

Fiber Optics Communications

Week 12

Optical Fiber transmission system design -2

Lecturer: Zelalem Hailu (Assistant Prof.)

***Addis Ababa Science and Technology University
Addis Ababa, Ethiopia***

Topics of Previous Lecture (Week-11)

Optical Fiber transmission system design -1

- Ideal Non-Amplified optical Fiber Transmission systems
- Optical Power and Number of Photons Received
- Bit Error Probability of Ideal Fiber Optics Transmission System
- Sensitivity and Loss Limited Maximum Reachable Distance
- Optical Power Budget
- Dispersion limited Maximum Reachable Distance

Week-12: Lecture Learning Outcomes

1. Explain the fundamental design principles of an optical fiber transmission system with optical signal amplification
2. Describe the operation and gain characteristics of Erbium-Doped Fiber Amplifiers (EDFA) and their role in long-haul optical communication systems
3. Discuss the nature and impact of Amplified Spontaneous Emission (ASE) noise and polarization effects in optical amplifiers
4. Compute and assess the Optical Signal-to-Noise Ratio (OSNR) and relate it to system quality metrics
5. Estimate the bit error probability in ASE-limited optical communication systems.
6. Determine the maximum reachable transmission distance constrained by ASE noise and OSNR degradation.

Week-12: Optical Fiber transmission system design -2

Outline

- Amplified Optical Fiber transmission system
- EDFA Gain and Noise Model
- ASE Noise and Polarization
- ASE Noise Power Spectral Density (PSD)
- Filtered Optical Signal
- Optical Signal to Noise Ratio (OSNR)
- Bit Error Probability
- ASE Limited Maximum Reachable Distance

Optical Fiber Transmission With Amplification

- As we have seen in the previous lecture, attenuation put a limitation in terms of transmission distance and bit error rate
- During transmission, the optical signal power is reduced due to optical fiber attenuation
- To improve the system performance the signal should be amplified using optical amplifiers to compensate the signal loss
- Erbium doped fiber amplifier (EDFA) is the most widely employed optical amplifier in practical fiber optics system
- EDFA has high gain typically in the range 20-40 dB
- However, EDFA will also generate a noise which is called **ASE** (**Amplified Spontaneous Emission**) Noise

Optical Fiber Transmission With Amplification

- In amplified optical fiber communication, Optical filter will be employed to filter out the ASE noise

