

WEEK 2: Yield spreads. The concept of bond pricing. Pricing bonds.

TOPICS:

- Understanding yield spreads
- Yields on non-Treasury securities
- Measuring yield spread
- Intermarket Sector Spreads and Intramarket Spreads
- Introduction to valuation of debt securities
- Pricing of vanilla bonds: 'clean' vs 'dirty' bond prices
- Yield to maturity estimation

1. Understanding yield spreads

The interest rate offered on a particular bond issue depends on the interest rate that can be earned on (1) risk-free instruments and (2) the perceived risks associated with the issue. We refer to the interest rates on risk-free instruments as the "level of interest rates." The actions of a country's central bank influence the level of interest rates as does the state of the country's economy. In the United States, the level of interest rates depends on the state of the economy, the interest rate policies implemented by the Board of Governors of the Federal Reserve Board, and the government's fiscal policies.

In implementing monetary policy, the Fed uses the following interest rate policy tools:

1. open market operations
2. the discount rate
3. bank reserve requirements
4. verbal persuasion to influence how bankers supply credit to businesses and consumers

Engaging in open market operations and changing the discount rate are the tools most often employed. Together, these tools can raise or lower the cost of funds in the economy. Open market operations do this through the Fed's buying and selling of U.S. Treasury securities. This action either adds funds to the market (when Treasury securities are purchased) or withdraws funds from the market (when Treasury securities are sold). Fed open market operations influence the federal funds rate, the rate at which banks borrow and lend funds from each other. The discount rate is the interest rate at which banks can borrow on a collateralized basis at the Fed's discount window. Increasing the discount rate makes the cost of funds more expensive for banks; the cost of funds is reduced when the discount rate is lowered. Changing bank reserve requirements is a less frequently used policy, as is the use of verbal persuasion to influence the supply of credit.

General risks that fixed income securities are exposed to: (1) interest rate risk, (2) call and prepayment risk, (3) yield curve risk, (4) reinvestment risk, (5) credit risk, (6) liquidity risk, (7) exchange-rate risk, (8) volatility risk, (9) inflation or purchasing power risk, and (10) event risk.

All fixed income securities, including Treasury securities, expose investors to interest rate risk. However, the degree of interest rate risk is not the same for all securities. The reason is that maturity and coupon rate affect how much the price changes when interest rates change.

One measure of a security's interest rate risk is its duration. Since Treasury securities, like other fixed income securities, have different durations, they have different exposures to interest rate risk as measured by duration.

Technically, yield curve risk and volatility risk are risks associated with Treasury securities. However, at this early stage of our understanding of fixed income analysis, we will not attempt to explain these risks. It is not necessary to understand these risks at this point in order to appreciate the material that follows in this section.

Because Treasury securities are non-callable, there is no reinvestment risk due to an issue being called. Treasury coupon securities carry reinvestment risk because in order to realize the yield offered on the security, the investor must reinvest the coupon payments received at an interest rate equal to the computed yield. So, all Treasury coupon securities are exposed to reinvestment risk. Treasury bills are not exposed to reinvestment risk because they are zero-coupon instruments. As for credit risk, the perception in the global financial community is that Treasury securities have no credit risk. In fact, when market participants and the popular press state that Treasury securities are "risk free," they are referring to credit risk.

Treasury securities are highly liquid. However, on-the-run and off-the-run Treasury securities trade with different degrees of liquidity. Consequently, the yields offered by on-the-run and off-the-run issues reflect different degrees of liquidity.

Since U.S. Treasury securities are dollar denominated, there is no exchange-rate risk for an investor whose domestic currency is the U.S. dollar. However, non-U.S. investors whose domestic currency is not the U.S. dollar are exposed to exchange-rate risk.

Fixed-rate Treasury securities are exposed to inflation risk. Treasury inflation protection securities (TIPS) have a coupon rate that is effectively adjusted for the rate of inflation and therefore have protection against inflation risk.

Yields on non-Treasury securities

Despite the imperfections of the Treasury yield curve as a benchmark for the minimum interest rate that an investor requires for investing in a non-Treasury security, it is commonplace to refer to the additional yield over the benchmark Treasury issue of the same maturity as the yield spread. In fact, because non-Treasury sectors of the fixed income market offer a yield spread to Treasury securities, non-Treasury sectors are commonly referred to as spread sectors and non-Treasury securities in these sectors are referred to as spread products.

Measuring Yield Spreads

While it is common to talk about spreads relative to a Treasury security of the same maturity, a yield spread between any two bond issues can be easily computed. In general, the yield spread between any two bond issues, bond X and bond Y, is computed as follows:

absolute yield spread = yield on bond X – yield on bond Y

where bond Y is considered the reference bond (or benchmark) against which bond X is measured. When a yield spread is computed in this manner it is referred to as an absolute yield spread and it is measured in basis points. For example, on February 8, 2002, the yield on the 10-year on-the-run Treasury issue was 4.88% and the yield on a single A rated 10-year industrial bond was 6.24%. If bond X is the 10-year industrial bond and bond Y is the 10-year on-the-run Treasury issue, the absolute yield spread was:

absolute yield spread = 6.24% – 4.88% = 1.36% or 136 basis points

Unless otherwise specified, yield spreads are typically measured in this way. Yield spreads can also be measured on a relative basis by taking the ratio of the yield spread to the yield of the reference bond. This is called a relative yield spread and is computed as shown below, assuming that the reference bond is bond Y:

