Fiber Optics Communications

Week 11 Optical Fiber transmission system design -1

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Topics of Previous Lecture (Week-10)

Optical Amplifiers

- General Operation Principle of Optical Amplifier
- Types of Optical Amplifiers
- Semiconductor Optical Amplifiers (SOA)
- Erbium Doped Fiber Amplifiers (EDFA)
- Raman Amplifiers (RA)

Week-11: Lecture Learning Outcomes

1. Explain the fundamental components and working principles of an optical fiber transmission system
2. Explain the concept of an ideal non-amplified optical link and its assumptions
3. Analyze the relationship between transmitted power, received power, and system loss.
4. Evaluate system performance parameters in the absence of optical amplification
5. Interpret the effect of received optical power on system detectability and signal quality
6. Derive the relationship between received power, noise, and BER in an ideal system
7. Compute the maximum achievable transmission distance for loss-limited fiber links
8. Calculate an optical power budget for a fiber optic link
9. Calculate dispersion-limited distance for a given bit rate

Week-11: Optical Fiber transmission system design -1

Outline

- Introduction: Optical Fiber transmission system
- Ideal Non-Amplified optical Fiber Transmission systems
- Optical Power and Number of Photons Received
- Bit Error Probability of Ideal Fiber Optics Transmission System
- Sensitivity and Loss Limited Maximum Reachable Distance
- Optical Power Budget
- Dispersion limited Maximum Reachable Distance

Optical Fiber transmission system design

- The preceding lectures have covered the fundamental characteristics of the basic building blocks of an optical fiber transmission system such as:

  - ❖ Fiber optics transmission medium
  - ❖ Optical sources
  - ❖ Optical detectors

- Now it is the time to put those basic building blocks together to form to form an optical fiber transmission link
- The simplest transmission link is a point-to-point optical fiber line that has a transmitter on one end and a receiver on the other end [1]
- Transmission distance, BER, and bitrate are the key design and performance parameters in analyzing optical fiber transmission link [2]

Ideal Non-Amplified Fiber Optics Transmission System

- Ideal non-amplified fiber optics transmission follows the following asumptions:
  - ❖ Ideal Optical sources
  - ❖ Ideal Optical detectors (quantum noise limited)
  - ❖ Ideal decision circuit (Electron Counter)
  - ❖ Perfect transmitter and receiver coupling (no connector or splice losses)
  - ❖ Ideal fiber optics transmission medium (only attenuation and dispersion limitations)
  - ❖ Point to point non amplified link
- **Ideal non-amplified fiber optics transmission** is a theoretical reference model used to understand intrinsic limitations and design trade-offs in practical optical communication systems
- It provides a baseline for evaluating the effects of losses, dispersion, and noise in real systems.

Ideal Non-Amplified Fiber Optics Transmission System

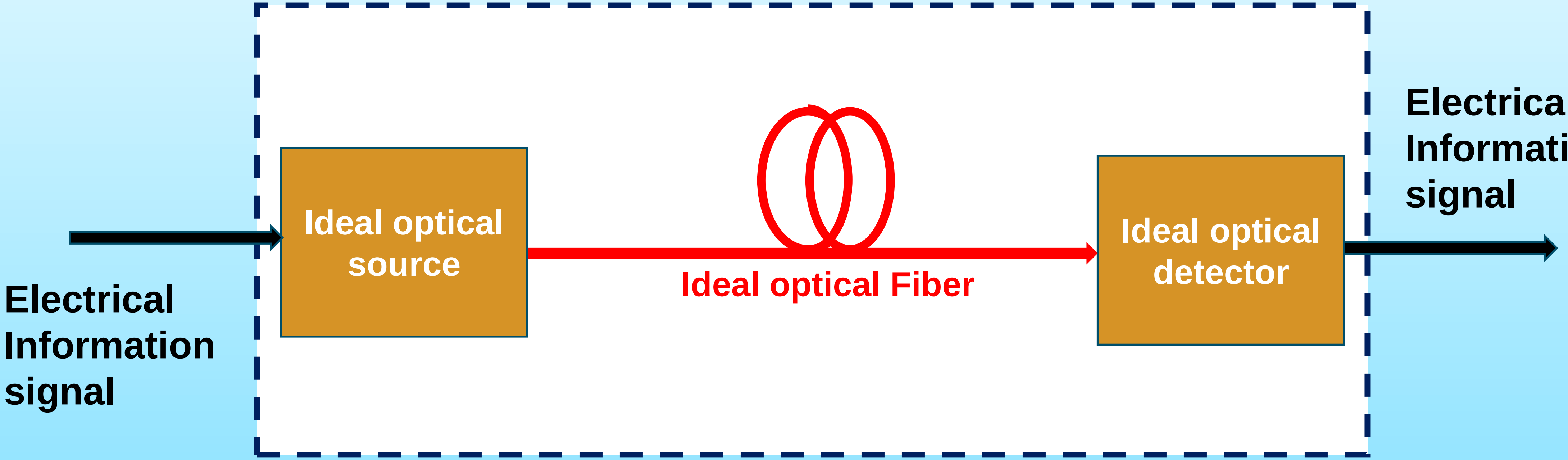


Figure 1: Point to point ideal optical fiber transmission system

Ideal Optical Source

- A device that converts electrical current (electrons) into light (photons) at a specific wavelength (λ)