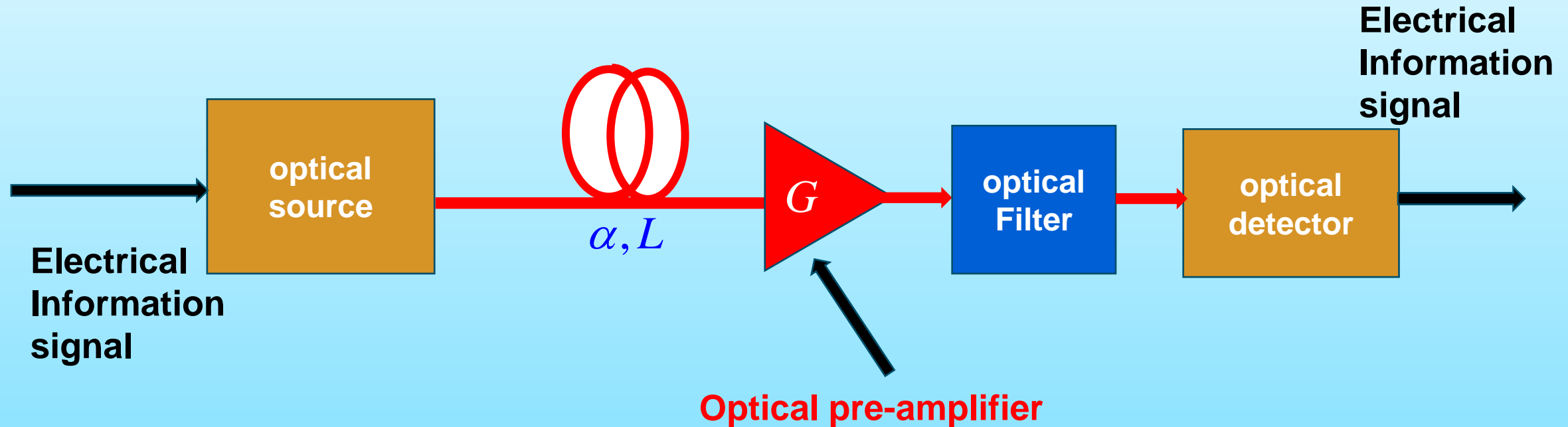


Figure 1: Optical fiber transmission system with amplification

EDFA Gain and Noise Model

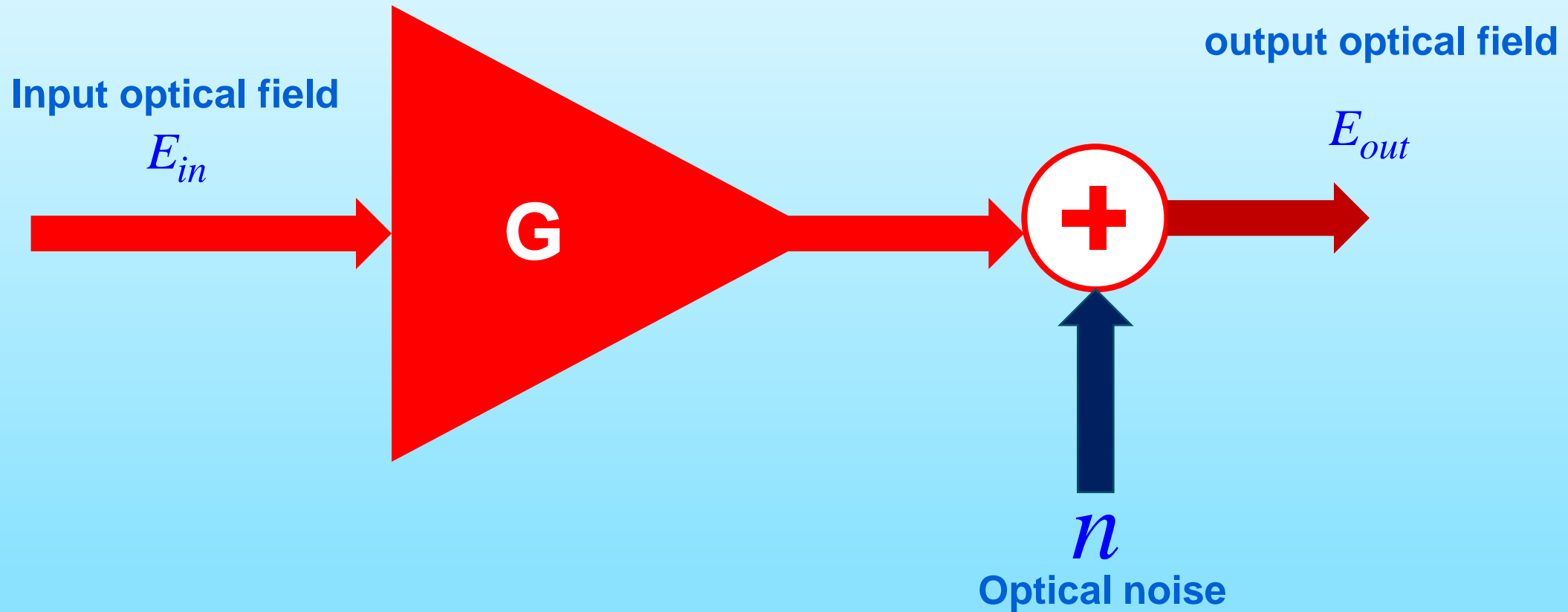


Figure 2: EDFA Gain and Noise Model

ASE Noise and Polarization

- Optical noise (ASE) introduced by an EDFA increases proportionally with the EDFA gain (G)
- Spontaneous emission is unpolarized and ASE noise is added equally across all polarizations
- Considering x and y polarization, the statistical characteristics of ASE noise in x and y polarization is similar.
- ASE noise is a **white Gaussian noise** which is random noise that has equal power across all frequencies
- The output field, $\vec{E}_{out}(t)$, of the EDFA can be written:

$$\vec{E}_{out}(t) = \underbrace{GE_x(t) + n_x(t)}_{\text{x- component: signal + ASE}} \hat{x} + \underbrace{\left[GE_y(t) + n_y(t) \right]}_{\text{y- component: signal + ASE}} \hat{y} \quad (1)$$

x- component: signal + ASE

y- component: signal + ASE

ASE Noise and Polarization

- Polarization beam splitter (PBS) can be used to clearly illustrate the ASE noise added in x and y polarization in the model representation of the EDFA

