- A moving average may be viewed as a low-pass filter
- A simple moving average of the last n observations is given by:

$$s_t = \frac{1}{n}\sum_{i=1}^{n} y_{t-i+1}$$

- By smoothing the time series, it can remove the effects of seasonality
- Note that it gives equal weight to both new and old observations
- Similar to climatology benchmark, where n can be estimated using in-sample data

Exponentially weighted moving average

- EWMA employs exponentially decreasing weights to discount the influence of old observations
- For example:

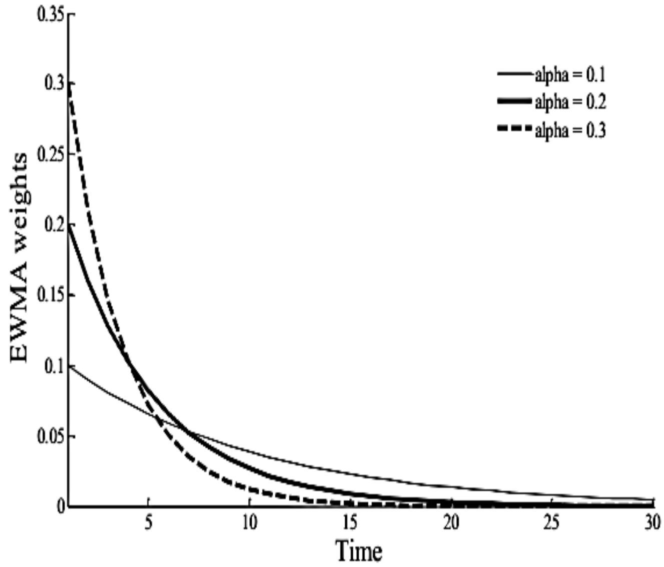
$$s_t = \frac{1}{2}y_t + \frac{1}{4}y_{t-1} + \frac{1}{8}y_{t-2} + \cdots$$

- This can be written as:

$$s_t = \alpha y_t + \alpha(1-\alpha)y_{t-1} + \alpha(1-\alpha)^2 y_{t-2} + \cdots$$
$$s_t = \alpha y_t + (1-\alpha)s_{t-1}$$

- The smoothing factor α ($0 \le \alpha \le 1$) determines the rate of decay of old information
- k-step ahead forecast: $\hat{y}_{t+k} = s_t$

EWMA weights



- Low α, low weight to latest observation, more smoothing, slow response to structural change, more suitable for rapidly fluctuating, noisy series
- High α, more weight to latest observation, less smoothing, quick response to structural change, more suitable for slight random fluctuating
- Common practise to initialize s_0=y_0, and estimate α by minimizing one-step ahead sum of squared errors, $min \sum_{i=1}^{n} e_i^2$

Exponential smoothing with a trend

- The Holt's exponential smoothing method can be used for a time series exhibiting trend
- Holt's method smoothes both level and trend:
- $S_t = \alpha y_t + (1-\alpha)(S_{t-1} + T_{t-1})$ $\quad level$
- $T_t = \beta(S_t - S_{t-1}) + (1-\beta)(T_{t-1})$ $\quad trend$
- $\hat{y}_{t+1} = S_t + T_t$ $\quad one-step\ ahead\ forecast$
- k-step ahead forecast: $\hat{y}_{t+k} = S_t + k \times T_t$
- To optimize the smoothing parameters α (0 ≤ α ≤1) and β (0 ≤β≤1) , vary the two parameters simultaneously over a two dimension search grid [0,1], and select the pair $\{\alpha_{opt}, \beta_{opt}\}$ that minimizes one-step ahead in-sample sum of squared errors

Exponential smoothing with a trend and seasonality

- The Holt's Winters exponential smoothing method can be used for a time series exhibiting trend and seasonality
- Holt's Winters method smoothes level, trend and seasonality:
- $S_t = \alpha(y_t - I_{t-s}) + (1-\alpha)(S_{t-1} + T_{t-1})$ *level*
- $T_t = \beta(S_t - S_{t-1}) + (1-\beta)(T_{t-1})$ *trend*
- $I_t = \gamma(y_t - S_t) + (1-\gamma)(I_{t-s})$ *seasonality*
- $\hat{y}_{t+1} = S_t + T_t + I_{t-s+1}$ *one – step ahead forecast*
- k-step ahead forecast: $\hat{y}_{t+k} = S_t + k \times T_t + I_{t-s+k}$
- s denotes length of the seasonal cycle
- Parameters are estimated by cross-validation

Modelling approaches – summary

- Benchmarks – Persistence and Climatology
- AR (uses lagged observations), ARMA (uses lagged observations and error terms), SARMA (extension of ARMA for seasonal time series)
- SETAR/MS-AR (piecewise linear models, switching depends on a threshold and a delay parameters in SETAR, and a latent variable in MS-AR)
- Analogue prediction method – for a current state, identify similar past states (analogues) and use their future trajectories as a forecast
- Exponential smoothing – weighted moving average, giving more weight to the more recent observations