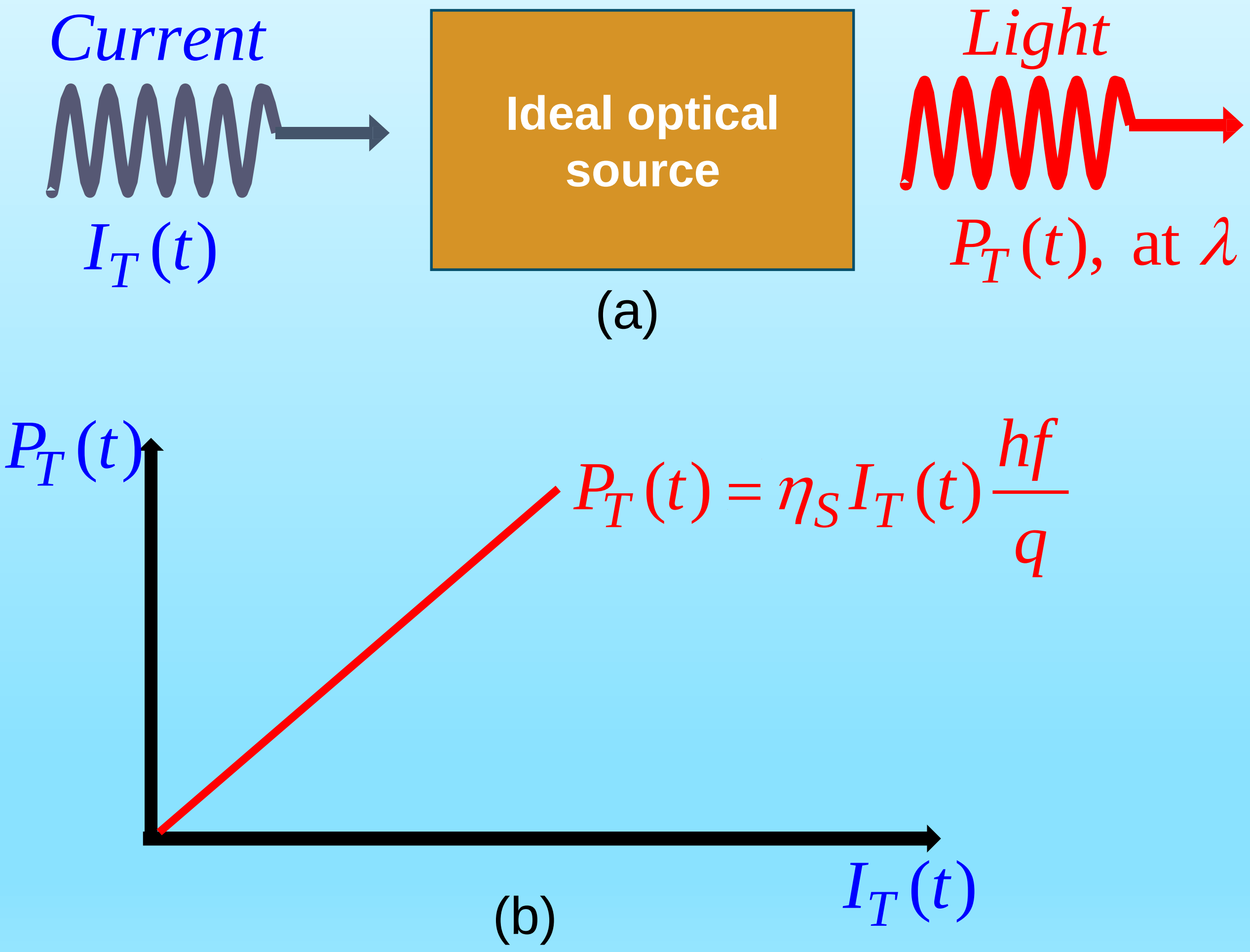
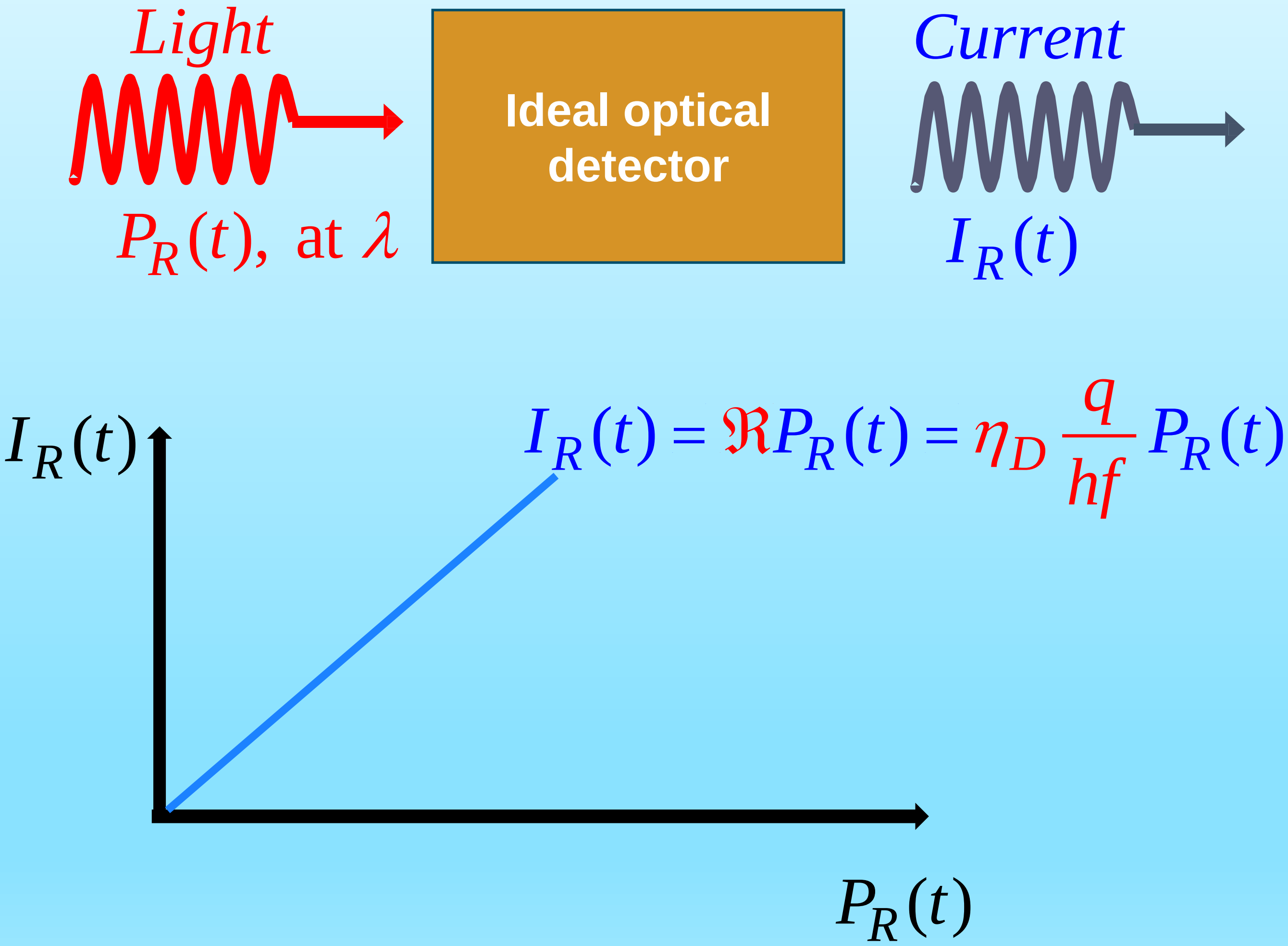


