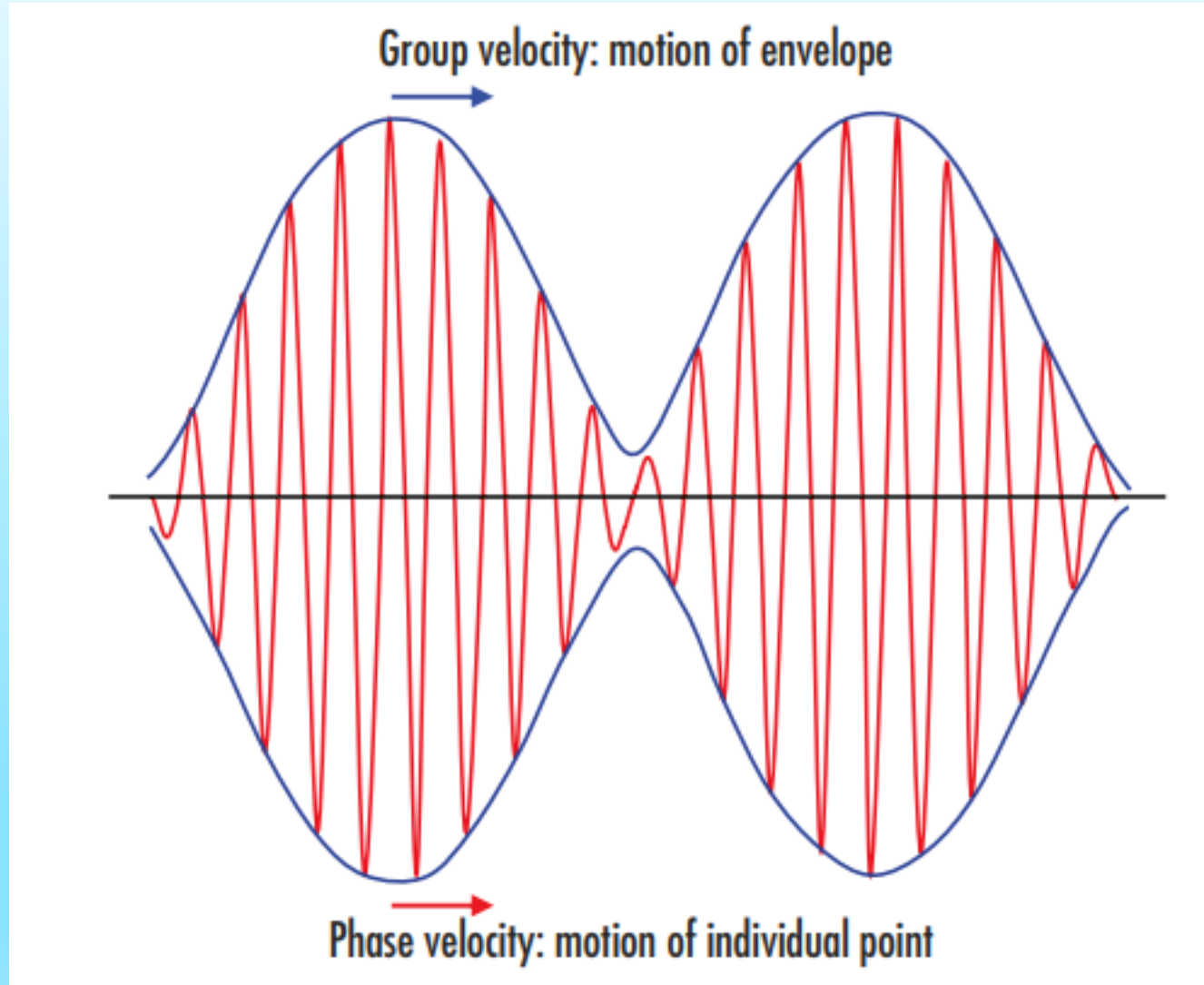


Figure 3: Group Velocity

Source: “*Ultrafast Dispersion*,” Edmund Optics.
<https://www.edmundoptics.eu/media/q4bloap1/ultrafast-dispersion-fig-2.png?rmode=max>