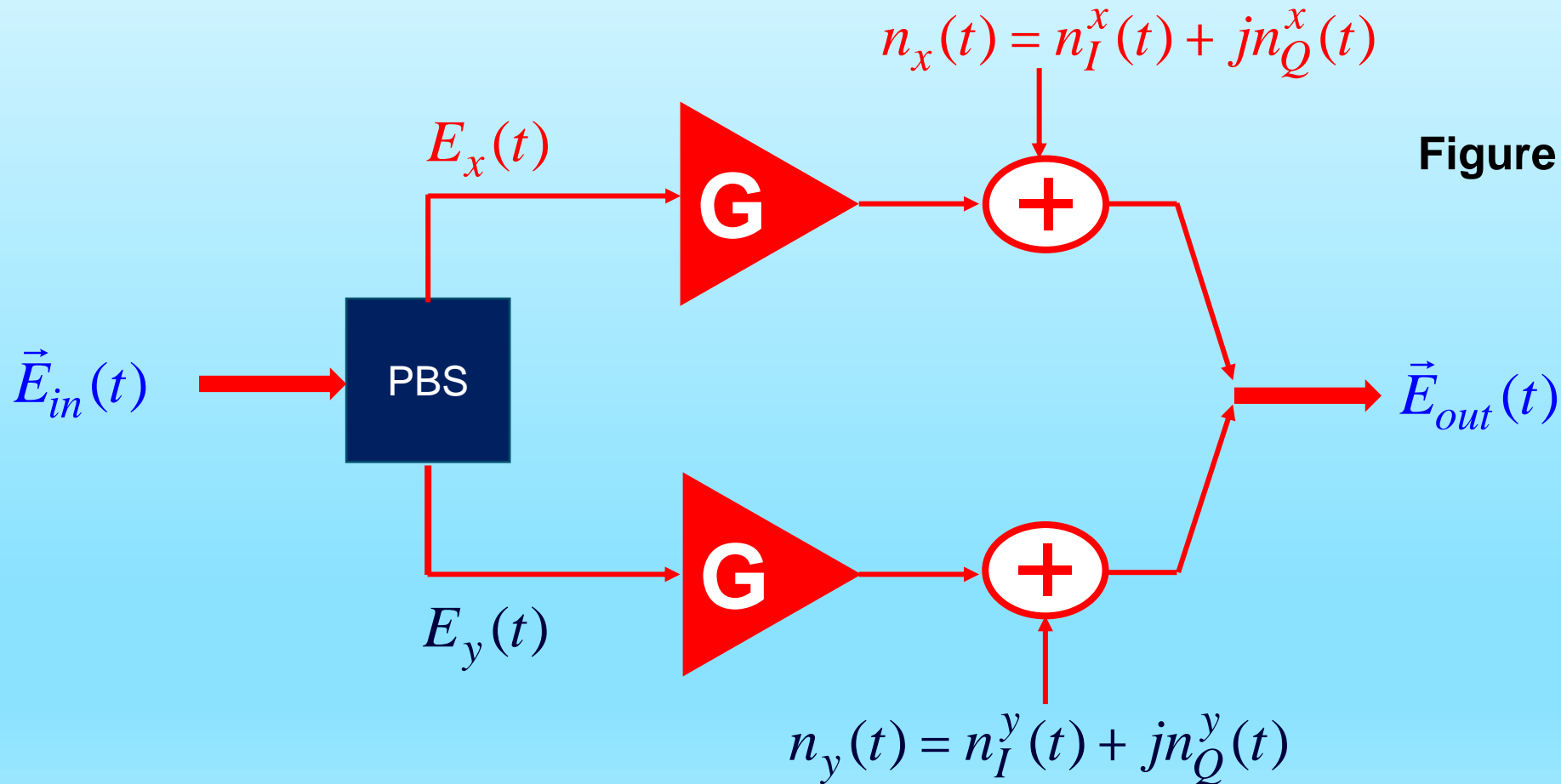


Figure 3: EDFA Noise and Polarization

ASE Noise and Polarization

- The x and y component of the signal at the output of EDFA

x-component

$$\vec{E}_{x\text{-out}}(t) = [GE_x(t) + n_x(t)]\hat{x} = [GE_x(t) + n_I^x(t) + jn_Q^x(t)]\hat{x} \quad (2)$$

y-component

$$\vec{E}_{y\text{-out}}(t) = [GE_y(t) + n_y(t)]\hat{y} = [GE_y(t) + n_I^y(t) + jn_Q^y(t)]\hat{y} \quad (3)$$

- The total output optical field at the output of EDFA

$$\begin{aligned} \vec{E}_{out}(t) &= \vec{E}_{x\text{-out}}(t) + \vec{E}_{y\text{-out}}(t) \\ &= G(E_x(t)\hat{x} + E_y(t)\hat{y}) + n_I^x(t)\hat{x} + n_I^y(t)\hat{y} + j(n_Q^x(t)\hat{x} + n_Q^y(t)\hat{y}) \end{aligned} \quad (4)$$

EDFA Gain Spectrum

- The light emitted by a laser is typically linearly polarized
- However, the optical fiber does not preserve the state of polarization
- the output polarization at the end of the fiber changes randomly with time
- The Gain spectral density of EDFA is typically flat in C-band (approximately between 1530 and 1560 nm)
- Without gain flattening, the EDFA gain varies with wavelength, typically higher around and below 1530 nm and lower toward 1565 nm
- This non-uniform gain causes channel power imbalance in WDM systems