Figure 2: Ideal source **(a)** input-output **(b)** P-I relation

Ideal Optical Detector

- A device that converts light (photons) at a specific wavelength (λ) electrical current (electrons)

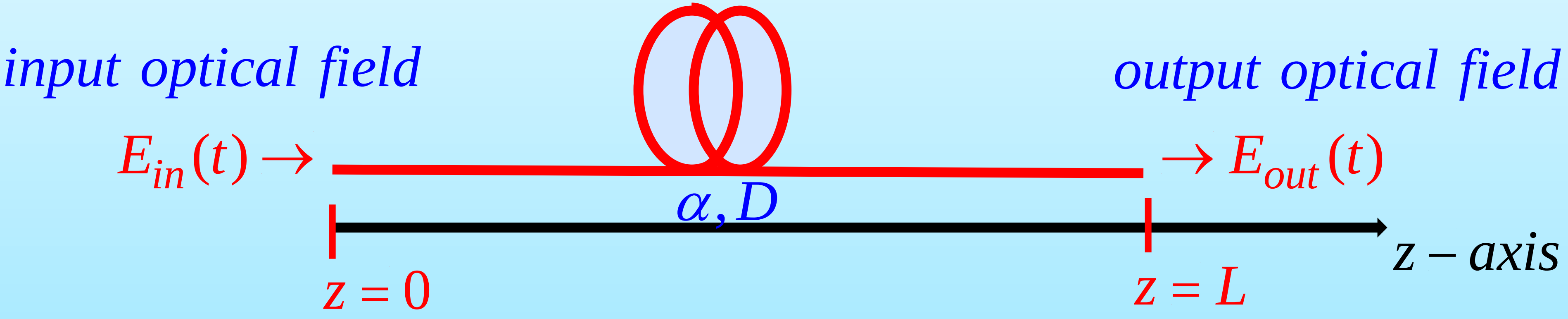


$$I_R(t) = \Re P_R(t) = \eta_D \frac{q}{hf} P_R(t)$$

Figure 3: Ideal photodetector **(a)** input-output **(b)** I-P relation

Ideal Optical Fiber

- Ideal optical fiber characterized by its attenuation and dispersion effect



- Input optical field to the optical fiber can be written as:

$$E_{in}(t) = \sqrt{P_T(t)}e^{j2\pi ft} \qquad (1)$$

- output optical field at the end of the optical fiber can be written as:

$$E_{out}(t) = \sqrt{P_T(t-t_g)e^{-\alpha L}}\,e^{j2\pi f(t-t_p)} \qquad (2)$$

Group delay

Attenuation (loss)

Phase delay

Ideal Optical Fiber

- The group delay (t_g) and the phase delay (t_p) in eq (2) can be written as:

$$t_g = \frac{L}{v_g} \qquad (3)$$

$$t_p = \frac{L}{v_p} \qquad (4)$$

Where:

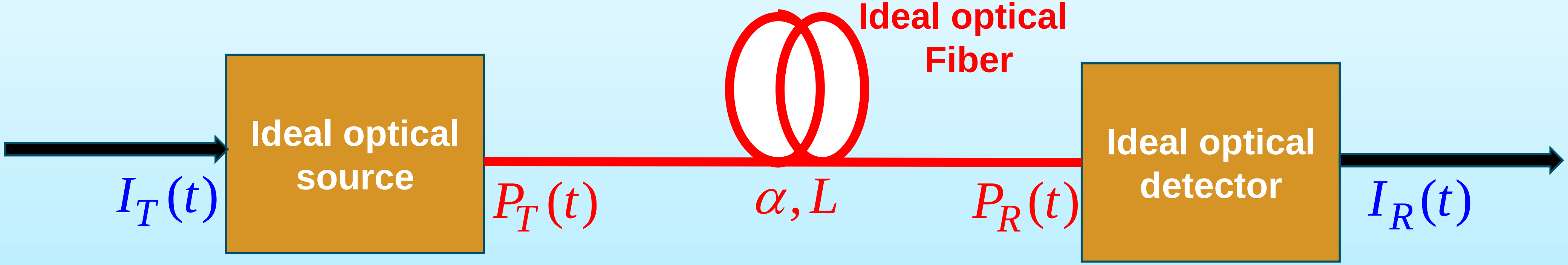
$v_g \rightarrow$ Group velocity

$v_p \rightarrow$ Phase velocity

- The group delay (t_g) and the phase delay (t_p) can be neglected to evaluate the effect of optical attenuation on the system performance

$$E_{out}(t) = \sqrt{P_T(t)e^{-\alpha L}}\, e^{j2\pi f(t)} \qquad (5)$$

Ideal Fiber Optics Transmission System



- The Transmitted optical signal power:

$$P_T(t) = \eta_s I_T(t)\frac{hf}{q} \qquad (6)$$

- The received optical signal power at the photodetector:

$$P_R(t) = P_T(t)e^{-\alpha L} = \eta_S I_T(t)\frac{hf}{q}e^{-\alpha L} \qquad (7)$$

- The output current from the photodetector:

$$I_R(t) = \eta_D \frac{q}{hf}P_R(t) = \eta_D \frac{q}{hf}\eta_S I_T(t)\frac{hf}{q}e^{-\alpha L} = \overbrace{\eta_S\eta_D}^{\text{Unity for ideal system}} I_T(t)e^{-\alpha L} \qquad (8)$$

Ideal Fiber Optics Transmission System

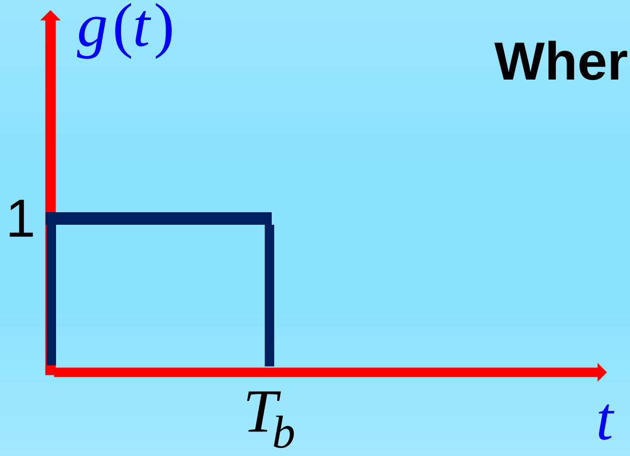
- The output current from the photodetector:

$$I_R(t) = \eta_S \eta_D I_T(t) e^{-\alpha L}$$

$$\boxed{= \eta_{SD} I_T(t) e^{-\alpha L}} \qquad (9)$$

- Considering a binary NRZ-OOK modulation, the transmitted sequence of optical pulse is:

$$P_T(t) = P_{T,peak} \sum_n b_n g(t - nT_b) = I_{T,peak} \eta_S \frac{hf}{q} \sum_n b_n g(t - nT_b) \qquad (10)$$