Relative yield spread = (yield on bond X – yield on bond Y) / yield on bond Y

Sometimes bonds are compared in terms of a yield ratio, the quotient of two bond yields, as shown below:

yield ratio = yield on bond X / yield on bond Y

Typically, in the U.S. bond market when these measures are computed, bond Y (the reference bond) is a Treasury issue. In that case, the equations for the yield spread measures are as follows:

absolute yield spread = yield on bond X – yield of on-the-run Treasury

relative yield spread = (yield on bond X – yield of on-the-run Treasury) / yield of on-the-run Treasury

yield ratio = yield on bond X / yield of on-the-run Treasury

For the above example comparing the yields on the 10-year single A rated industrial bond and the 10-year on-the-run Treasury, the relative yield spread and yield ratio are computed below:

absolute yield spread = 6.24% – 4.88% = 1.36% = 136 basis points

relative yield spread = (6.24% – 4.88%) / 4.88% = 0.279 = 27.9%

yield ratio = 6.24% / 4.88% = 1.279

The reason for computing yield spreads in terms of a relative yield spread or a yield ratio is that the magnitude of the yield spread is affected by the level of interest rates. For example, in 1957 the yield on Treasuries was about 3%. At that time, the absolute yield spread between triple B rated utility bonds and Treasuries was 40 basis points. This was a relative yield spread of 13% (0.40% divided by 3%). However, when the yield on Treasuries exceeded 10% in 1985, an absolute yield spread of 40 basis points would have meant a relative yield spread of only 4% (0.40% divided by 10%). Consequently, in 1985 an absolute yield spread greater than 40 basis points would have been required in order to produce a similar relative yield spread.

We will **focus on** the yield spread as most commonly measured, the **absolute yield spread**. So, when we refer to yield spread, we mean absolute yield spread.

Whether we measure the yield spread as an absolute yield spread, a relative yield spread, or a yield ratio, the question to answer is what causes the yield spread between two bond issues. Basically, active bond portfolio strategies involve assessing the factors that cause the yield spread, forecasting how that yield spread may change over an investment horizon, and taking a position to capitalize on that forecast.

Intermarket Sector Spreads and Intramarket Spreads

The bond market is classified into sectors based on the type of issuer. In the United States, these sectors include the U.S. government sector, the U.S. government agencies sector, the municipal sector, the corporate sector, the mortgage-backed securities sector, the asset-backed securities sector, and the foreign (sovereign, supranational, and corporate) sector. Different sectors are generally perceived as offering different risks and rewards.

The major market sectors are further divided into sub-sectors reflecting common economic characteristics. For example, within the corporate sector, the subsectors are: (1) industrial companies, (2) utility companies, (3) finance companies, and (4) banks. In the market for asset-backed securities, the sub-sectors are based on the type of collateral backing the security.

The major types are securities backed by pools of (1) credit card receivables, (2) home equity loans, (3) automobile loans, (4) manufactured housing loans, and (5) student loans. Excluding the Treasury market sector, the other market sectors have a wide range of issuers, each with different abilities to satisfy their contractual obligations. Therefore, a key feature of a debt obligation is the nature of the issuer.

The yield spread between the yields offered in two sectors of the bond market with the same maturity is referred to as an intermarket sector spread. The most common intermarket sector spread calculated by market participants is the yield spread between a non-Treasury sector and Treasury securities with the same maturity.

The yield spread between two issues within a market sector is called an **intramarket sector spread**. As with Treasury securities, a yield curve can be estimated for a given issuer.

The yield spread typically increases with maturity. The yield spreads for a given issuer can be added to the yield for the corresponding maturity of the on-the-run Treasury issue. The resulting yield curve is then an issuer's on-the-run yield curve.