EDFA Gain Spectrum

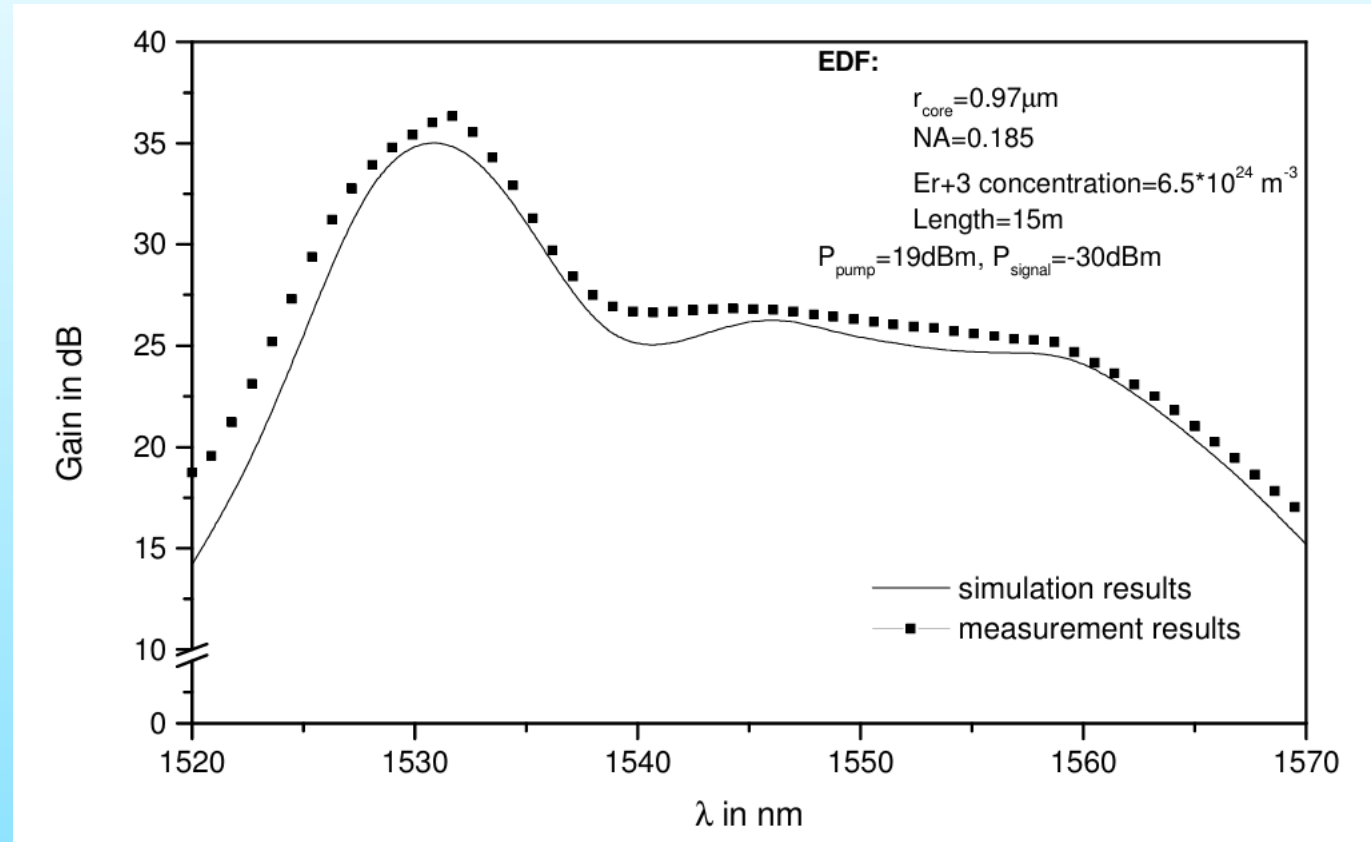


Figure 3: EDFA Gain Spectrum

Source: M. M. Kozak, R. Caspary and U. B. Unrau, "Computer aided EDFA design, simulation and optimization," Proceedings of 2001 3rd International Conference on Transparent Optical Networks (IEEE Cat. No.01EX488), 2001. https://www.researchgate.net/profile/Marcin-Kozak/publication/3904255/figure/fig1/AS:668979236442112@1536508487195/Results-of-the-Giles-test-using-EDFA-Design-R-and-VPIComponentMaker-R-The-parameters_W640.jpg

EDFA Gain Spectrum

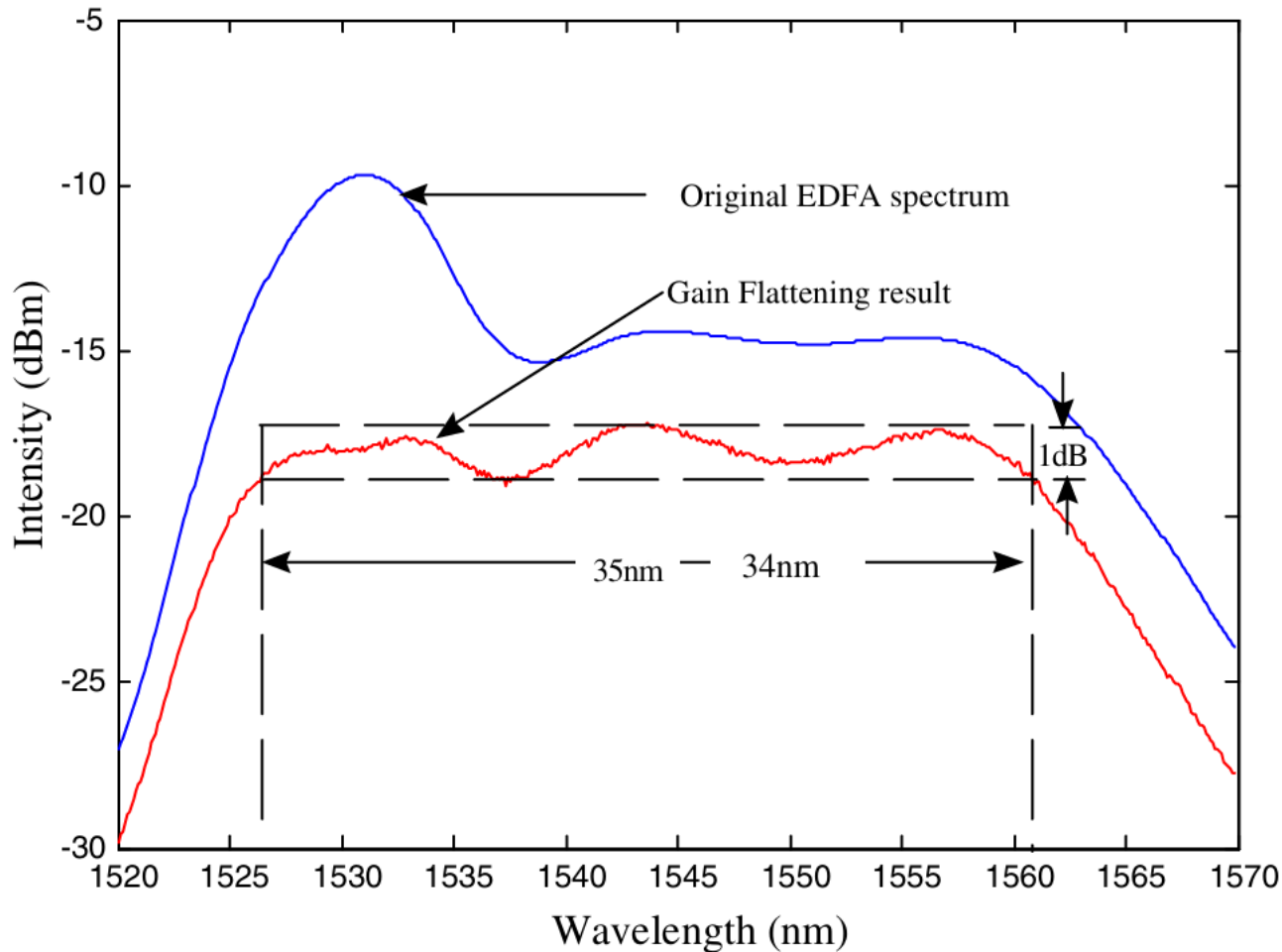


Figure 4: EDFA Gain Flattening

Source: N. Ni, C.C. Chan, K.M. Tan, S.C. Tjin, X.Y. Dong, "Broad-band EDFA gain flattening by using an embedded long-period fiber grating filter," *Optics Communications*, Volume 271, Issue 2, 2007.
<https://ars.els-cdn.com/content/image/1-s2.0-S0030401806010583-gr8.jpg>

ASE Noise Power Spectral Density (PSD)

- Quantum mechanics and experimental results show that the ASE noise power spectral density (PSD) is given by [1]:

$$\frac{N_0}{2} = \frac{hf}{2} (G - 1) n_{sp} \quad (5)$$

Where:

N_0 ASE noise power per unit bandwidth

G EDFA gain

n_{sp} The spontaneous emission factor

- The spontaneous emission factor is always greater than unity (1)

ASE Noise Power Spectral Density (PSD)