Where: $b_n : \{0,1\}$ binary bits

$g(t)$ optical pulse shape

T_b single bit duration

Ideal Fiber Optics Transmission System

- Similarly, the output electrical pulse from the photodetector is:

$$I_R(t) = \eta_{SD} I_{T,peak}\; e^{-\alpha L} \sum_n b_n g(t - nT_b) \tag{11}$$

- However, there is quantum (shoot) noise even in ideal fiber optic transmission and the real received electrical pulse is given by:

$$I_R(t) = \eta_{SD} I_{T,peak}\; e^{-\alpha L} \sum_n b_n g(t - nT_b) \; + \underset{\uparrow}{n(t)} \tag{12}$$

Quantum noise
(Shoot noise)

Ideal Fiber Optics Transmission System

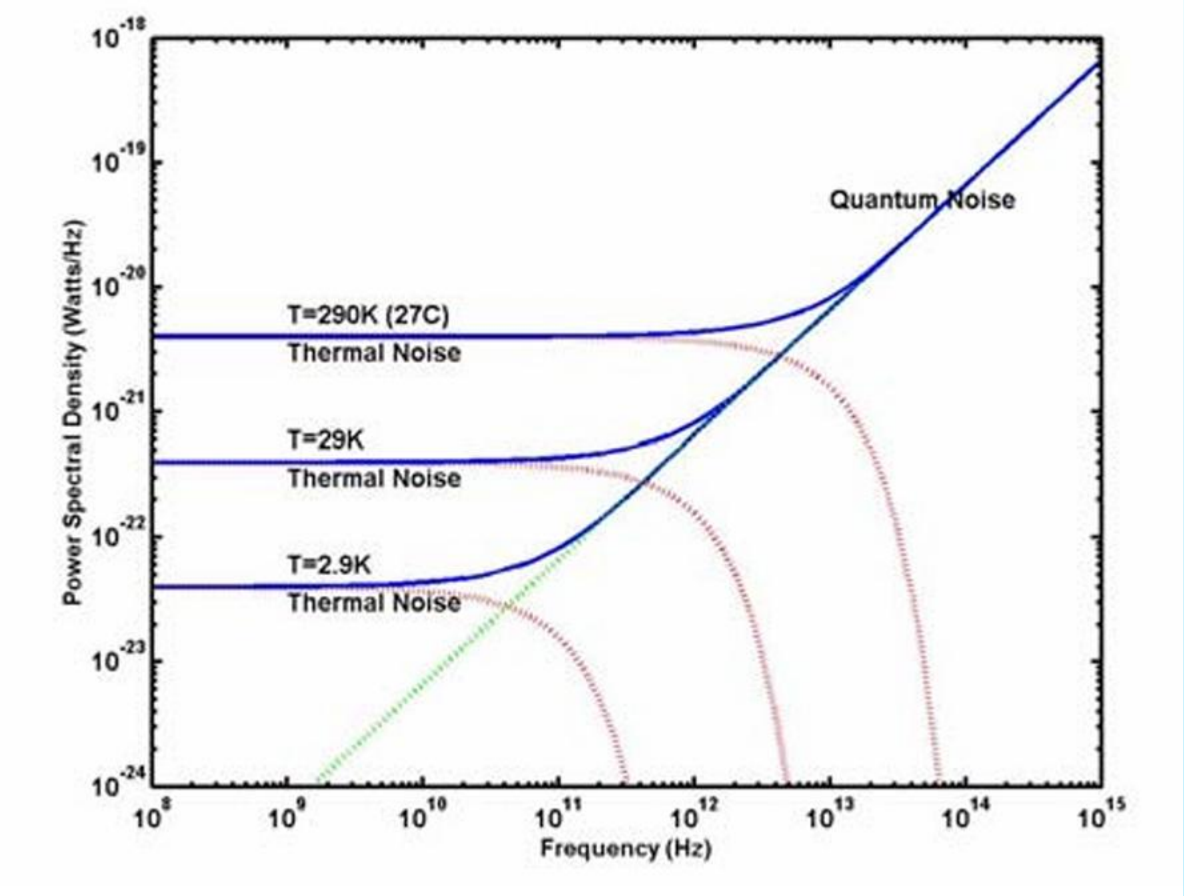


Figure 4: Quantum and thermal noise at different frequencies

Source: E. Yuceturk, S. C. Esener, D. Huang, and T. Sze, "Comparative study of very short distance electrical and optical interconnects based on channel characteristics," in Optics in Computing, A. Sawchuk, ed., Vol. 90 of OSA Trends in Optics and Photonics (Optica Publishing Group, 2003), paper OWA3. https://www.researchgate.net/profile/Sadik-Esener/publication/228565077/figure/fig1/AS:668963189047304@1536504661576/Power-density-of-thermal-noise-and-quantum-noise-versus-frequency-3-Blue-Total-noise.png

Optical Power and Number of Photons Received

- Average number of photons received in a time Δt is given by:

$$N_R(\Delta t) = \frac{E_R}{hf} = \frac{P_R \Delta t}{hf} \qquad (13)$$

Where:

$N_R(\Delta t)$	Average number of photons received in a time Δt
E_R	Energy of photons received in a time Δt
P_R	Received optical power

- Photon arrival at the photodetector has statistical nature and follows Poisson distribution.