Factors Contributing to Optical Signal Dispersion

- For an optical wave traveling in z-direction, the z-component of the propagation constant β is the function of frequency ω .
- β changes slowly with angular frequency and various dispersion effects can be identified by expanding β in a Taylor series around the central frequency ω_0 as [2]:

$$\beta(\omega) \approx \beta_0(\omega_0) + \beta_1(\omega_0)(\omega - \omega_0) + \frac{1}{2} \beta_2(\omega_0)(\omega - \omega_0)^2 + \frac{1}{6} \beta_3(\omega_0)(\omega - \omega_0)^3 \quad (7)$$

- $\beta_m(\omega_0)$, for $m = 0, 1, 2, 3, \dots$ in Eq(7) is the m order derivative of β with respect to ω evaluated at $\omega = \omega_0$ as:

$$\beta_m = \left(\frac{\partial^m \beta}{\partial \omega^m} \right)_{\omega=\omega_0} \quad (8)$$

Factors Contributing to Optical Signal Dispersion

- From Eq (8) we have:

For $m = 0$

$$\beta_0$$

For $m = 1$

$$\beta_1 = \frac{\partial \beta}{\partial \omega} = \frac{1}{V_g}$$

For $m = 2$

$$\beta_2 = \frac{\partial^2 \beta}{\partial \omega^2}$$

- When the above two terms (β_0, β_1) multiplied with the signal propagation distance L we have:

$\beta_0 L \rightarrow$ Phase Shift

$\beta_1 L = \frac{L}{V_g} = \tau_g \rightarrow$ Group Delay

$\beta_2 \rightarrow$ Chromatic/Group Velocity Dispersion (GVD)

$D = -\frac{2\pi c}{\lambda^2} \beta_2 \rightarrow$ Dispersion



Chromatic Dispersion

Chromatic Dispersion

- No optical source can emit monochromatic light and the emitted light contains different spectral components with different wavelengths.
- Those different spectral components of the pulse travel at slightly different group velocities which introduces a pulse broadening phenomena called **Chromatic** or **Group Velocity Dispersion (GVD)**
- Chromatic dispersion exists in both Single-mode and multimode fibers
- Chromatic dispersion/GVD/Intra-Modal Dispersion can be divided as:

- **Material Dispersion**
- **Waveguide Dispersion**

Chromatic Dispersion

- For optical signal propagating a distance L in the fiber each spectral component travels independently, experiencing a time (group) delay τ_g which is given by:

$$\tau_g = \frac{L}{V_g} = L \frac{\partial \beta}{\partial \omega} = \frac{L}{c} \frac{\partial \beta}{\partial k} = -\frac{L \lambda^2}{2\pi c} \frac{\partial \beta}{\partial \lambda} \quad (9)$$

$$\begin{array}{l} \text{from } \omega = 2\pi f = 2\pi \frac{c}{\lambda} = ck \\ \frac{\partial \omega}{\partial k} = \frac{\partial}{\partial k}(ck) \\ \partial \omega = c \partial k \end{array} \quad \begin{array}{l} k = \frac{2\pi}{\lambda} \\ \frac{\partial k}{\partial \lambda} = 2\pi \frac{\partial}{\partial \lambda} \left(\frac{1}{\lambda} \right) = -\frac{2\pi}{\lambda^2} \\ \partial k = -\left(\frac{2\pi}{\lambda^2} \right) \partial \lambda \end{array}$$

- Since group delay varies with wavelength as revealed in Eq (9), each spectral component of a particular mode takes different propagation time to travel the same distance.

Chromatic Dispersion

- These unequal time delays taken by different spectral components cause the optical pulse to spread in time as it propagates through the fiber
- The different colors in Figure (4) represents spectral components with different wavelength