Figure 4: ASE noise PSD

ASE Noise Power Spectral Density (PSD)

- From eq (5) it is noted that the PSD of ASE noise increase as n_{sp} increases
- In addition, PSD of ASE noise increase as frequency increases and ASE noise is not purely white
- However, it is considered as white noise in theoretical analysis
- The power spectral density of both the in phase and quadrature components of ASE noise are equal $\rightarrow N_0 / 2$

Filtered Optical Signal

- Assuming the transmitted optical power is $P_T(t)$ and the transfer function of the optical filter is $h_o(t)$

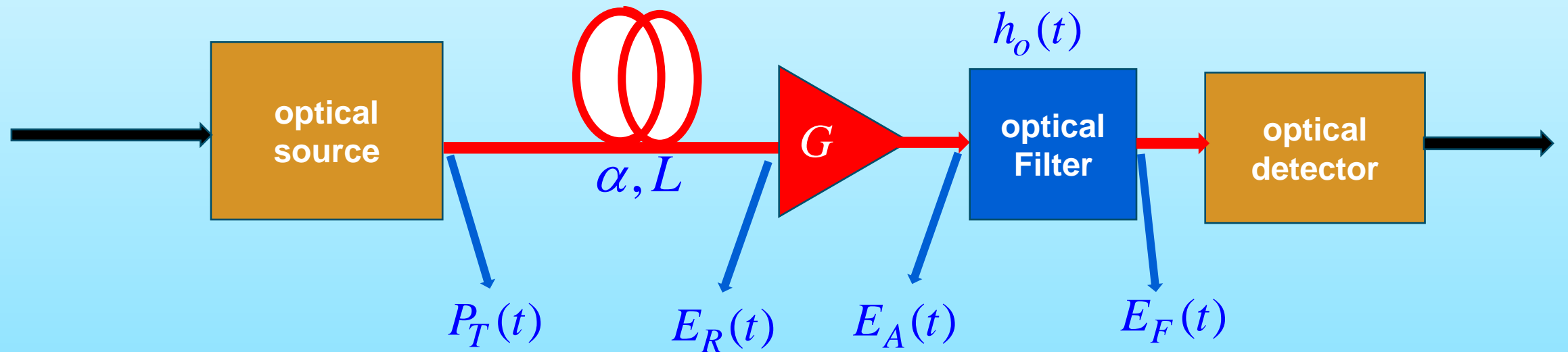


Figure 5: Amplified optical transmission system with optical filter

Filtered Optical Signal

- Considering on-off keying modulation, $P_T(t)$ can be written as:

$$P_T(t) = P_{T,peak} \sum_n b_n g(t - nT_b) = 2\bar{P}_T \sum_n b_n g(t - nT_b) \quad (6)$$

Where: $b_n : \{0,1\}$ binary bits

$g(t)$ optical pulse shape

T_b single bit duration

$P_{T,peak}$ Transmitted peak optical power when bit 1 is transmitted

$\bar{P}_T = \frac{1}{2} P_{T,peak}$ Average transmitted optical power

- The transmitted optical field $E_T(t)$ can also be written:

$$E_T(t) = E_{T,peak} \sum_n b_n f(t - nT_b) = \sqrt{P_{T,peak}} \sum_n b_n \sqrt{g(t - nT_b)} \quad (7)$$

Filtered Optical Signal

- The received optical field at the input of EDFA, $E_R(t)$, is given by:

$$E_R(t) = \sqrt{P_{T,peak} e^{-\alpha L}} \sum_n b_n \sqrt{g(t - nT_b)} \quad (8)$$

- The amplified optical field $E_A(t)$ by EDFA can also be written:


$$E_A(t) = \sqrt{P_{T,peak} e^{-\alpha L} G} \sum_n b_n \sqrt{g(t - nT_b)} + n(t) \quad (9)$$

- The optical field $E_F(t)$ after the optical filter:

$$\begin{aligned} E_F(t) &= E_A(t) * h_o(t) \\ &= \left[\sqrt{P_{T,peak} e^{-\alpha L} G} \sum_n b_n \sqrt{g(t - nT_b)} + n(t) \right] * h_o(t) \quad (10) \end{aligned}$$

Filtered Optical Signal

- From eq(10), $E_F(t)$ can be written as:

$$\begin{aligned} E_F(t) &= \left[\sqrt{P_{T,peak}} e^{-\alpha L} G \sum_n b_n \sqrt{g(t - nT_b)} \right] * h_o(t) + n(t) * h_o(t) \\ &= \underbrace{\left[\sqrt{P_{T,peak}} e^{-\alpha L} G \sum_n b_n \sqrt{g(t - nT_b)} \right] * h_o(t)}_{\text{Filtered Optical signal}} + \underbrace{n(t) * h_o(t)}_{\text{Filtered ASE noise}} \end{aligned} \quad (11)$$




Optical Signal to Noise Ratio (OSNR)

OSNR

- OSNR is one of the key performance metric in fiber optics communication system
- OSNR is given by [2]:

$$OSNR = \frac{\bar{P}_S}{P_N} \quad (12)$$

Where:

\bar{P}_S Is the useful signal average optical power at the receiver

P_N The ASE noise power measured at the output of an optical filter having a specified noise equivalent bandwidth

- The ASE noise affect the OSNR values; OSNR reduces as ASE power increases

OSNR

- The noise power which is measured at the output of optical filter is given by:

$$P_N = (2)(2) \frac{N_0}{2} B_N \quad (13)$$

Accounts for
the in-phase
and quadrature
components

Accounts for
x and y
polarizations

Where:

B_N The noise equivalent bandwidth

- Assuming the optical filter is matched filter, the widely used convention is as follows:

$$B_N = R_B \quad (14)$$

Bitrate of the system

OSNR

- For dual polarization The OSNR can also be written as:

$$OSNR = \frac{\bar{P}_S}{P_N} = \frac{\bar{P}_S}{(2)(2)\left(\frac{N_0}{2}\right)R_B} = \frac{\bar{P}_S}{2N_0R_B} \quad (15)$$