Optical Power and Number of Photons Received

- Probability of receiving k photons in a time Δt at the photodetector is given by:

$$P(N_R(\Delta t) = k) = \frac{(\lambda \Delta t)^k e^{-\lambda \Delta t}}{k!} \qquad (14)$$

Where:

λ Average number of photons received per unit time

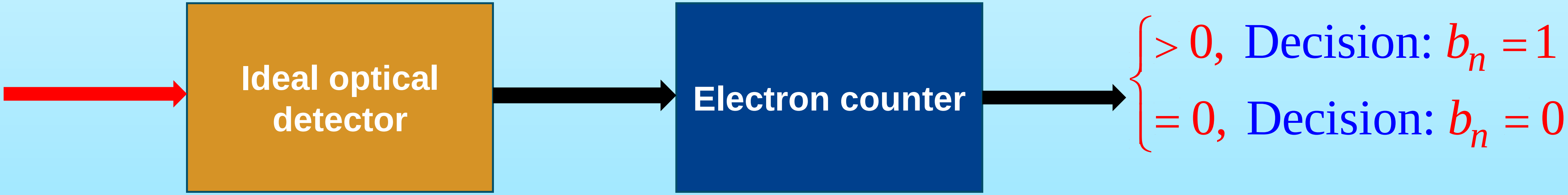
- From eq (13) and eq (14), the average number of photons received in a time Δt is given by

$$N_R(\Delta t) = \lambda \Delta t = \frac{P_R \Delta t}{hf} \qquad (15)$$

Bit Error Probability of Ideal Fiber Optics Transmission System

Bit Error Probability

- In ideal fiber optic transmission, we consider ideal decision procedure which is electron counter at the receiver



$$\begin{cases} >0, & \text{Decision: } b_n = 1 \\ =0, & \text{Decision: } b_n = 0 \end{cases}$$

Figure 5: Ideal Receiver

- The quantum efficiency of the ideal optical detector is unity, $\eta_D = 1$, Hence

Each received photon → **Generates an electron**

Bit Error Probability

- Bit error probability is the probability of receiving bit 1 when bit 0 is transmitted and the probability of receiving bit 0 when bit 1 transmitted:

$$P(e) = P(1\,RX\,/\,0\,TX)P(0\,TX) + P(0\,RX\,/\,1\,TX)P(1\,TX) \qquad (16)$$

- Considering equiprobable bit transmission, $P(0\,TX) = P(1\,TX) = \frac{1}{2}$, we will have:

:

$$P(e) = \frac{1}{2}P(1\,RX\,/\,0\,TX) + \frac{1}{2}P(0\,RX\,/\,1\,TX) \qquad (17)$$

- Since there are no noise source which generate electrons in the absence of received photon, we will have , $P(1\,RX\,/\,0\,TX) = 0$, and the bit error probability becomes

$$P(e) = \frac{1}{2}P(0\,RX\,/\,1\,TX) \qquad (18)$$

Bit Error Probability

- Eq (18) reveals that there is a probability of receiving bit 0 when bit 1 is transmitted due to the effect of **attenuation (fiber optic loss)**

$$P(e) = \frac{1}{2} P(0\ RX\ /1\ TX) = \frac{1}{2} P(N_R(T_b) = 0 / b_n = 1)$$

$$= \left(P(N_R(\Delta t) = k) = \frac{(\lambda \Delta t)^k e^{-\lambda \Delta t}}{k!} \right)_{k=0, \Delta t = T_b}$$

$$= e^{-\lambda T_b} = e^{-\frac{P_{R,peak} T_b}{hf}} = e^{-N_R(T_b)} \qquad (19)$$

Where:

$N_R(T_b)$ Average number of photons received when bit 1 transmitted

Bit Error Probability

- If we consider equiprobable transmission of bit 1 and bit 0, the average received photon number, $\bar{N}_R(T_b)$ becomes:

$$\bar{N}_R(T_b) = N_R(T_b)P(1\ TX) + 0 * P(1\ TX) = \frac{1}{2}N_R(T_b) \qquad (20)$$