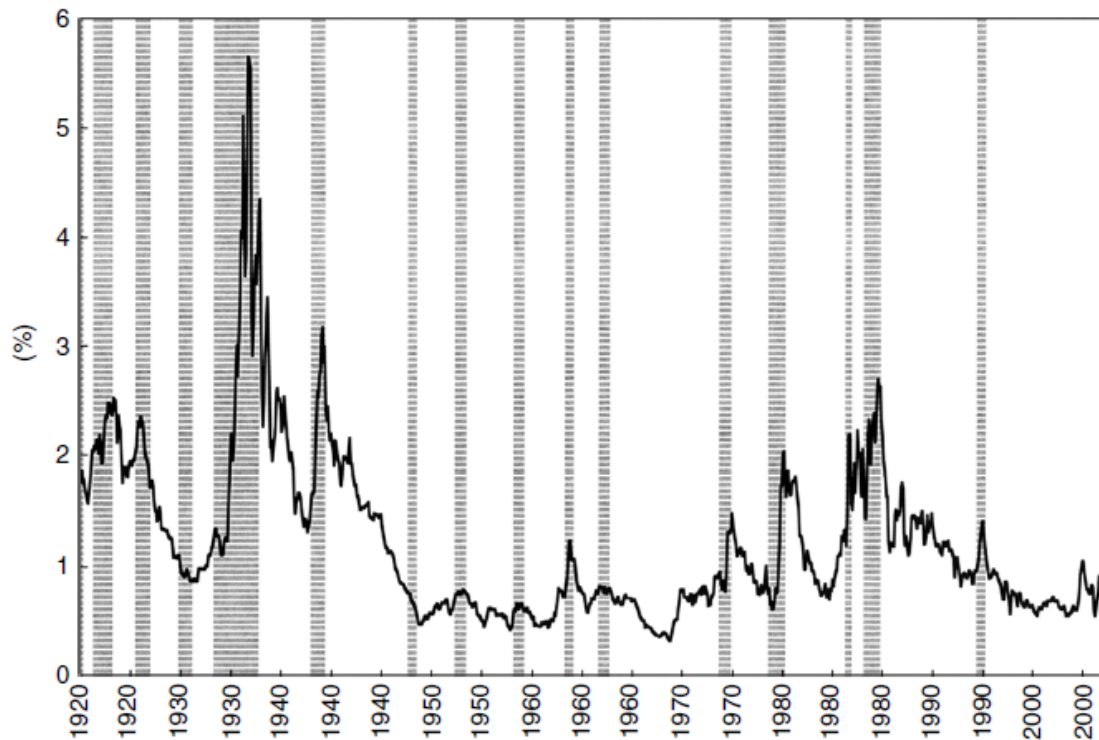
The factors other than maturity that affect the intermarket and intramarket yield spreads are (1) the relative credit risk of the two issues, (2) the presence of embedded options, (3) the liquidity of the two issues, and (4) the taxability of interest received by investors.

Credit Spreads

The yield spread between non-Treasury securities and Treasury securities that are identical in all respects except for credit rating is referred to as a credit spread or quality spread. "Identical in all respects except credit rating" means that the maturities are the same and that there are no embedded options.

It is argued that credit spreads between corporates and Treasuries change systematically with changes in the economy. Credit spreads widen (i.e., become larger) in a declining or contracting economy and narrow (i.e., become smaller) during economic expansion. The economic rationale is that, in a declining or contracting economy, corporations experience declines in revenue and cash flow, making it more difficult for corporate issuers to service their contractual debt obligations. To induce investors to hold spread products as credit quality deteriorates, the credit spread widens. The widening occurs as investors sell off corporates and invest the proceeds in Treasury securities (popularly referred to as a "flight to quality"). The converse is that, during economic expansion and

EXHIBIT 4 Credit Spreads Between Baa and Aaa Corporate Bonds Over the Business Cycle Since 1919



Shaded areas = economic recession as defined by the NBER.

Source: Exhibit 1 in Leland E. Crabbe and Frank J. Fabozzi, *Managing a Corporate Portfolio* (Hoboken, NJ: John Wiley & Sons, 2002), p. 154.

brisk economic activity, revenue and cash flow increase, increasing the likelihood that corporate issuers will have the capacity to service their contractual debt obligations.

The chart below provides evidence of the impact of the business cycle on credit spreads since 1919. The credit spread in the exhibit is the difference between Baa rated and Aaa rated corporate bonds; the shaded areas in the exhibit represent periods of economic recession as defined by the National Bureau of Economic Research (NBER). In general, corporate credit spreads tightened during the early stages of economic expansion, and spreads widened sharply during economic recessions. In fact, spreads typically begin to widen before the official beginning of an economic recession.

Some market observers use the yield spread between issuers in cyclical and non-cyclical industry sectors as a proxy for yield spreads due to expected economic conditions. The rationale is as follows. While companies in both cyclical and non-cyclical industries are adversely affected by expectations of a recession, the impact is greater for cyclical industries. As a result, the yield spread between issuers in cyclical and non-cyclical industry sectors will widen with expectations of a contracting economy.

Including Embedded Options

It is not uncommon for a bond issue to include a provision that gives either the issuer and/or the bondholder an option to take some action against the other party. The most common type of option in a bond issue is the call provision that grants the issuer the right to retire the debt, fully or partially, before the scheduled maturity date.

The presence of an embedded option has an effect on both the yield spread of an issue relative to a Treasury security and the yield spread relative to otherwise comparable issues that do not have an embedded option. In general, investors require a larger yield spread to a comparable Treasury security for an issue with an embedded option that is favourable to the issuer (e.g. a call option) than for an issue without such an option. In contrast, market participants require a smaller yield spread to a comparable Treasury security for an issue with an embedded option that is favourable to the investor (e.g., put option or conversion option).

In fact, for a bond with an option favourable to an investor, the interest rate may be less than that on a comparable Treasury security.

Even for callable bonds, the yield spread depends on the type of call feature. For a callable bond with a deferred call, the longer the deferred call period, the greater the call protection provided to the investor. Thus, all other factors equal, the longer the deferred call period, the lower the yield spread attributable to the call feature.

In general, when a yield spread is cited for an issue that is callable, part of the spread reflects the risk associated with the embedded option. Reported yield spreads do not adjust for embedded options. The raw yield spreads are sometimes referred to as **nominal spreads**—nominal in the sense that the value of embedded options has not been removed in computing an adjusted yield spread. The yield spread that adjusts for the embedded option is **option adjusted spread (OAS)**.

