- For example: given a AR model, $y_t = 0.7y_{t-1} + \varepsilon_t$, $y_{t-1} = 0.7y_{t-2} + \varepsilon_{t-1}$, we can write $y_t = 0.49y_{t-2} + 0.7\varepsilon_{t-1} + \varepsilon_t$. This indicates autocorrelations are significant for first few orders and die away exponentially (Tails off). For PACF, after accounting of y_{t-1}, additional effect of y_{t-2} is 0 (p spikes, then cuts off to zero)

- For example: given a MA model, $y_t = 0.7\varepsilon_{t-1} + \varepsilon_t$, $y_{t-1} = 0.7\varepsilon_{t-2} + \varepsilon_{t-1}$. This indicates autocorrelation is significant for first few order but is zero thereafter (q spikes, then cuts off to zero). For PACF, the above term can be re-written as $y_t = 0.7y_{t-1} - 0.49\varepsilon_{t-2} + \varepsilon_t$ (tails off)

Type of model	**ACF**	**PACF**
AR(p)	Tails off	p spikes, then cuts off to zero
MA(q)	q spikes, then cuts off to zero	Tails off
Mixed ARMA	Mix of the above	Mix of the above

Seasonal ARMA

- Seasonality – regular pattern in a time series that repeats itself, say every, s periods.
- For example, monthly temperature time series would show annual seasonality (s=12)
- To predict temperature for March this year (y_t), will have to refer to the same month from last year (y_{t-s})
- Seasonality and the length of seasonal cycle (s) can be identified by plotting the time series, using ACF, PACF plots
- Differencing is commonly used to remove seasonal effects:

$$\nabla_s y_t = y_t - y_{t-s}$$

Seasonal ARMA (SARMA)

- SARMA models can be employed to accommodate seasonality, and can be represented as (for a single seasonal cycle of period s :

$$\Upsilon_p(L)X_{P_2}(L^s)(y_t - c) = \Omega_q(L)\Gamma_{Q_2}(L^s)\varepsilon_t$$

- c is a constant
- ε_t is the model error
- L is the lag operator,
- Υ_p and X_{P_2} are polynomial functions for the AR terms having order p and P_2, respectively
- The polynomial functions for the MA terms are denoted by Ω_q and Γ_{Q_2}, having corresponding model order given by q and Q_2
- Additional AR and MA terns can be included to accommodate multiple seasonal cycle (for example, daily, weekly and annual cycle in electricity demand data)

Regime Switching Models

- Commonly used for time series exhibiting different regimes of activity
- Examples – Business cycles (growth and recession), Finance (high and low volatility), Epidemiology (inhibition and outbreak)
- Problem – How do we model periods of distinct activity?
- Solution – One approach is that we build a separate model for each regime. Say if there are two regimes, we use two models (i.e., one model for each regime)
- More precisely, this phenomenon of shifts in regime requires that the data generating process (DGP) is modelled using two or more sub-models, perhaps having the same model structure but a different set of model parameters
- Common to employ piecewise linear models, which assume that each regime can be described using a traditional linear model, AR(p)

Regime Switching Models