Probability

Lecture 4

Probability of a Single event

$$probability = \frac{Number\ of\ favorable\ outcomes}{Number\ of\ possible\ equally - likely\ outcomes}$$

Probability of a Single event

Probability of a Single event

Die 1	Die 2	Total	Die 1	Die 2	Total	Die 1	Die 2	Total
1	1	2	3	1	4	5	1	6
1	2	3	3	2	5	5	2	7
1	3	4	3	3	6	5	3	8
1	4	5	3	4	7	5	4	9
1	5	6	3	5	8	5	5	10
1	6	7	3	6	9	5	6	11
2	1	3	4	1	5	6	1	7
2	2	4	4	2	6	6	2	8
2	3	5	4	3	7	6	3	9
2	4	6	4	4	8	6	4	10
2	5	7	4	5	9	6	5	11
2	6	8	4	6	10	6	6	12

Probability of Two (or more) Independent Events

Probability of A and B

$$P(A \text{ and } B) = P(A) \times P(B)$$

Probability of A or B

$$P\,(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

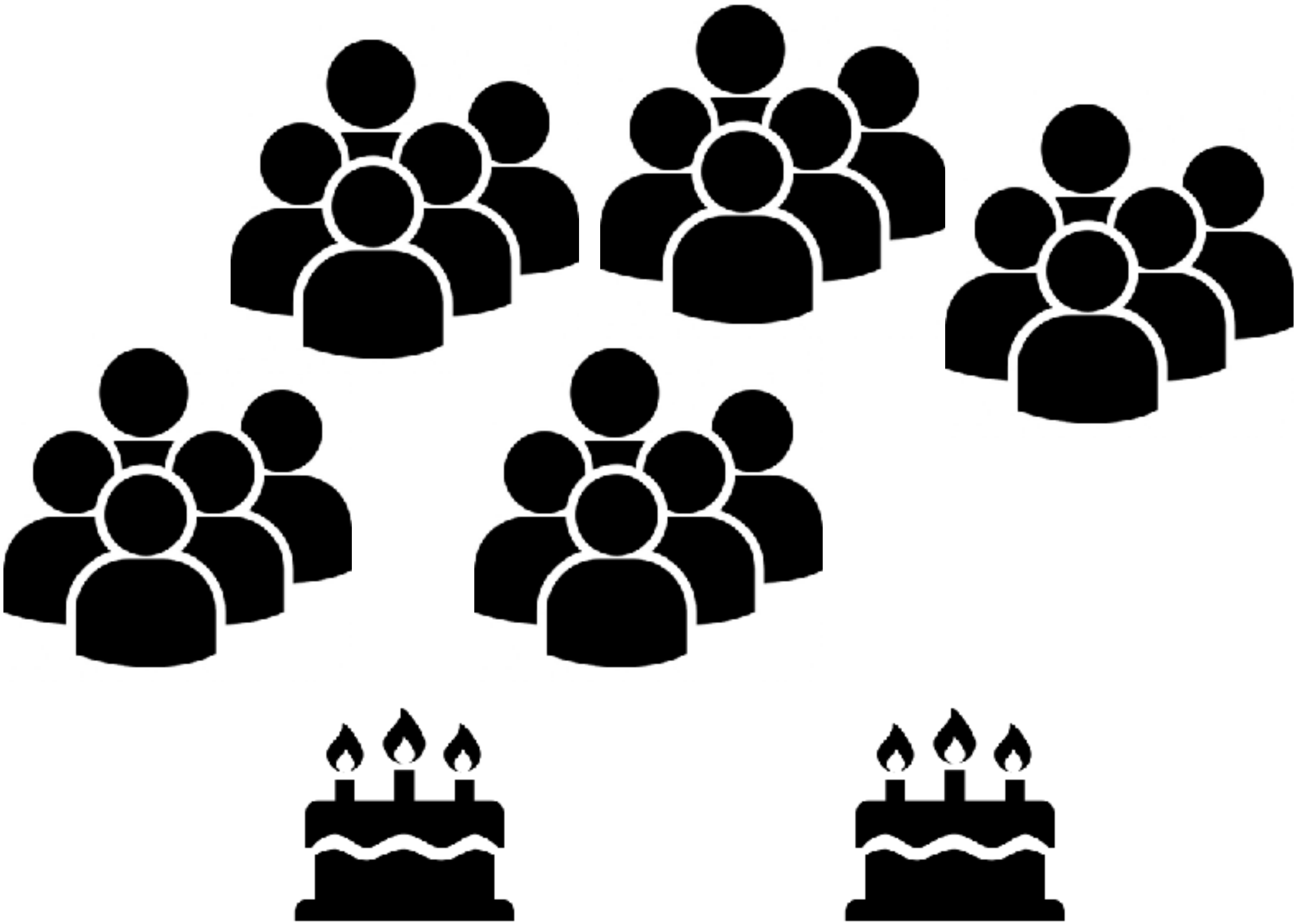
$$P(6 \text{ or head}) = P(6) + P(\text{head}) - P(6 \text{ and head}) =$$