INTRODUCTION TO THE VALUATION OF DEBT SECURITIES

Valuation is the process of determining the fair value of a financial asset. The process is also referred to as “valuing” or “pricing” a financial asset. In this chapter, we will explain the general principles of fixed income security valuation. In this chapter, we will limit our discussion to the valuation of option-free bonds.

General principles of valuation

The fundamental principle of financial asset valuation is that its value is equal to the present value of its expected cash flows. This principle applies regardless of the financial asset. Thus, the valuation of a financial asset involves the following three steps:

Step 1: Estimate the expected cash flows.

Step 2: Determine the appropriate interest rate or interest rates that should be used to discount the cash flows.

Step 3: Calculate the present value of the expected cash flows found in step 1 using the interest rate or interest rates determined in step 2.

Estimating Cash Flows

Cash flow is simply the cash that is expected to be received in the future from an investment. In the case of a fixed income security, it does not make any difference whether the cash flow is interest income or payment of principal. The cash flows of a security are the collection of each period’s cash

flow. Holding aside the risk of default, the cash flows for few fixed income securities are simple to project. Non-callable U.S. Treasury securities have known cash flows.

For Treasury coupon securities, the cash flows are the coupon interest payments every six months up to and including the maturity date and the principal payment at the maturity date. At times, investors will find it difficult to estimate the cash flows when they purchase a fixed income security. For example, if

1. the issuer or the investor has the option to change the contractual due date for the payment of the principal, or
2. the coupon payment is reset periodically by a formula based on some value or values of reference rates, prices, or exchange rates, or
3. the investor has the choice to convert or exchange the security into common stock.

Callable bonds, puttable bonds, mortgage-backed securities, and asset-backed securities are examples of (1). Floating-rate securities are an example of (2). Convertible bonds and exchangeable bonds are examples of (3).

For securities that fall into the first category, future interest rate movements are the key factor to determine if the option will be exercised. Specifically, if interest rates fall far enough, the issuer can sell a new issue of bonds at the lower interest rate and use the proceeds to pay off (call) the older bonds that have the higher coupon rate. (This assumes that the interest savings are larger than the costs involved in refunding.) Similarly, for a loan, if rates fall enough that the interest savings outweigh the refinancing costs, the borrower has an incentive to refinance.

For a puttable bond, the investor will put the issue if interest rates rise enough to drive the market price below the put price (i.e., the price at which it must be repurchased by the issuer).

What this means is that to properly estimate the cash flows of a fixed income security, it is necessary to incorporate into the analysis how, in the future, changes in interest rates and other factors affecting the embedded option may affect cash flows.

Determining the Appropriate Rate or Rates

Once the cash flows for a fixed income security are estimated, the next step is to determine the appropriate interest rate to be used to discount the cash flows. As we did in the previous chapter, we will use the terms interest rate and yield interchangeably. The minimum interest rate that an investor should require is the yield available in the marketplace on a default-free cash flow. In the United States, this is the yield on a U.S. Treasury security. This is one of the reasons that the Treasury market is closely watched. What is the minimum interest rate U.S. investors demand? At this point, we can assume that it is the yield on the on-the-run Treasury security with the same as the security being valued. We will qualify this shortly.

For a security that is not issued by the U.S. government, investors will require a yield premium over the yield available on an on-the-run Treasury issue. This yield premium reflects the additional risks that the investor accepts. For each cash flow estimated, the same interest rate can be used to calculate the present value. However, since each cash flow is unique, it is more appropriate to value each cash flow using an interest rate specific to that cash flow's maturity. In the traditional approach to valuation a single interest rate is used. In the next session (week), we will see that the proper

approach to valuation uses multiple interest rates each specific to a particular cash flow. In that section, we will also demonstrate why this must be the case.

Discounting the Expected Cash Flows

Given expected (estimated) cash flows and the appropriate interest rate or interest rates to be used to discount the cash flows, the final step in the valuation process is to value the cash flows.

What is the value of a single cash flow to be received in the future? It is the amount of money that must be invested today to generate that future value. The resulting value is called the present value of a cash flow. (It is also called the discounted value.) The present value of a cash flow will depend on (1) when a cash flow will be received (i.e., the timing of a cash flow) and (2) the interest rate used to calculate the present value. The interest rate used is called the discount rate.

First, we calculate the present value for each expected cash flow. Then, to determine the value of the security, we calculate the sum of the present values (i.e., for all of the security's expected cash flows).

