CONTINOUS ASSESSMENT TEST

ATTEMPT ALL QUETIONS

QUESTION ONE: 15 Marks

1. Let W be a Brownian motion and $Z_t = \max_{s \le t} W_s$, $z_t = \min_{s \le t} W_s$.

   (a) Show that the *conditional* distribution function for Z_t, given that

   $$W_t = x,$$

   is given by

   $$F_{[Z_t|W_t=x]}(y) = 1 - e^{-2y(y-x)/t}$$

   for $t > \max(0, x)$ and zero otherwise.

   Use this to construct a method of sampling from the distribution of the maximum of a Brownian motion given its terminal value.

   (b) Show that the conditional distribution function for W, given that $Z_t = y > 0$, is

   $$f_{[W_t|Z_t=y]}(x) = \left(\frac{2\,y - x}{t}\right) \exp\left(\frac{(x-y)\,(3\,y-x)}{2\,t}\right)$$

   for $x < y$ and zero otherwise.

   (c) Note that if W_t is a Brownian motion then so too is $-W_t$ and then use the fact that

   $$\begin{aligned} z_t &= \min_{0 \le \tau \le t}(W_\tau) \\ &= -\max_{0 \le \tau \le t}(-W_\tau) \end{aligned}$$

   to find the analogous distribution and density functions for z_t.

QUESTIONS TWO: 15 Marks

A lookback strike put option with expiry T is sampled discretely, with a single sample date $T_1 < T$ (in addition to the start date, $t = 0$). Show that the payoff is

$$\max(\max(S_0, S_{T_1}) - S_T, 0).$$

In the standard Black–Scholes model without dividends show that the option can be valued as a multi-stage option with intermediate time T_1 payoff

$$AS_{T_1} + \max(P_{\mathrm{v}}(T_1, S_{T_1}; T, S_0) - AS_{T_1}, 0),$$

where the last term is the time-T_1 value of a vanilla put with strike S_0 and expiry T, and where A is a constant which you should find.[1]

Generalise the above to the case when $S_t = S_0 e^{L_t}$, where (L_t) is an $(\mathcal{F}_t)$-adapted process with independent increments. Specify A as an expectation.

[1] This is a complicated version of a compound option. There is no nice formula for the initial value.