$$= (1/6) + (1/2) - (1/6)(1/2) = 7/12$$

Probability of A or B

(not getting a 6) AND (not getting a head)

Birthday Problem



Permutations and Combinations

Possible orders

Number	First	Second	Third
1	red	yellow	green
2	red	green	yellow
3	yellow	red	green
4	yellow	green	red
5	green	red	yellow
6	green	yellow	red

Number of orders = $n!$

Permutations and Combinations

Multiplication orders



3 x 6 x 4 = 72

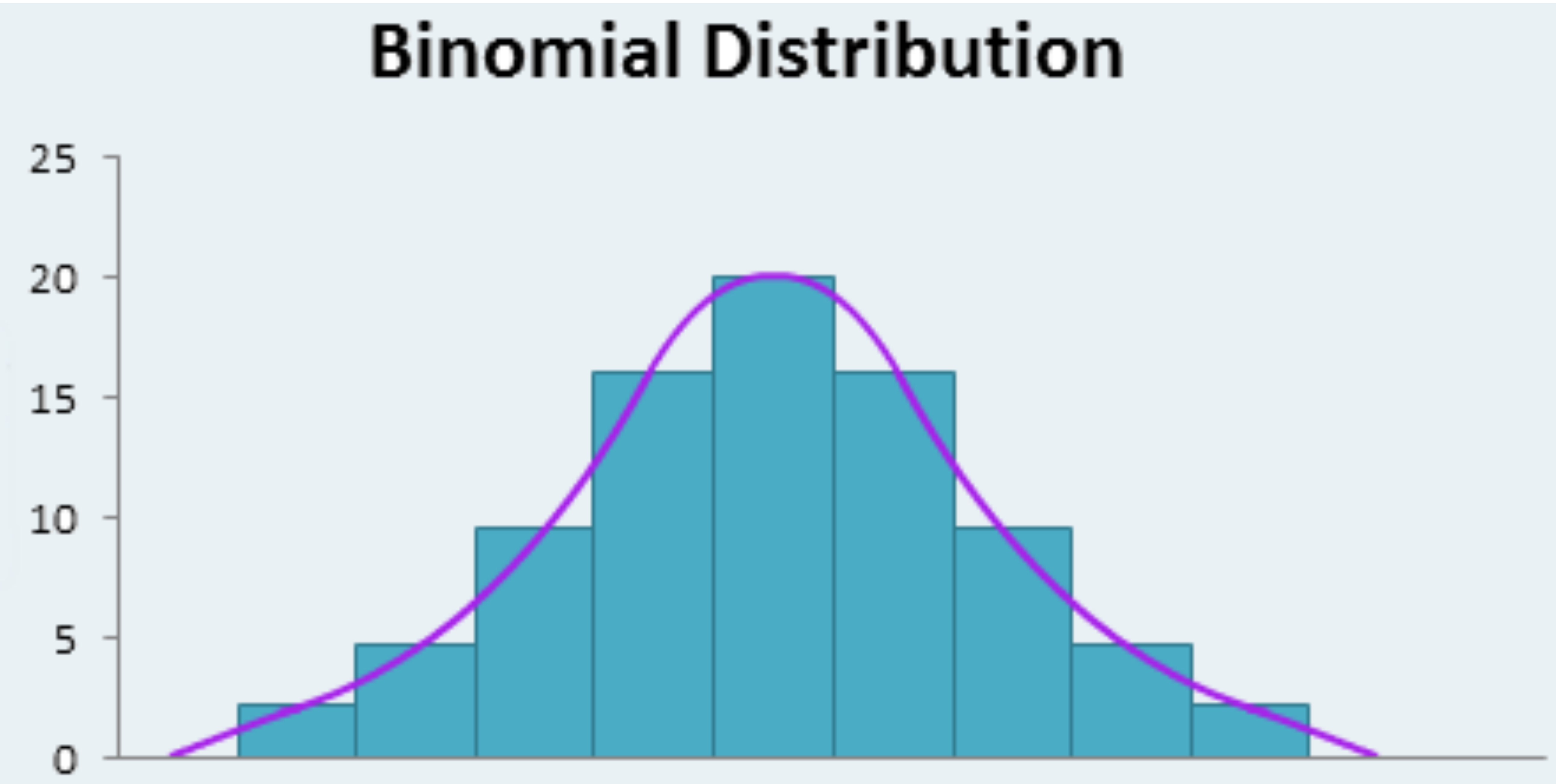
Permutations

Twelve possible orders

Number	First	Second
1	red	yellow
2	red	green
3	red	brown
4	yellow	red
5	yellow	green
6	yellow	brown
7	green	red
8	green	yellow
9	green	brown
10	brown	red
11	brown	yellow
12	brown	green

$${}_{n}P_{r} = \frac{n!}{(n-r)!}$$

$${}_{4}P_{2} = \frac{4!}{(4-2)!} = \frac{4 \text{ x } 3 \text{ x } 2 \text{ x } 1}{2 \text{ x } 1} = 12$$