If a discount rate i can be earned on any sum invested today, the present value of the expected cash flow to be received t years from now is:

$$\text{present value}_t = \frac{\text{expected cash flow in period } t}{(1 + i)^t}$$

The value of a financial asset is then the sum of the present value of all the expected cash flows. That is, assuming that there are N expected cash flows:

$$\text{value} = \text{present value}_1 + \text{present value}_2 + \dots + \text{present value}_N$$

To illustrate the present value formula, consider a simple bond that matures in four years, has a coupon rate of 10%, and has a maturity value of \$100. For simplicity, let's assume the bond pays interest annually and a discount rate of 8% should be used to calculate the present value of each cash flow. The cash flow for this bond is:

Year	Cash flow
1	\$10
2	10
3	10
4	110

The present value of each cash flow is:

$$\text{Year 1: present value}_1 = \frac{\$10}{(1.08)^1} = \$9.2593$$

$$\text{Year 2: present value}_2 = \frac{\$10}{(1.08)^2} = \$8.5734$$

$$\text{Year 3: present value}_3 = \frac{\$10}{(1.08)^3} = \$7.9383$$

$$\text{Year 4: present value}_4 = \frac{\$110}{(1.08)^4} = \$80.8533$$

The value of this security is then the sum of the present values of the four cash flows. That is, the present value is \$106.6243 (\$9.2593 + \$8.5734 + \$7.9383 + \$80.8533).

Present Value Properties

An important property about the present value can be seen from the above illustration. For the first three years, the cash flow is the same (\$10) and the discount rate is the same (8%). ***The present value decreases as we go further into the future. This is an important property of the present value: for a given discount rate, the further into the future a cash flow is received, the lower its present value. This can be seen in the present value formula. As t increases, present value(t) decreases.***

Suppose that instead of a discount rate of 8%, 12% discount rate is used for each cash flow. Then, the present value of each cash flow is:

$$\text{Year 1: present value}_1 = \frac{\$10}{(1.12)^1} = \$8.9286$$

$$\text{Year 2: present value}_2 = \frac{\$10}{(1.12)^2} = \$7.9719$$

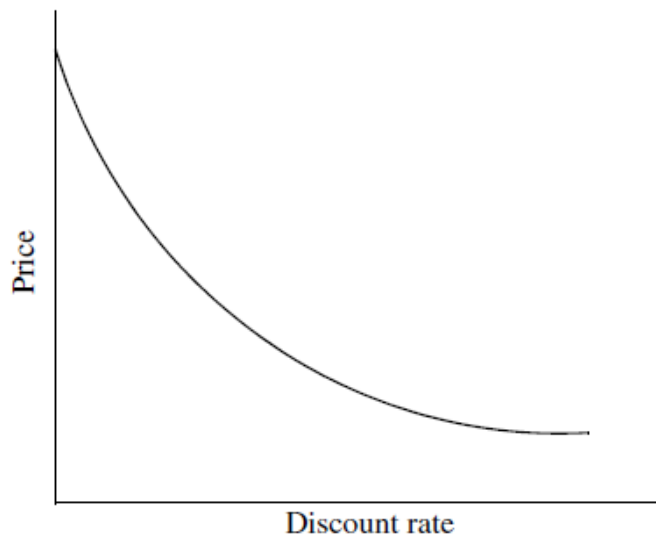
$$\text{Year 3: present value}_3 = \frac{\$10}{(1.12)^3} = \$7.1178$$

$$\text{Year 4: present value}_4 = \frac{\$110}{(1.12)^4} = \$69.9070$$

The value of this security is then \$93.9253 (\$8.9286 + \$7.9719 + \$7.1178 + \$69.9070). The security's value is lower if a 12% discount rate is used compared to an 8% discount rate (\$93.9253 versus \$106.6243). This is another general property of present value: the higher the discount rate, the lower the present value. Since the value of a security is the present value of the expected cash flows, this property carries over to the value of a security: the higher the discount rate, the lower a security's value. The reverse is also true: the lower the discount rate, the higher a security's value.

Exhibit 1 shows, for an option-free bond, this inverse relationship between a security's value and the discount rate. The shape of the curve in Exhibit 1 is referred to as convex. By convex, it is meant the curve is bowed in from the origin. As you will learn in the next session, this convexity or bowed shape has implications for the price volatility of a bond when interest rates change. What is important to understand is that the relationship is not linear.

EXHIBIT 1 Price/Discount Rate Relationship for an Option-Free Bond



Maximum price = sum of undiscounted cash flows

Relationship between Coupon Rate, Discount Rate, and Price Relative to Par Value

The following relationship is valid for YTM, coupon and price:

coupon rate = yield required by market, therefore price = par value

coupon rate < yield required by market, therefore price < par value (discount)

coupon rate > yield required by market, therefore price > par value (premium)

Now that we know how to value a bond, we can demonstrate the relationship. The coupon rate on our hypothetical bond is 10%. When an 8% discount rate is used, the bond's value is \$106.6243. That is, the price is greater than par value (premium). This is because the coupon rate (10%) is greater than the required yield (the 8% discount rate). We also showed that when the discount rate is 12% (i.e., greater than the coupon rate of 10%), the price of the bond is \$93.9253. That is, the bond's value is less than par value when the coupon rate is less than the required yield (discount). When the discount rate is the same as the coupon rate, 10%, the bond's value is **equal to par value** as shown below:

Year	Cash flow	Present value at 10%
1	\$10	\$9.0909
2	10	8.2645
3	10	7.5131
4	110	75.1315
	Total	\$100.0000

Change in a Bond's Value as it Moves Toward Maturity

As a bond moves closer to its maturity date, its value changes. More specifically, assuming that the discount rate does not change, a bond's value:

1. decreases over time if the bond is selling at a premium

2. increases over time if the bond is selling at a discount

3. is unchanged if the bond is selling at par value

At the maturity date, the bond's value is equal to its par value. So, over time as the bond moves toward its maturity date, its price will move to its par value—a characteristic sometimes referred to as a “pull to par value.”