- For single polarization The OSNR can also be written as:

$$OSNR = \frac{\bar{P}_S}{P_N} = \frac{\bar{P}_S}{(2)(1)\left(\frac{N_0}{2}\right)R_B} = \frac{\bar{P}_S}{N_0R_B} \quad (16)$$

↑
Only one polarization
(either x or y polarization)

OSNR

- Alternatively, the OSNR can be written as

For Dual Polarization:
$$OSNR = \frac{\bar{P}_S}{2N_0R_B} = \frac{G\bar{P}_R}{2R_B[hf(G-1)n_{sp}]} \quad (17)$$

For single Polarization:
$$OSNR = \frac{\bar{P}_S}{N_0R_B} = \frac{G\bar{P}_R}{R_B[hf(G-1)n_{sp}]} \quad (18)$$

Where:

\bar{P}_R Is the signal average optical power received at input of EDFA

Bit Error Probability

- The approximated bit error probability of the system can be written as a function of the OSNR as:

$$P(e) \cong \frac{1}{2} e^{-0.98(OSNR)} \quad (19)$$

- The ASE noise induce performance limitation in number of ways as :

ASE Noise–Induced Limitations

- ❖ Reduced Optical Signal-to-Noise Ratio (OSNR)
- ❖ Increased Bit Error Probability
- ❖ Reduce transmission distance
- ❖ Impacts Modulation Formats: High-order modulation becomes more sensitive to ASE noise.)

Bit Error Probability Vs Bit Error Rate (BER)

- Bit error probability and bit error rate are related parameters
- **Bit error probability:** is theoretical measure that shows the probability that any given bit is received in error
- Bit error probability usually derived from analytical models based on the signal and noise statistics
- **Bit Error Rate (BER):** A measured quantity of the ratio of erroneous bits to the total transmitted bits

$$BER = \frac{\text{Number of received erroneous bits}}{\text{Total transmitted bits}} \quad (20)$$

Bit Error Probability

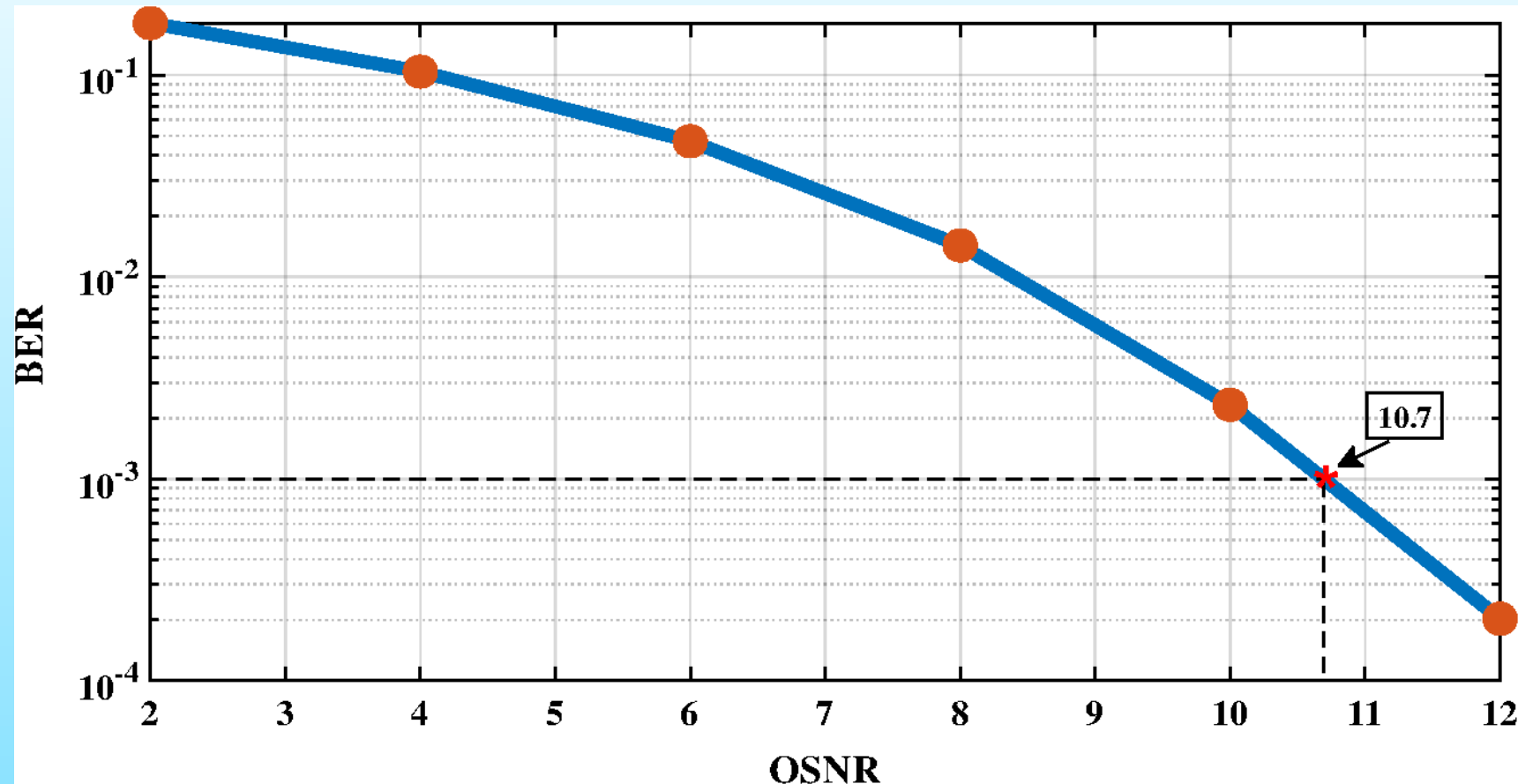


Figure 6: BER Vs OSNR

Source: S. Debnath, S. Das, N. Sinha, and B. Bhowmik, "Generation of an optical 8-amplitude phase-shift-keying (8-APSK) modulator using silicon microrings in add-drop configuration," *Optical Engineering*, vol. 63, no. 3, p. 035110, Mar. 2024.