Binomial Distribution
25
20
15
10
5
0

Binomial Distributions

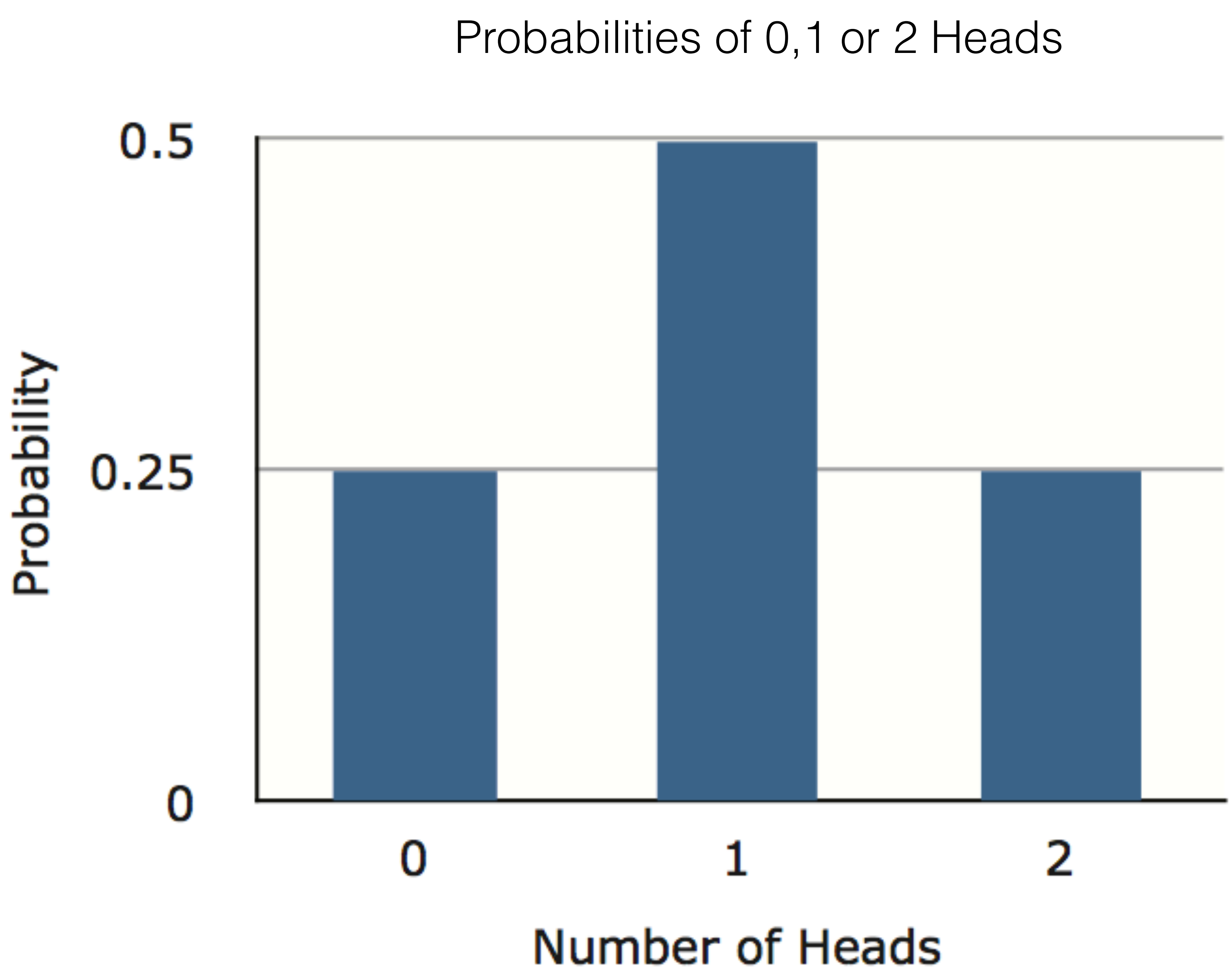
Four possible outcomes

Outcome	First Flip	Second Flip
1	Heads	Heads
2	Heads	Tails
3	Tails	Heads
4	Tails	Tails

Probabilities of Getting 0,1 or 2 Heads

Number of Heads	Probability
0	1/4
1	1/2
2	1/4

Binomial Distributions



Formula for Binomial Distributions

$$P(x) = \frac{N!}{x!\,(N-X)!}\pi^{x}(1-\pi)^{N-x}$$

$$P(0) = \frac{2!}{0!\,(2-0)!}(.5^{0})(1-.5)^{2-0} = \frac{2}{2}(1)(.25) = 0.25$$

$$P(1) = \frac{2!}{1!\,(2-1)!}(.5^{1})(1-.5)^{2-1} = \frac{2}{1}(.5)(.5) = 0.50$$

$$P(2) = \frac{2!}{2!\,(2-2)!}(.5^{2})(1-.5)^{2-2} = \frac{2}{2}(.25)(1) = 0.25$$

Mean and Standard Deviation of Binomial Distributions

$\mu = N\pi$

$\mu = N\pi = (12)(0.5) = 6$

$\sigma^2 = N\pi(1-\pi)$

$\sigma^2 = N\pi(1-\pi) = (12)(0.5)(1.0 - 0.5) = 3.0.$

Mean and Standard Deviation of Binomial Distributions

$$p = \quad p = \frac{e^{-\mu}\mu^{x}}{x!} \; .072$$

e is the base of natural logarithms (2.7183)
μ is the mean number of “successes”
x is the number of “successes” in question

Multinomial Distribution

$$p = \frac{n!}{(n_1!)(n_2!)(n_3!)} p_1^{n_1} p_2^{n_2} p_3^{n_3}$$

p is the probability,

n is the total number of events