To illustrate what happens to a bond selling at a premium, consider once again the 4-year 10% coupon bond. When the discount rate is 8%, the bond's price is 106.6243. Suppose that one year later, the discount rate is still 8%. There are only three cash flows remaining since the bond is now a 3-year security. The cash flow and the present value of the cash flows are given below:

Year	Cash flow	Present value at 8%
1	\$10	\$9.2593
2	10	8.5734
3	110	87.3215
	Total	\$105.1542

The price has declined from \$106.6243 to \$105.1542.

Now suppose that the bond's price is initially below par value. For example, as stated earlier, if the discount rate is 12%, the 4-year 10% coupon bond's value is \$93.9253. Assuming the discount rate remains at 12%, one year later the cash flow and the present value of the cash flow would be as shown:

Year	Cash flow	Present value at 12%
1	\$10	\$8.9286
2	10	7.9719
3	110	78.2958
	Total	\$95.1963

The bond's price increases from \$93.9253 to \$95.1963.

To understand how the price of a bond changes as it moves towards maturity, consider the following three 20-year bonds for which the yield required by the market is 8%: a premium bond (10% coupon), a discount bond (6% coupon), and a par bond (8% coupon). To simplify the example, it is assumed that each bond pays interest annually. Exhibit 2 shows the price of each bond as it moves toward maturity, assuming that the 8% yield required by the market does not change. The premium bond with an initial price of 119.6363 decreases in price until it reaches par value at the maturity date. The discount bond with an initial price of 80.3637 increases in price until it reaches par value at the maturity date.

In practice, over time the discount rate will change. So, the bond's value will change due to both the change in the discount rate and the change in the cash flow as the bond moves toward maturity. For

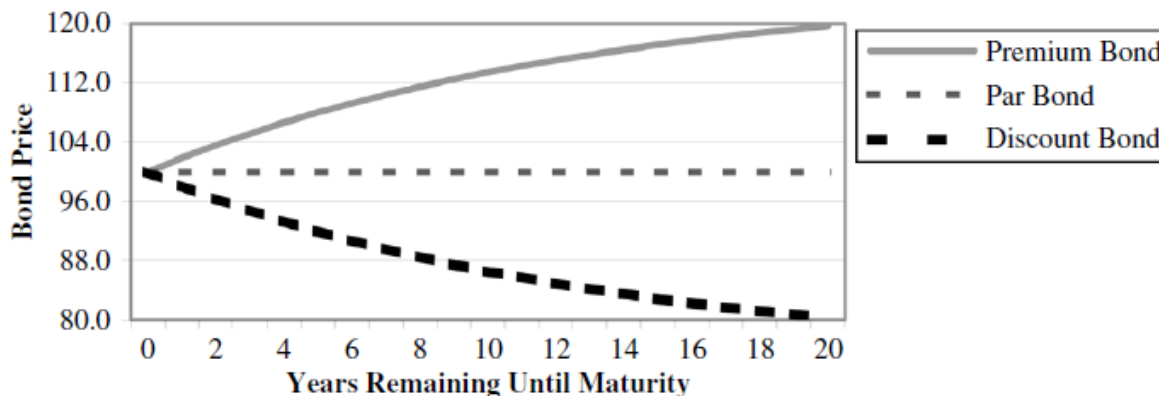
example, again suppose that the discount rate for the 4-year 10% coupon is 8% so that the bond is selling for \$106.6243. One year later, suppose that the discount rate appropriate for a 3-year 10% coupon bond increases from 8% to 9%. Then the cash flow and present value of the cash flows are shown below:

Year	Cash flow	Present value at 9%
1	\$10	\$9.1743
2	10	8.4168
3	110	84.9402
	Total	\$102.5313

The bond's price will decline from \$106.6243 to \$102.5313. As shown earlier, if the discount rate did not increase, the price would have declined to only \$105.1542. The price decline of \$4.0930 (\$106.6243 – \$102.5313) can be decomposed as follows:

Price change attributable to moving to maturity (no change in discount rate)	\$1.4701 (106.6243 – 105.1542)
Price change attribute to an increase in the discount rate from 8% to 9%	<u>\$2.6229 (105.1542 – 102.5313)</u>
Total price change	\$4.0930

The Effect of Time on a Bond's Price



Valuing Semiannual Cash Flows

For most bonds, the coupon payments are semiannual. This does not introduce any complexities into the calculation. The procedure is to simply adjust the coupon payments by dividing the annual coupon payment by 2 and adjust the discount rate by dividing the annual discount rate by 2. The time period t in the present value formula is treated in terms of 6-month periods rather than years.

For example, consider once again the 4-year 10% coupon bond with a maturity value of \$100. The cash flow for the first 3.5 years is equal to \$5 (\$10/2). The last cash flow is equal to the final coupon payment (\$5) plus the maturity value (\$100). So the last cash flow is \$105.