- Therefore, using eq (19) and eq (20) the error probability can also be written as:

$$P(e) = \frac{1}{2}e^{-2\bar{N}_R(T_b)} \qquad (21)$$

Sensitivity and Loss Limited Maximum Reachable Distance

Sensitivity

- The required bit error probability for optical fiber communication system spans in the range: $\rightarrow 10^{-9}$ - 10^{-12}
- **Sensitivity:** refers to the minimum optical power (or photons per bit) required at the receiver to achieve the desired bit error probability
- Considering the required bit error probability for ideal fiber optic communication is $P(e)=10^{-9}$, the required average number of photons to achieve $P(e)=10^{-9}$ is :

$$P(e)=\frac{1}{2}e^{-2\bar{N}_R(T_b)}=10^{-9} \qquad (22)$$

- From Eq(22), we have:

$$\bar{N}_R(T_b)=\frac{1}{2}\ln\left(\frac{10^9}{2}\right)=10\ photons/bit \qquad (23)$$

Sensitivity

- The required average optical power per bit can also be calculated as:

$$\bar{P}_R(T_b) = \bar{N}_R(T_b)\frac{hf}{T_b} = \bar{N}_R(T_b)hfR_b = 10hfR_b \tag{24}$$

Where:

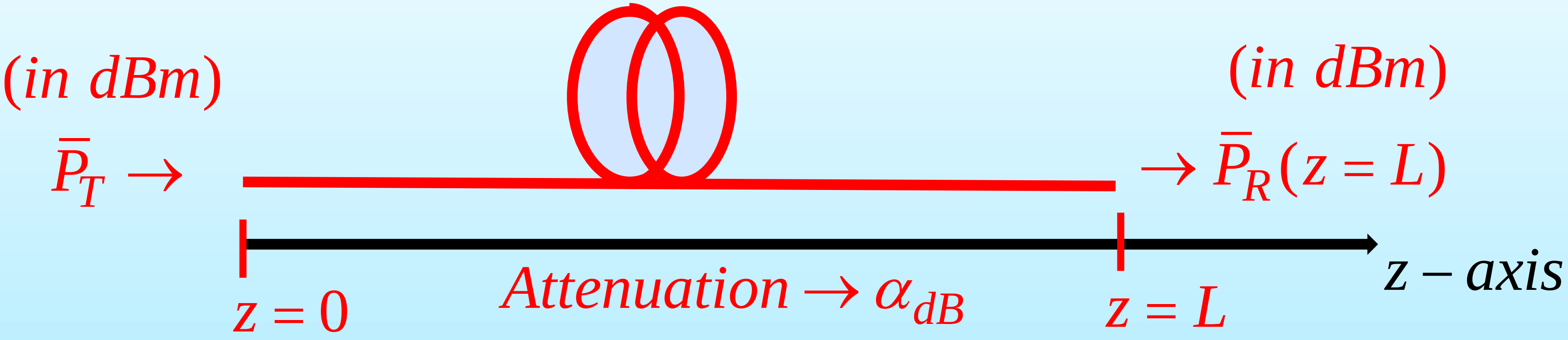
$\bar{P}_R(T_b)$ The required average optical power per bit

$R_b = \dfrac{1}{T_b}$ The bitrate of the system

- The sensitivity of ideal receiver is the minimum required average optical power per bit $(\bar{P}_{R\text{-}sen}(T_b))$ for $P(e) = 10^{-9}$:

$$\bar{P}_{R\text{-}sen}(T_b) = 10hfR_b \tag{25}$$

Loss limited Maximum Reachable Distance



(in dBm)

$\bar{P}_T \rightarrow$

(in dBm)

$\rightarrow \bar{P}_R(z = L)$

$z = 0$

$Attenuation \rightarrow \alpha_{dB}$

$z = L$

$z - axis$

- The received average optical power in dBm $(\bar{P}_R(L))$ at a given point of z=L is:

$$\bar{P}_R(L) = \bar{P}_T - \alpha_{dB} L \qquad (26)$$

- From eq (26), the transmission distance L is:

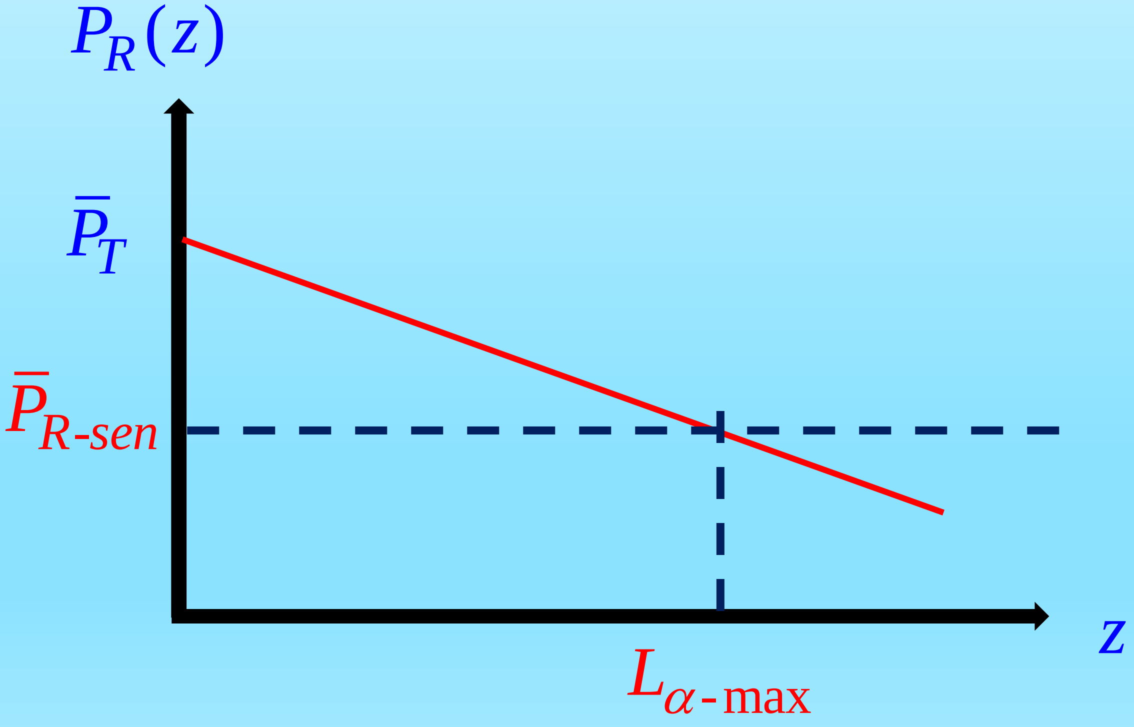
$$L = \frac{\bar{P}_T - \bar{P}_R(L)}{\alpha_{dB}} \qquad (27)$$

$$for\ P(e) = 10^{-9} \rightarrow \bar{P}_R \geq \bar{P}_{R\text{-}sen}$$

Loss limited Maximum Reachable Distance

- From eq(27) we will obtain the optical fiber attenuation limited maximum transmission distance$\left(L_{\alpha\text{-max}}\right)$When $\rightarrow \bar{P}_R = \bar{P}_{R\text{-}sen}$

$$L_{\alpha\text{-max}} = \frac{\bar{P}_T - \bar{P}_{R\text{-}sen}}{\alpha_{dB}} \qquad (28)$$



Optical Power Budget

- Let,

$\bar{P}_T \rightarrow$ the available average transmitted optical power

$\bar{P}_{R\text{-}sen} \rightarrow$ the sensitivity to achieve the require bit error probability

- The power budget of the system is given by:

$$Power\ budget = \bar{P}_T - \bar{P}_{R\text{-}sen} \qquad (29)$$

Dispersion limited Maximum Reachable Distance

- For functional fiber optics communication system, the differential group delay $(\Delta\tau_g)$ should satisfy the following condition

$$|\Delta\tau_g| \leq \frac{T_B}{2} = \frac{1}{2R_B} \tag{30}$$

Where: $T_B = \frac{1}{R_B}$

Bit duration

Bitrate

- For spectral width $(\Delta\lambda)$ between two optical signal spectra, the deferential group delay can be written in terms **of dispersion parameter.**

Dispersion limited Maximum Reachable Distance

$$\Delta\tau_g = DL(\Delta\lambda) \qquad (31)$$

Where:

D Is dispersion

L Is the transmission distance

- Using eq(30) and eq(31), we have:

$$|\Delta\tau_g| = |D| L(\Delta\lambda) \leq \frac{1}{2R_B} \qquad (32)$$

- Solving for L:

$$L \leq \frac{1}{2|D|(\Delta\lambda)R_B} \qquad (33)$$

Dispersion limited Maximum Reachable Distance

- The maximum dispersion limited transmission distance can be found from eq (33) when the equal sign holds:

$$L_{D\text{-max}} = \frac{1}{2|D|(\Delta\lambda)R_B} \tag{34}$$

Dispersion Limited Maximum Transmission Distance

- The maximum transmission distance $(L_{\max})$ considering both attenuation and dispersion limitation is :

$$L_{\max} = \min\{L_{\alpha\text{-max}}, L_{D\text{-max}}\} \tag{35}$$

Summary

- **Fiber optic transmission system key design parameters:**
    - ✓ Transmitted optical power, received optical power, power budget, bit error probability
- **Ideal Fiber optics transmission system components:**
    - ✓ Ideal optical source, ideal photo detector, ideal optical fiber, ideal decision circuit (electron counter)
    - ✓ **Noise Consideration:** Quantum Noise
- **Maximum Transmission distance of ideal system limited by:**
    - ✓ Level of transmitted power
    - ✓ Sensitivity of the system
    - ✓ Fiber loss
    - ✓ Fiber dispersion

References

[1] Govind P. Agrawal,‘’ Fiber-Optic Communication Systems’’, John Wiley & Sons, Pp.183, 2002.

[2] Gerd Keiser, *“Fiber Optic Communications”,* Springer, Pp.305, 2021.

Thank You !