https://www.spiedigitallibrary.org/ContentImages/Journals/OPEGAR/63/3/035110/FigureImages/OE_63_3_035110_f012.png

ASE Limited Maximum Reachable Distance

- The maximum reachable distance of the fiber optic system is also limited by the ASE noise.
- The maximum allowable BER (BER_{\max}) should satisfy the following condition

$$P(e) \cong \frac{1}{2} e^{-0.98(OSNR)} \leq BER_{\max} \quad (21)$$

- By solving eq (21), we have

$$OSNR \geq 1.02 \ln \left(\frac{1}{2BER_{\max}} \right) \quad (22)$$

- From the above eq (22), the maximum reachable distance can be obtained when the left and the right term are equal as:

ASE Limited Maximum Reachable Distance

$$OSNR = 1.02 \ln \left(\frac{1}{2BER_{\max}} \right) \quad (23)$$

- From the above eq (23):

$$OSNR = \frac{G\bar{P}_R}{2hfn_{sp}R_B(G-1)} = \frac{e^{-\alpha L_{\max}}\bar{P}_T}{2hfn_{sp}R_B} = 1.02 \ln \left(\frac{1}{2BER_{\max}} \right) \quad (24)$$

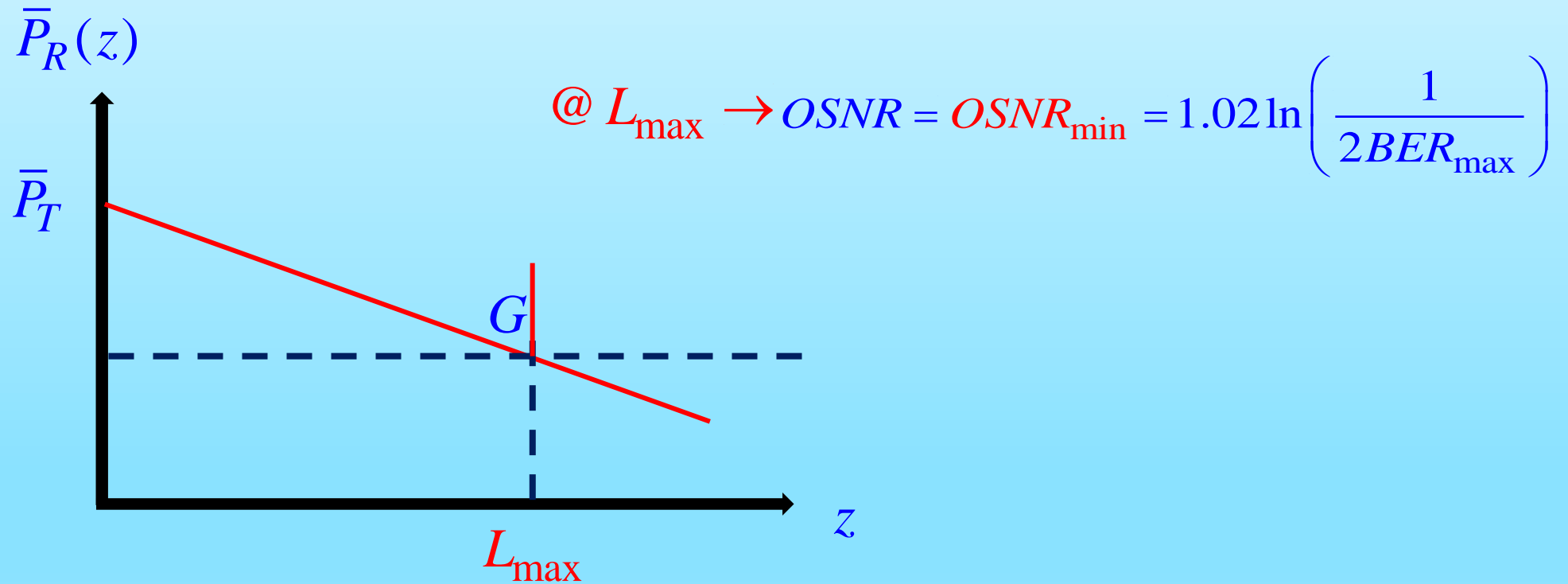
Where: $G \approx G-1$

- From the above eq (24):

$$L_{\max} = \frac{1}{2\alpha} \left[\ln(\bar{P}_T) - \left[\ln \left(2.04hfn_{sp}R_B \left(\ln \left(\frac{1}{2BER_{\max}} \right) \right) \right) \right] \right] \quad (25)$$

ASE Limited Maximum Reachable Distance

- Once the maximum transmission distance (L_{\max}) is reached for the given average transmitted optical power (\bar{P}_T), it will be difficult to increase the transmission distance beyond L_{\max} unless the number of amplifier is increased



ASE Limited Maximum Reachable Distance

- However, it is possible to increase the transmission distance beyond L_{\max} if chain of in-line amplifier are used.