Now the tricky part. If an annual discount rate of 8% is used, how do we obtain the semiannual discount rate? We will simply use one-half the annual rate, 4% (or 8%/2). The reader should have a

problem with this: a 4% semiannual rate is not an 8% effective annual rate. That is correct. However, as we will see in the next chapter, the convention in the bond market is to quote annual interest rates that are just double semiannual rates. This will be explained more fully in the next chapter. Don't let this throw you off here. For now, just accept the fact that one-half an annual discount rate is used to obtain a semiannual discount rate in the balance of the chapter.

Given the cash flows and the semiannual discount rate of 4%, the present value of each cash flow is shown below:

$$\begin{aligned}
 \text{Period 1: present value}_1 &= \frac{\$5}{(1.04)^1} = \$4.8077 \\
 \text{Period 2: present value}_2 &= \frac{\$5}{(1.04)^2} = \$4.6228 \\
 \text{Period 3: present value}_3 &= \frac{\$5}{(1.04)^3} = \$4.4450 \\
 \text{Period 4: present value}_4 &= \frac{\$5}{(1.04)^4} = \$4.2740 \\
 \text{Period 5: present value}_5 &= \frac{\$5}{(1.04)^5} = \$4.1096 \\
 \text{Period 6: present value}_6 &= \frac{\$5}{(1.04)^6} = \$3.9516 \\
 \text{Period 7: present value}_7 &= \frac{\$5}{(1.04)^7} = \$3.7996 \\
 \text{Period 8: present value}_8 &= \frac{\$105}{(1.04)^8} = \$76.7225
 \end{aligned}$$

The security's value is equal to the sum of the present value of the eight cash flows, \$106.7327. Notice that this price is greater than the price when coupon payments are annual (\$106.6243). This is because one-half the annual coupon payment is received six months sooner than when payments are annual. This produces a higher present value for the semiannual coupon payments relative to the annual coupon payments.

The value of a non-amortizing bond can be divided into two components: (1) the present value of the coupon payments and (2) the present value of the maturity value. For a fixed-rate coupon bond, the coupon payments represent an annuity. A short-cut formula can be used to compute the value of a bond when using a single discount rate: compute the present value of the annuity and then add the present value of the maturity value. The present value of an annuity is equal to:

$$\text{annuity payment} \times \left[\frac{1 - \frac{1}{(1+i)^{\text{no. of periods}}}}{i} \right]$$

For a bond with annual interest payments, i is the annual discount rate and the "no. of periods" is equal to the number of years. Applying this formula to a semiannual-pay bond, the annuity payment is one half the annual coupon payment and the number of periods is double the number of years to maturity. So, the present value of the coupon payments can be expressed as:

$$\text{semiannual coupon payment} \times \left[\frac{1 - \frac{1}{(1+i)^{\text{no. of years} \times 2}}}{i} \right]$$

where i is the semiannual discount rate (annual rate/2). Notice that in the formula, we use the number of years multiplied by 2 since a period in our illustration is six months.

The present value of the maturity value is equal to:

$$\text{present value of maturity value} = \frac{\$100}{(1+i)^{\text{no. of years} \times 2}}$$

To illustrate this computation, consider once again the 4-year 10% coupon bond with an annual discount rate of 8% and a semiannual discount rate of one half this rate (4%) for the reason cited earlier. Then:

semiannual coupon payment = \$5

semiannual discount rate(i) = 4%

number of years = 4

then the present value of the coupon payments is:

$$\$5 \times \left[\frac{1 - \frac{1}{(1.04)^{4 \times 2}}}{0.04} \right] = \$33.6637$$

To determine the price, the present value of the maturity value must be added to the present value of the coupon payments. The present value of the maturity value is:

$$\text{present value of maturity value} = \frac{\$100}{(1.04)^{4 \times 2}} = \$73.0690$$

The price is then \$106.7327 (\$33.6637 + \$73.0690). This agrees with our previous calculation for the price of this bond.

Valuing a Zero-Coupon Bond

For a zero-coupon bond, there is only one cash flow—the maturity value. The value of a zero-coupon bond that matures N years from now is:

$$\frac{\text{maturity value}}{(1+i)^{\text{no. of years} \times 2}}$$

where i is the semiannual discount rate.

It may seem surprising that the number of periods is double the number of years to maturity. In computing the value of a zero-coupon bond, the number of 6-month periods (i.e., “no. of years $\times 2$ ”) is used in the denominator of the formula. The rationale is that the pricing of a zero-coupon bond should be consistent with the pricing of a semiannual coupon bond. Therefore, the use of 6-month periods is required in order to have uniformity between the present value calculations.