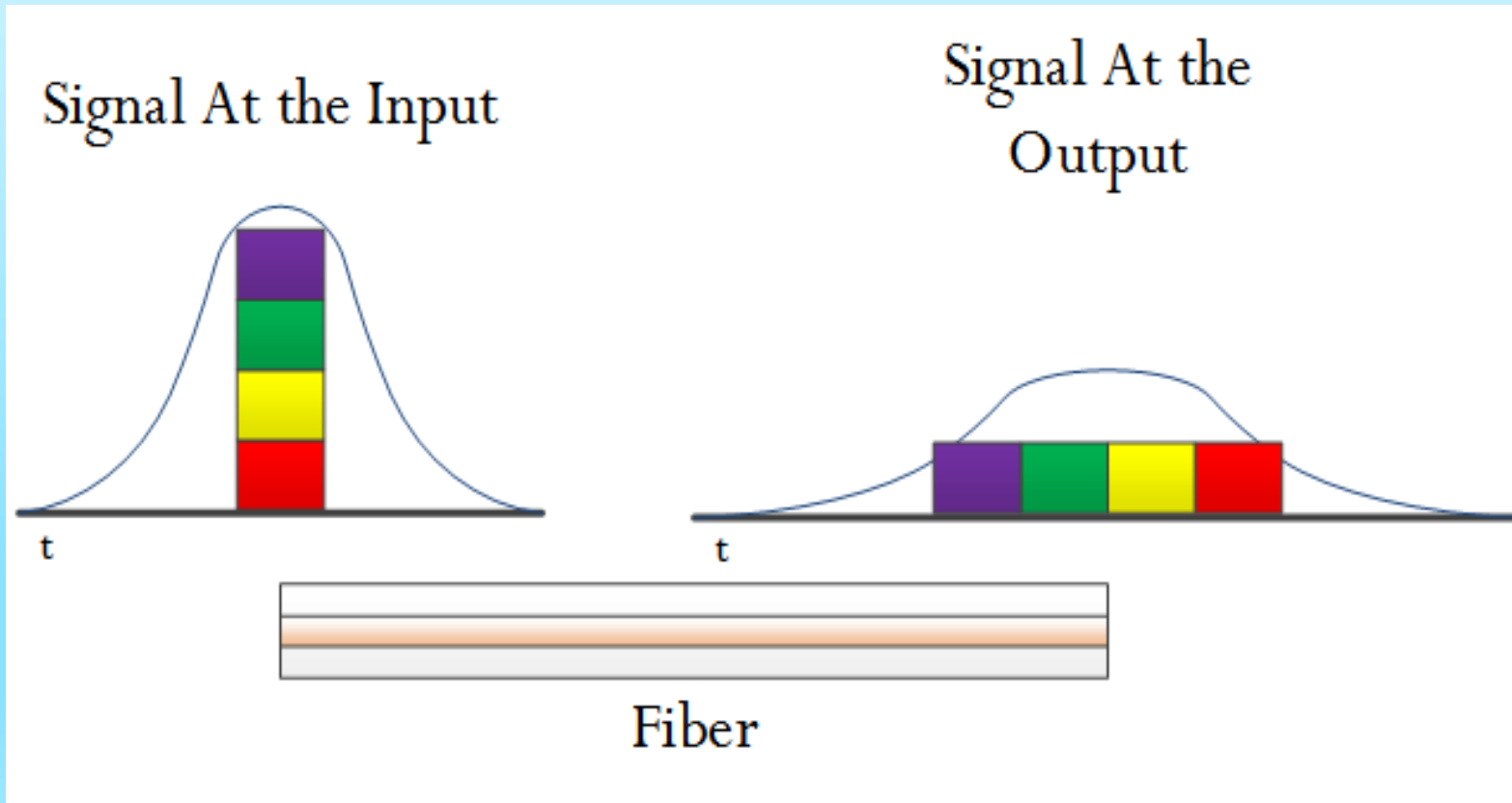


Figure 4: Chromatic Dispersion

Source: M. Biegert, "Chromatic Dispersion with 10 Gigabit Optical Transports," Math Encounters Blog, 9-Sep-2016.
<https://i0.wp.com/www.mathscinotes.com/wp-content/uploads/2016/09/DispersionExample.png?w=640&ssl=1>

Chromatic Dispersion

- The pulse spreading caused by group delay variation is the key parameter which should be taken in to account.
- For narrow spectral sources, the delay difference per unit wavelength is approximated by:

$$\frac{\partial \tau_g}{\partial \lambda} \rightarrow \text{Delay difference per unit wavelength}$$

- For spectral components spaced $\Delta\lambda$ around the central wavelength λ_0 , the total delay difference over distance L is δ_τ :

$$\delta_\tau = \frac{\partial \tau_g}{\partial \lambda} \Delta\lambda = \Delta\lambda \frac{\partial}{\partial \lambda} \left(-\frac{L\lambda^2}{2\pi c} \frac{\partial \beta}{\partial \lambda} \right) = -\frac{\Delta\lambda L}{2\pi c} \left(2\lambda \frac{\partial \beta}{\partial \lambda} + \lambda^2 \frac{\partial^2 \beta}{\partial \lambda^2} \right) \quad (10)$$