n_1 is the number of times Outcome 1 occurs,

n_2 is the number of times Outcome 2 occurs,

n_3 is the number of times Outcome 3 occurs,

p_1 is the probability of Outcome 1,

p_2 is the probability of Outcome 2 and

p_3 is the probability of Outcome 3.

Hypergeometric Distribution

$$p = \frac{{}_{k}C_{x}\;{}_{(N-k)}C_{(n-x)}}{{}_{N}C_{n}}$$

k is the number of “successes” in the population,
x is the number of “successes” in the sample

N is the size of the population

n is the number sampled

p is the probability of obtaining exactly x successes

${}_{k}C_{x}$ is the number of combinations of k things taken x at a time

Hypergeometric Distribution

$$p = \frac{{}_{4}C_{2}\,{}_{(52-4)}C_{(3-2)}}{{}_{52}C_{3}}$$

$$p = \frac{\frac{4!}{2!2!}\,\frac{48!}{47!1!}}{\frac{52!}{49!3!}} = 0.013$$

Base Rates

Diagnosing disease X

True Condition			
No Disease 980,000		Disease 20,000	
Test Result		**Test Result**	
Positive 88,200	Negative 891,800	Positive 19,800	Negative 200

Bayes' Theorem

$$P(D|T) = \frac{P(T|D)P(D)}{P(T|D)P(D) + P(T|D')P(D')}$$

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P(T|D)   =  0.99
P(T|D')  =  0.09
P(D)     =  0.02
P(D')    =  0.98
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The end theorem