To illustrate the application of the formula, the value of a 5-year zero-coupon bond with a maturity value of \$100 discounted at an 8% interest rate is \$67.5564, as shown below:

$$i = 0.04 (= 0.08/2)$$

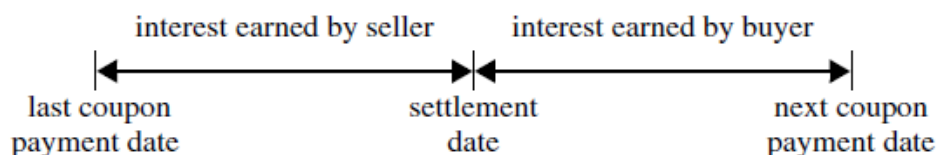
$$N = 5$$

$$\frac{\$100}{(1.04)^{5 \times 2}} = \$67.5564$$

Valuing a Bond Between Coupon Payments

For coupon-paying bonds, a complication arises when we try to price a bond between coupon payments. The amount that the buyer pays the seller in such cases is the present value of the cash flow. But one of the cash flows, the very next cash flow, encompasses two components as shown below:

1. interest earned by the seller
2. interest earned by the buyer



The interest earned by the seller is the interest that has accrued between the last coupon payment date and the settlement date. This interest is called accrued interest. At the time of purchase, the buyer must compensate the seller for the accrued interest. The buyer recovers the accrued interest when the next coupon payment is received.

When the price of a bond is computed using the present value calculations described earlier, it is computed with accrued interest embodied in the price. This price is referred to as the full price. (Some market participants refer to it as the dirty price). It is the full price that the buyer pays the seller. From the full price, the accrued interest must be deducted to determine the price of the bond, sometimes referred to as the clean price.

Below, we show how the present value formula is modified to compute the full price when a bond is purchased between coupon periods.

1. Computing the Full Price To compute the full price, it is first necessary to determine the fractional periods between the settlement date and the next coupon payment date. This is determined as follows:

$$w \text{ periods} = \frac{\text{days between settlement date and next coupon payment date}}{\text{days in coupon period}}$$

Then the present value of the expected cash flow to be received t periods from now using a discount rate i assuming the first coupon payment is w periods from now is:

$$\text{present value}_t = \frac{\text{expected cash flow}}{(1 + i)^{t-1+w}}$$

This procedure for calculating the present value when a security is purchased between coupon payments is called the "Street method."

To illustrate the calculation, suppose that there are five semiannual coupon payments remaining for a 10% coupon bond. Also assume the following:

1. 78 days between the settlement date and the next coupon payment date
2. 182 days in the coupon period

Then w is 0.4286 periods ($= 78/182$). The present value of each cash flow assuming that each is discounted at 8% annual discount rate is:

$$\text{Period 1: present value}_1 = \frac{\$5}{(1.04)^{0.4286}} = \$4.9167$$

$$\text{Period 2: present value}_2 = \frac{\$5}{(1.04)^{1.4286}} = \$4.7276$$

$$\text{Period 3: present value}_3 = \frac{\$5}{(1.04)^{2.4286}} = \$4.5457$$

$$\text{Period 4: present value}_4 = \frac{\$5}{(1.04)^{3.4286}} = \$4.3709$$

$$\text{Period 5: present value}_5 = \frac{\$105}{(1.04)^{4.4286}} = \$88.2583$$

The full price is the sum of the present value of the cash flows, which is \$106.8192. Remember that the full price includes the accrued interest that the buyer is paying the seller.

2. Computing the Accrued Interest and the Clean Price

To find the price without accrued interest, called the clean price or simply price, the accrued interest must be computed. To determine the accrued interest, it is first necessary to determine the number of days in the accrued interest period. The number of days in the accrued interest period is determined as follows:

$$\text{days in accrued interest period} = \text{days in coupon period} - \text{days between settlement and next coupon payment}$$

The percentage of the next semiannual coupon payment that the seller has earned as accrued interest is found as follows:

$$\frac{\text{days in accrued interest period}}{\text{days in coupon period}}$$

So, for example, returning to our illustration where the full price was computed, since there are 182 days in the coupon period and there are 78 days from the settlement date to the next coupon payment, the days in the accrued interest period is 182 minus 78, or 104 days. Therefore, the percentage of the coupon payment that is accrued interest is:

$$104/182 = 0.5714 = 57.14\%$$

This is the same percentage found by simply subtracting w from 1. In our illustration, w was 0.4286. Then $1 - 0.4286 = 0.5714$.

Given the value of w , the amount of accrued interest (AI) is equal to:

$$\text{AI} = \text{semiannual coupon payment} \times (1 - w)$$

So, for the 10% coupon bond whose full price we computed, since the semiannual coupon payment per \$100 of par value is \$5 and w is 0.4286, the accrued interest is:

$$\$5 \times (1 - 0.4286) = \$2.8570$$

The clean price is then:

full price – accrued interest

In our illustration, the clean price is $\$106.8192 - \$2.8570 = \$103.9622$.

3. Day Count Conventions

The practice for calculating the number of days between two dates depends on day count conventions used in the bond market. The convention differs by the type of security. Day count conventions are also used to calculate the number of days in the numerator and denominator of the ratio w . The accrued interest (AI) assuming semiannual payments is calculated as follows:

$$\text{AI} = \frac{\text{annual coupon}}{2} \times \frac{\text{days in AI period}}{\text{days in coupon period}}$$

Yield to maturity estimation

Although finding YTM is an iterative process that requires repeat calculations until the correct YTM is determined, it is possible to estimate the YTM. We can use the following formula to do so:

$$\text{Approx YTM} = \frac{C + \frac{F-P}{n}}{\frac{F+P}{2}}$$

C = Coupon/Interest Payment

F = Face Value

P = Price

n = years to maturity