Chromatic Dispersion

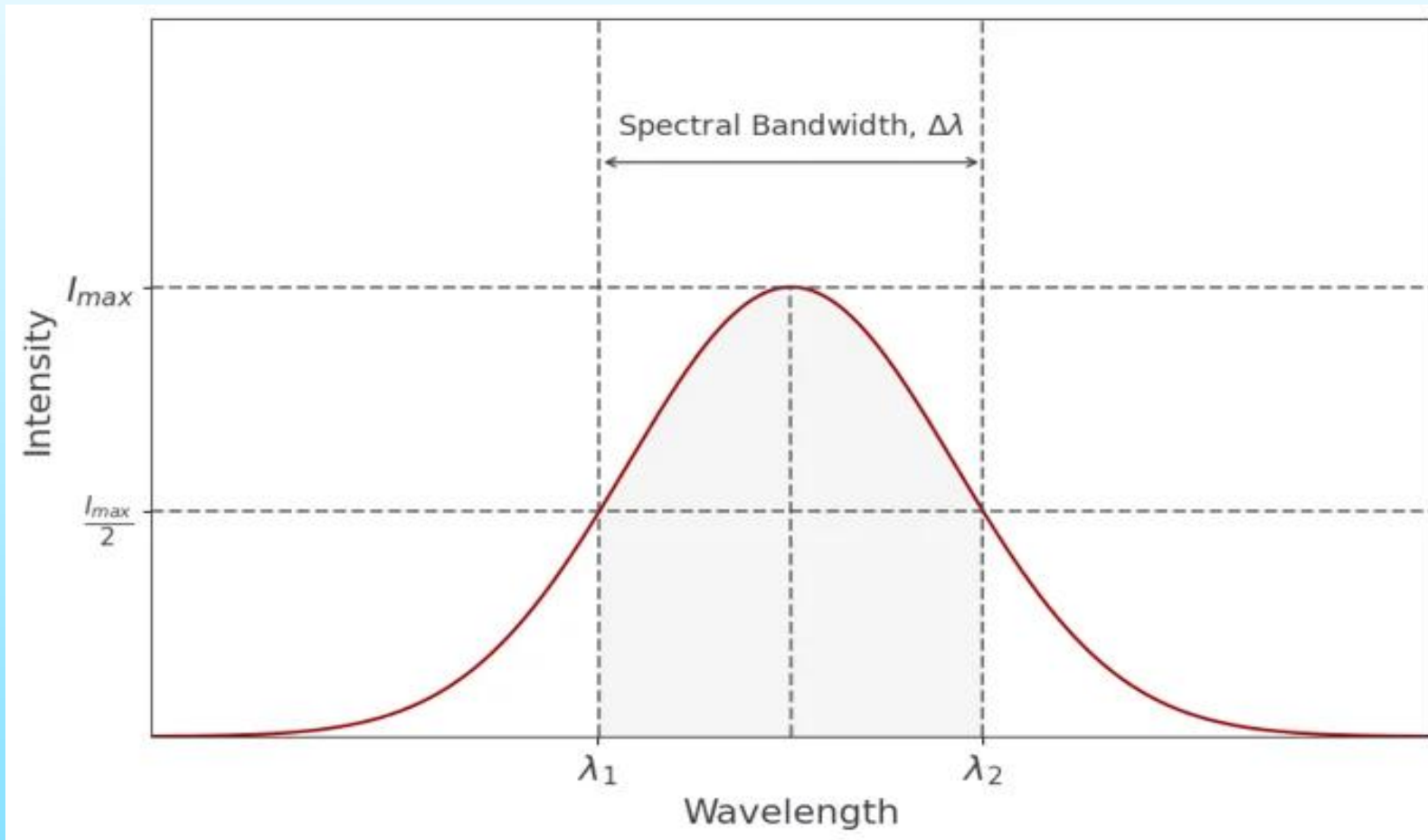


Figure 5: Spectral width of optical pulse

Source: "What Is Spectral Bandwidth within UV-Vis Spectroscopy?," Copenhagen Nanosystems Knowledge Center, 2023. <https://knowledge.cphnano.com/hs-fs/hubfs/spectral-bandwidth.png?width=685&height=424&name=spectral-bandwidth.png>

Chromatic Dispersion

- The total delay difference δ_τ can also be written in terms of angular frequency ω as:

$$\delta_\tau = \frac{\partial \tau_g}{\partial \omega} \Delta\omega = L\Delta\omega \frac{\partial}{\partial \omega} \left(\frac{\partial \beta}{\partial \omega} \right) = L\Delta\omega \frac{\partial^2 \beta}{\partial \omega^2} = L\Delta\omega \beta_2 \quad (11)$$