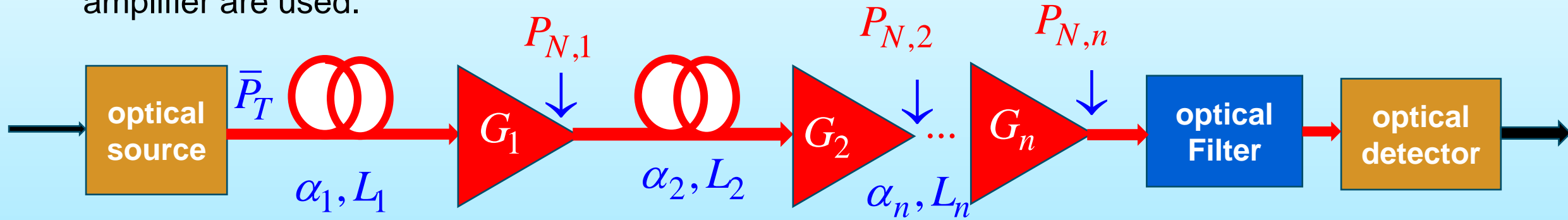
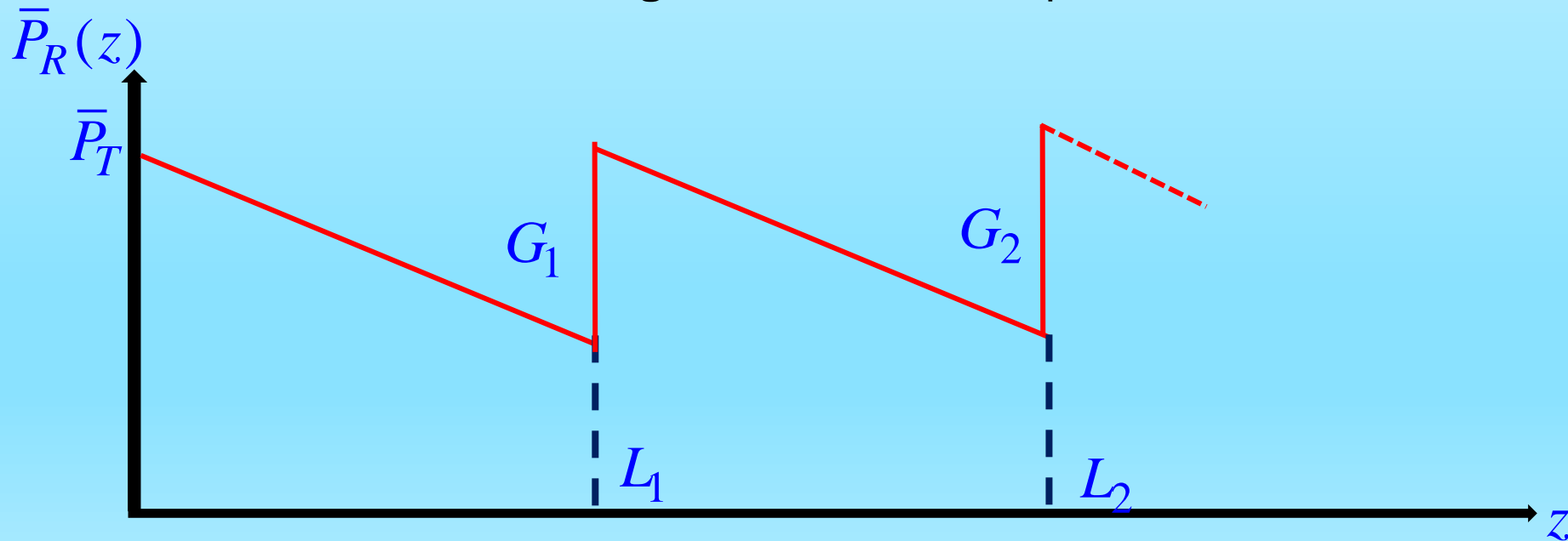


Figure 7: Chain of amplifier



ASE Limited Maximum Reachable Distance

- The optical transmission system with chain amplifier can be model as follows

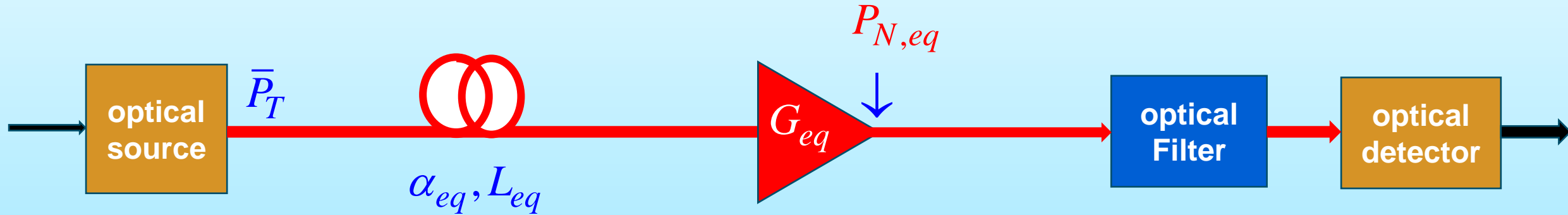


Figure 8: simplified model of Chain of amplifier

Where: $L_{eq} = \sum_n L_n$, $\alpha_{eq} = \frac{1}{L_{eq}} \sum_n \alpha_n L_n$, $G_{eq} = \prod_n G_n$

- The OSNR is also given by

$$OSNR = \frac{\bar{P}_S}{P_{N,eq}} = \frac{G_{eq} e^{-\alpha_{eq} L_{eq}} \bar{P}_T}{P_{N,eq}} \quad (26)$$

Summary

- **Amplified Optical fiber transmission system:**
 - ✓ Usage of optical amplifier to compensate optical power reduction
 - ✓ **Dominant Noise:** ASE noise
 - ✓ **ASE Noise:** Proportional to gain, random polarization
- **Optical Filter:**
 - ✓ Optical filter is used to filter out ASE noise
 - ✓ Noise Equivalent bandwidth is equal to the optical filter bandwidth
- **ASE Noise induce limitation on:**
 - ✓ **OSNR**
 - ✓ **Bit error probability**
 - ✓ **Transmission distance**

References

- [1] Gerd Keiser, "*Fiber Optic Communications*", Springer, Pp.456, 2021.
- [2] Shiva Kumar and M. Jamal Deen, "Fiber Optic Communication Systems", Wiley, Pp.317, 2014.



Thank You!