↑
GVD Parameter

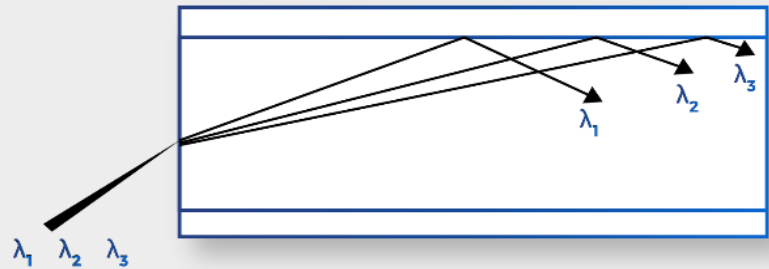
- The Dispersion Parameter D is the pulse spread per unit length per unit wavelength which is given by:

$$\begin{aligned} D &= \frac{\delta_\tau}{L\Delta\lambda} = \frac{1}{L\Delta\lambda} \left(\frac{\partial \tau_g}{\partial \lambda} \Delta\lambda \right) = \frac{1}{L} \left(\frac{\partial \tau_g}{\partial \lambda} \right) = \frac{1}{L} \frac{\partial}{\partial \lambda} \left(L \frac{\partial \beta}{\partial \omega} \right) = \frac{\partial}{\partial \lambda} \left(\frac{\partial \beta}{\partial \omega} \right) \\ &= -\frac{2\pi c}{\lambda^2} \left(\frac{\partial^2 \beta}{\partial \omega^2} \right) = -\frac{2\pi c}{\lambda^2} \beta_2 \end{aligned} \quad (12)$$

- The dispersion parameter in Eq (2) is a result of **material and waveguide** dispersion

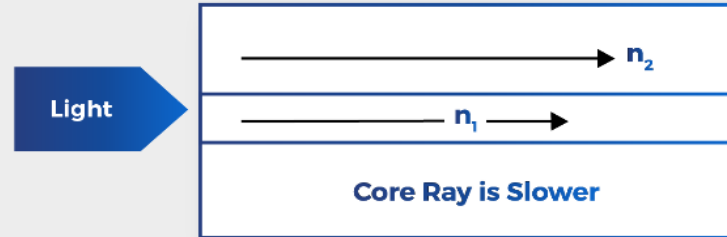
Chromatic Dispersion

Chromatic Dispersion is further divided into two sub-categories



Material Dispersion

Material dispersion arises due to the inherent properties of the fiber core material, where different wavelengths of light experience different velocities as they travel through the core. This type of dispersion can cause group delay distortion, thus affecting the transmitted data.



Waveguide Dispersion

Waveguide dispersion in optical fibers occurs because the signal travels in both the core and cladding, causing the mode field diameter to vary with wavelength due to the refractive index difference. This change in diameter affects the average dispersion index, leading to variations in signal velocity with wavelength.

Figure 6: Material and Waveguide dispersion

Source: "Dispersion in Optical Fiber-Understanding its Impact on Communication," HFCL, 2023.

<https://www.hfcl.com/wp-content/uploads/2023/09/images/Types-of-Chromatic-Dispersion.png>

Material Dispersion

- Material dispersion arises due to the variation of refraction index as a function of the optical wavelength:

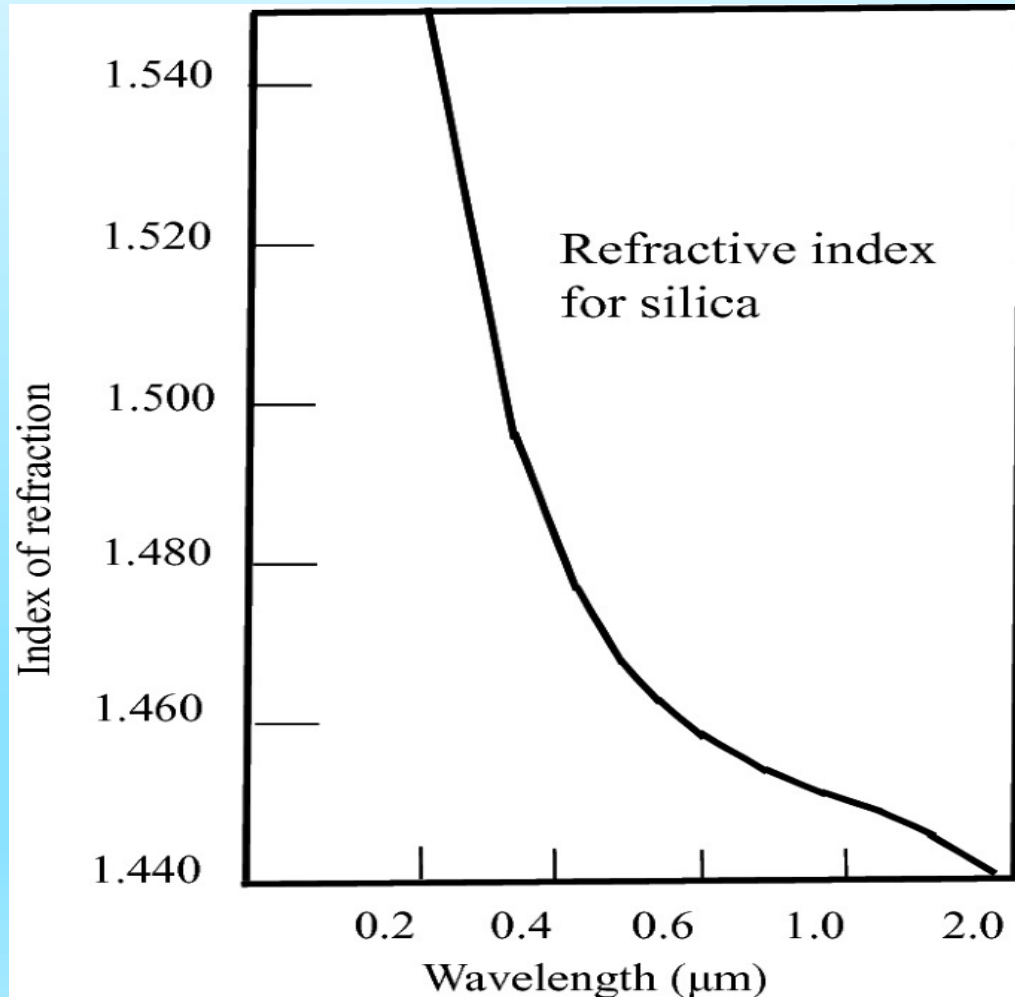


Figure 7: Variation of refraction index of silica as a function of the optical wavelength

Source: G. Keiser, "Chromatic Dispersion in Single-Mode Fiber," in *Optical Signal Attenuation and Dispersion*, G. Keiser, Ed., Singapore: Springer, 2021, Fig. 9. https://media.springernature.com/lw685/springer-static/image/chp%3A10.1007%2F978-981-33-4665-9_3/MediaObjects/495048_1_En_3_Fig9_HTML.png

Material Dispersion

- Different spectral components of a given mode travel at different speeds based on their wavelength
- Material dispersion is an intramodal effect, most significant in single-mode fibers.
- Considering a plane wave propagating in optical medium that has a refractive index ($n(\lambda)$), the propagation constant (β) is given by:

$$\beta = \frac{2\pi n(\lambda)}{\lambda} \quad (13)$$

- Substituting Eq (13) in previous Eq (9) for β , we can obtain the group delay (τ_{mat}) introduced by material dispersion as:

$$\tau_{mat} = -\frac{L\lambda^2}{2\pi c} \frac{\partial \beta}{\partial \lambda} = -\frac{L\lambda^2}{2\pi c} \frac{\partial}{\partial \lambda} \left(\frac{2\pi n(\lambda)}{\lambda} \right) = \frac{L}{c} \left(n - \lambda \frac{\partial n}{\partial \lambda} \right) \quad (14)$$

Material Dispersion

- Pulse spread (σ_{mat}) when the optical source at the transmitter has a spectral width (σ_λ) is given by:

$$\begin{aligned}\sigma_{mat} &= \left| \frac{\partial \tau_{mat}}{\partial \lambda} \right| \sigma_\lambda = \left| \frac{\partial}{\partial \lambda} \left(\frac{L}{c} \left(n - \lambda \frac{\partial n}{\partial \lambda} \right) \right) \right| \sigma_\lambda \\ &= L \sigma_\lambda \left| \frac{\lambda}{c} \frac{\partial^2 n}{\partial \lambda^2} \right| = L \sigma_\lambda |D_{mat}(\lambda)|\end{aligned}\quad (15)$$

Where:

$D_{mat}(\lambda)$ Is the material induced dispersion

Waveguide Dispersion

- Waveguide dispersion effect on pulse spreading is approximated by assuming wavelength-independent refractive index.
- The group delay (τ_{wg}) introduced by the waveguide dispersion is given by:

$$\tau_{wg} = \frac{L}{c} \frac{\partial \beta}{\partial k} = \frac{L}{c} \left(n_2 + n_2 \Delta \frac{\partial(kb)}{\partial k} \right) \quad (16)$$

Where:

n_2 is refractive index of the cladding

Δ is refractive index difference of core and cladding

k is free space propagation constant

b is normalized propagation constant

Waveguide Dispersion

- For small values of the refractive index difference (Δ), the normalized propagation constant (b) can be written as:

$$b = \frac{\frac{\beta}{k} - n_2}{n_1 - n_2} \quad (17)$$

Where:

n_2 is refractive index of the cladding

n_1 is refractive index of the core

k is free space propagation constant

Polarization-Mode Dispersion

- Polarization-mode dispersion arises due to the optical fiber **Birefringence**
- **Birefringence:** arises from factors like core irregularities or internal stresses
- The optical signal traveling down the fiber has two orthogonal polarizations
- If the fiber has birefringence, the two polarizations travel at different group velocity in the fiber and the two polarizations arrives at the end of the fiber at different times [3].
- Let V_{gx} and V_{gy} are the group velocities of the two polarizations, the differential group delay ($\Delta\tau_{PMD}$), after propagating a total distance L is given by:

$$\Delta\tau_{PMD} = \left| \frac{L}{V_{gx}} - \frac{L}{V_{gy}} \right| \quad (18)$$

Polarization-Mode Dispersion

- Polarization-mode dispersion (D_{PMD}) can also be approximated by:

$$D_{PMD} \approx \frac{\Delta\tau_{PMD}}{\sqrt{L}} \quad (19)$$

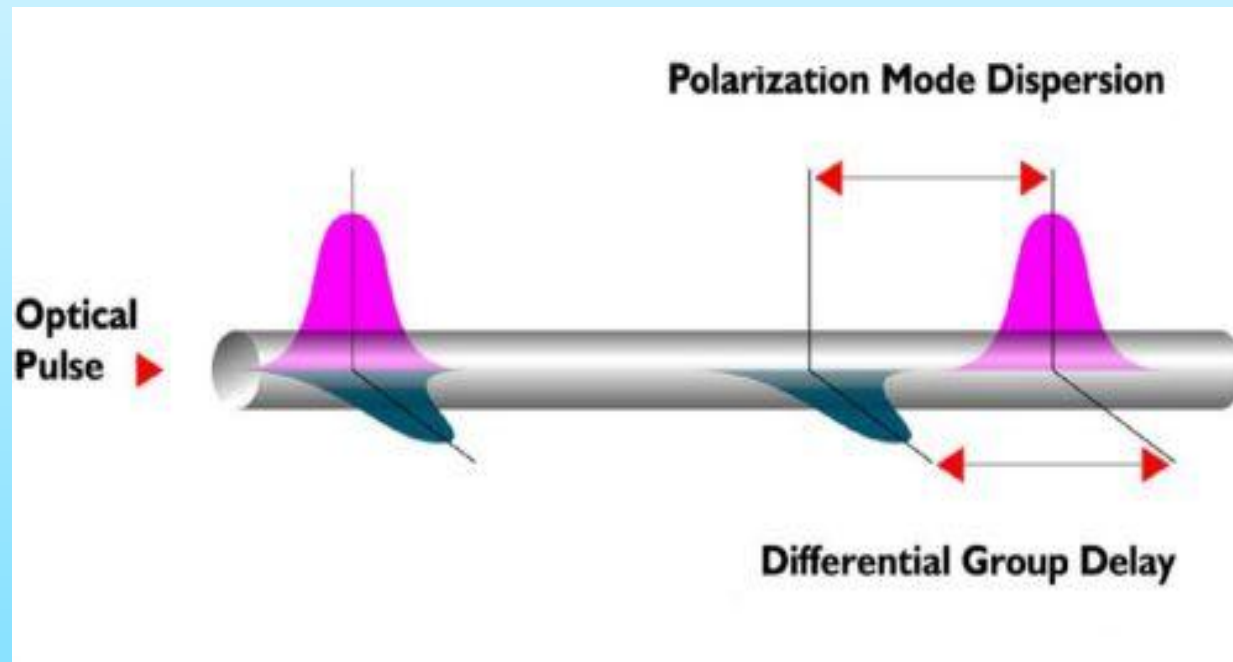


Figure 8: Polarization-mode dispersion

Source: "Polarization Mode Dispersion (PMD)," Technical University of Hamburg-Harburg (TUHH), Optics and Quantum Electronics Research Group (OKT).
<https://www.tuhh.de/okt/Forschung/Pmd/PMD-Illustration.jpg>

Modal Dispersion

- Modal dispersion depends on the path of each light ray and it occurs only in multi-mode fiber.
- Some rays travel through the fiber center (axial mode), while others bounce repeatedly at the core–cladding boundary.
- The modal delay (ΔT) is given by:

$$\Delta T = T_{\max} - T_{\min} \quad (19)$$

Where:

T_{\min} is the time taken by primary mode

T_{\max} is the time taken by higher-order mode

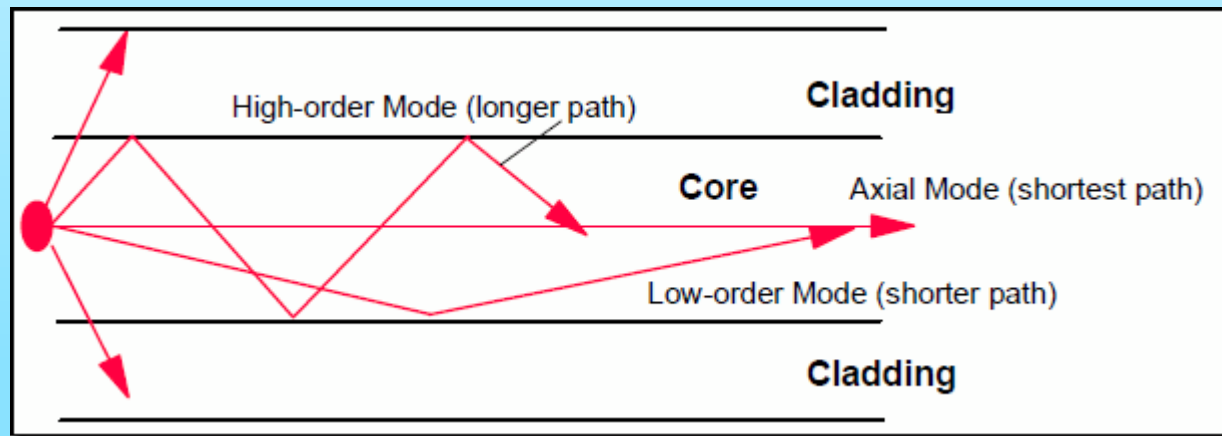


Figure 9: Modal dispersion

Source: “Fiber Dispersion and Optical Dispersion – An Overview,” Fiber Optics 4 Sale, 21-Jun-2009.
<https://s3-us-west-1.amazonaws.com/foscoshopify/graphics/uploads/2009/06/modaldispersion-thumb.gif>

Summary

- **Optical Signal Dispersion:**
 - ✓ **Definition:** Broadening of optical pulses as they travel through a fiber.
 - ✓ **Impact:** Limits bandwidth and maximum transmission distance.
 - ✓ **Cause:** Different spectral components of the signal travel at different speeds.
- **Types of Optical Signal Dispersion:**
 - ✓ **Chromatic Dispersion:** Wavelength-dependent speed variations.
 - ✓ **Polarization-Mode Dispersion (PMD):** Different polarizations travel at different speeds.
 - ✓ **Modal Dispersion:** Different rays in multimode fiber travel at different speeds.
- **Phase Velocity and Group Velocity:**
 - ✓ **Phase Velocity:** Speed of a particular phase of a wave.
 - ✓ **Group Velocity:** Speed of the overall pulse envelope.
 - ✓ **Relevance:** Pulse spreading occurs when group velocity varies with wavelength (dispersion)

References

- [1] Le Nguyen Binh, “Optical Fiber Communication Systems with MATLAB® and Simulink® Models”, CRC Press, Pp.60, 2015.
- [2] Gerd Keiser, “*Fiber Optic Communications*”, Springer, Pp.113, 2021.
- [3] [4] Shiva Kumar and M. Jamal Deen, “Fiber Optic Communication Systems”, Wiley, Pp.78, 2014.



Thank You !